

MATHEMATICS

Chapter 7: Congruence of Triangle



Congruence of Triangles

1. Two figures having exactly same shape and size are said to be congruent.
2. Two plane figures are congruent, if each one when superimposed on the other, covers the other exactly.
3. Congruence of some geometrical figures:
 - i. Two line segments are congruent if they have the same length.
 - ii. Two angles are congruent if they have the same measure.
 - iii. Two squares are congruent if they have the same side length.
 - iv. Two rectangles are congruent if they have the same length and same breadth.
 - v. Two circles are congruent if they have the same radius.
4. Two triangles are said to be congruent, if pairs of corresponding sides and the corresponding angles are equal.

If $\triangle ABC$ and $\triangle PQR$ are congruent, then we write it as $\triangle ABC \cong \triangle PQR$.

5. The congruency of $\triangle ABC$ and $\triangle PQR$ under the matching $\triangle ABC \leftrightarrow \triangle PQR$ is written as $\triangle ABC \cong \triangle PQR$.

This means the vertices A, B, C are matched with the vertices P, Q, R respectively.

6. SSS congruence condition: If three sides of a triangle are equal to the three sides of another triangle, then the two triangles are congruent.
7. SAS congruence condition: If two sides and the included angle of one triangle are respectively equal to the two sides and the included angle of another triangle, then the two triangles are congruent.
8. ASA congruence condition: If two angles and the included side of one triangle are respectively equal to the two angles and the included side of another triangle, then the two triangles are congruent.
9. RHS congruence condition: If the hypotenuse and a side of one right angled triangle are equal to the hypotenuse and a side of another right-angled triangle, then two triangles are congruent.
10. Two congruent figures are equal in area but two figures having the same area need not be congruent.

Congruent Figures

Congruent figures are exactly equal in size and shape.

In Geometry, congruence is a term used to define two objects that have same dimensions and shape. Moreover, if one shape can be exactly like the other on turning, flipping and/or sliding it, then the two shapes are said to be congruent to each other. So two congruent

figures drawn on a piece of paper can be cut out and placed over one another to match up perfectly. When we say a figure A is congruent to a figure B, symbolically it can be written as figure A \cong figure B. In this article, we will discuss congruence of plane figures and line segments and congruent angles.

Congruence of Plane Figures

You are given two plane figures. Using the method of superposition, here are the steps to find whether they are congruent to each other or not:



Congruent Figures

- Trace a copy of figure 1 using a tracing paper.
- Cut it out of the paper and place it over figure 2.
- You may slide or move or even turn over the paper. If they cover each other completely, they are congruent. We write figure 1 \cong figure 2.

Congruence Among Line Segments

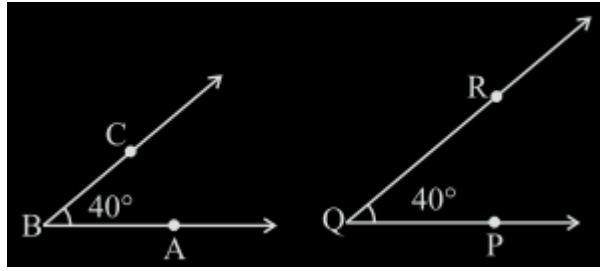
Two line segments are said to be congruent to each other if they have equal lengths. They may or may not align at the same angle with an axis or position in plane. To find whether one line segment is congruent to the other or not, we use the same method of superposition as discussed above. Thus if the two ends of the two line segments lie on one another, they are congruent.



Congruent Line Segments

Congruent Angles

If two angles have the same measure in degrees, they are congruent angles. The angles may or may not lie in the same position or orientation on plane. The method of superposition (as described above) can be used to check if given angles are congruent angles or not.



Congruent Angles

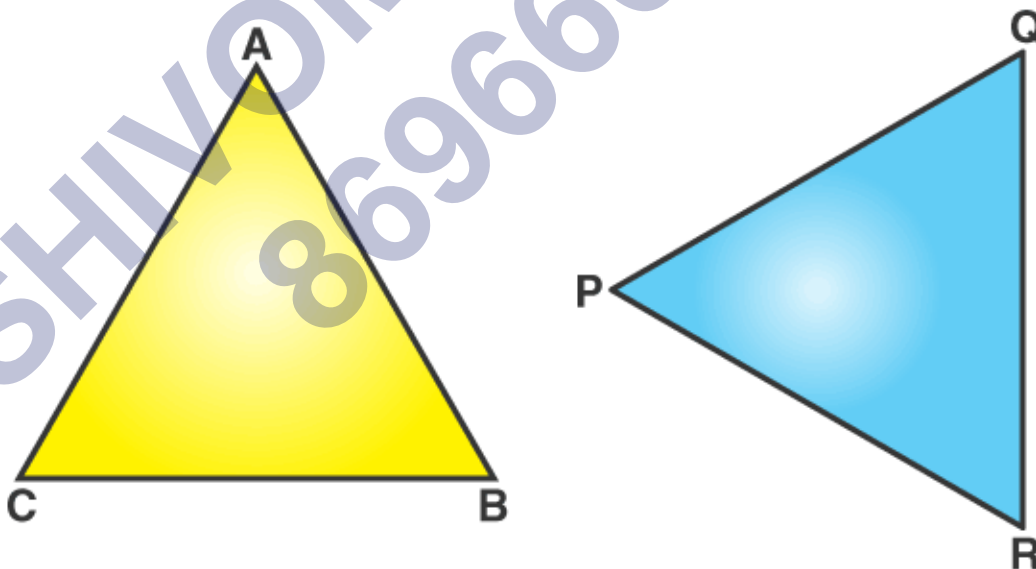
Thus, $\angle PQR \cong \angle ABC$ because the two angles measure 40 degrees. It is to be noted here that the length of BC is not equal to the length of QR. But congruence of angles does not depend on the length of the arms of the angles; it depends on the measures of the angles only. Thus, $\angle PQR$ and $\angle ABC$ are congruent angles.

Congruent Triangles

If all the sides and angles of a triangle are equal to the corresponding sides and angles of another triangle, then both the triangles are said to be congruent.

The meaning of congruent in Maths is addressed to those figures and shapes that can be repositioned or flipped to coincide with the other shapes. These shapes can be reflected to coincide with similar shapes.

Two shapes are congruent if they have the same shape and size. We can also say if two shapes are congruent, then the mirror image of one shape is same as the other.



In the above figure, $\triangle ABC$ and $\triangle PQR$ are congruent triangles. This means,

Vertices: A and P, B and Q, and C and R are the same.

Sides: $AB = PQ$, $QR = BC$ and $AC = PR$;

Angles: $\angle A = \angle P$, $\angle B = \angle Q$, and $\angle C = \angle R$.

Congruent triangles are triangles having corresponding sides and angles to be equal.

Congruence is denoted by the symbol " \cong ". They have the same area and the same perimeter.

CPCT Full Form

CPCT is the term we come across when we learn about the congruent triangle. CPCT means "Corresponding Parts of Congruent Triangles". As we know that the corresponding parts of congruent triangles are equal. While dealing with the concepts related to triangles and solving questions, we often make use of the abbreviation cpct in short words instead of full form.

CPCT Rules in Maths

The full form of CPCT is Corresponding parts of Congruent triangles. Congruence can be predicted without actually measuring the sides and angles of a triangle. Different rules of congruency are as follows.

SSS (Side-Side-Side)

SAS (Side-Angle-Side)

ASA (Angle-Side-Angle)

AAS (Angle-Angle-Side)

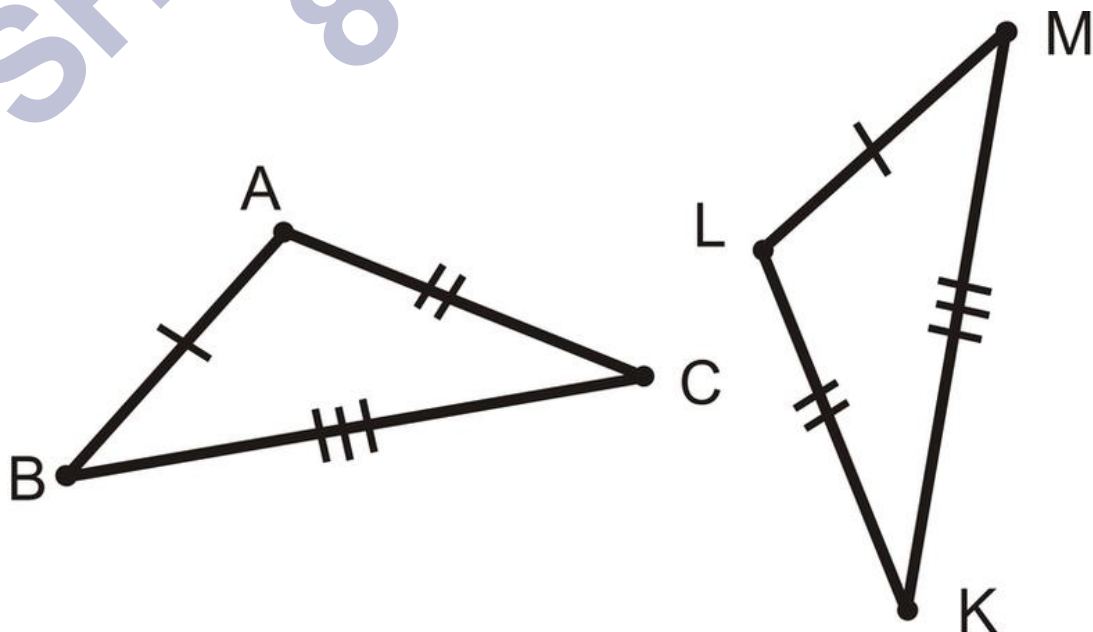
RHS (Right angle-Hypotenuse-Side)

Criteria for Congruency

SSS Criteria for Congruency

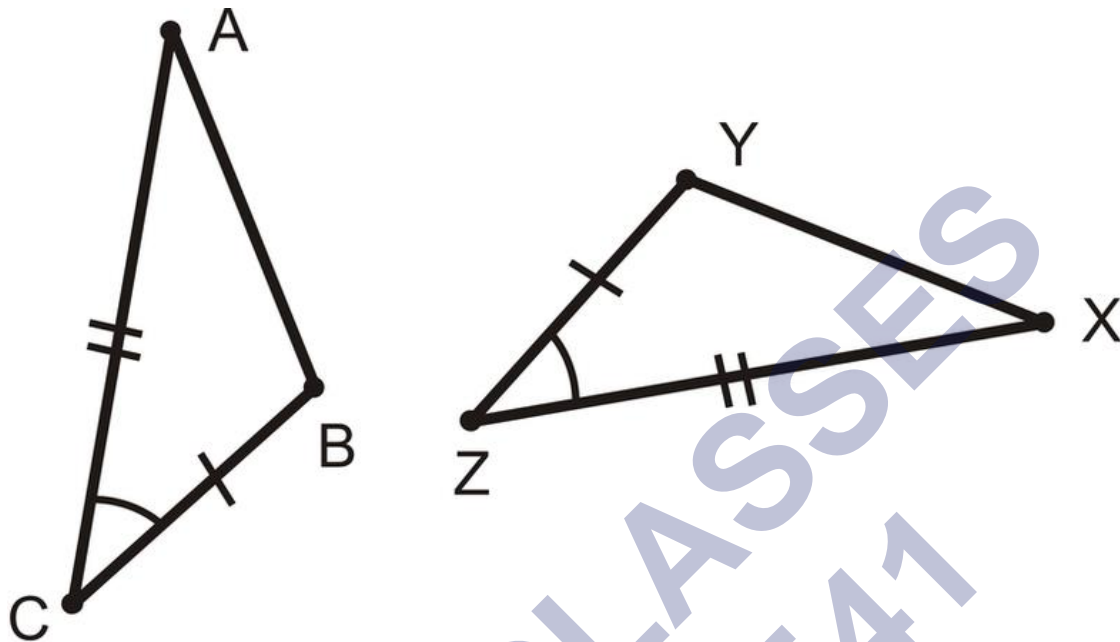
If under a given correspondence, the three sides of one triangle are equal to the three corresponding sides of another triangle, then the triangles are congruent.

Theorem: In two triangles, if the three sides of one triangle are equal to the corresponding three sides (SSS) of the other triangle, then the two triangles are congruent.



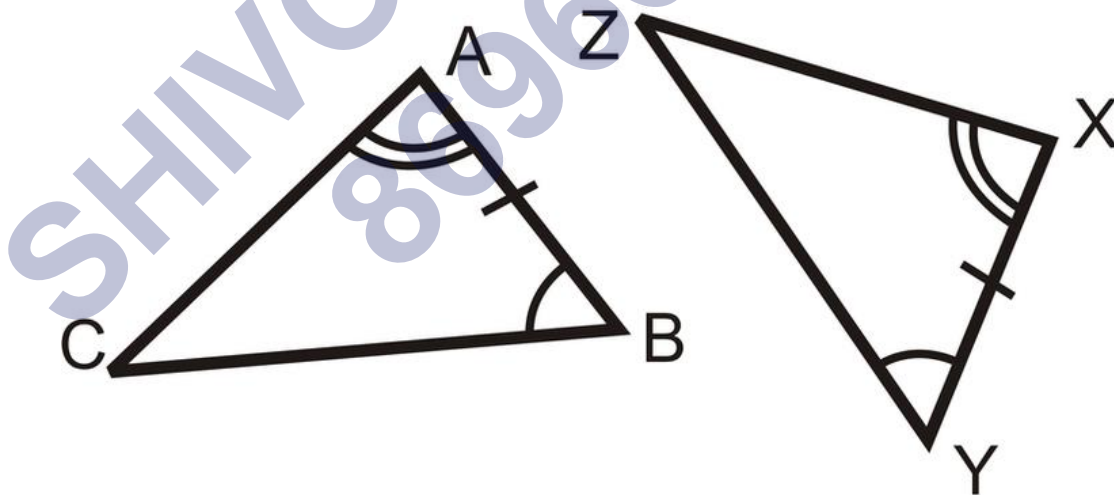
SAS Criteria for Congruency

If under a correspondence, two sides and the angle included between them of a triangle are equal to two corresponding sides and the angle included between them of another triangle, then the triangles are congruent.



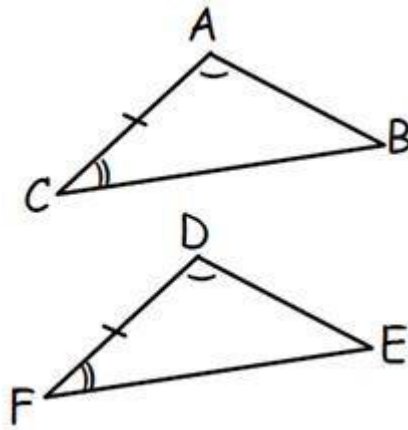
ASA Criteria for Congruency

If under a correspondence, two angles and the included side of a triangle are equal to two corresponding angles and the included side of another triangle, then the triangles are congruent.



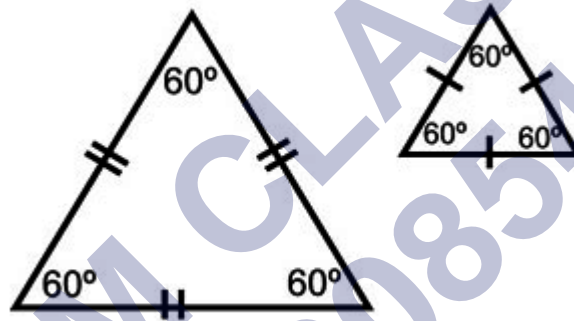
AAS Criteria for Congruency

AAS Rule: Triangles are congruent if two pairs of corresponding angles and a pair of opposite sides are equal in both triangles.

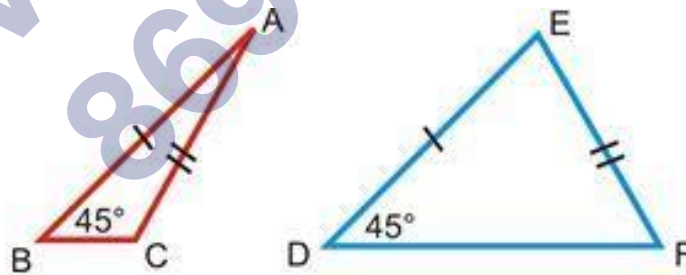


Why SSA and AAA congruency rules are not valid?

Two triangles with equal corresponding angles need not be congruent. In such a correspondence, one of them can be an enlarged copy of the other. Therefore, AAA congruency is not valid.



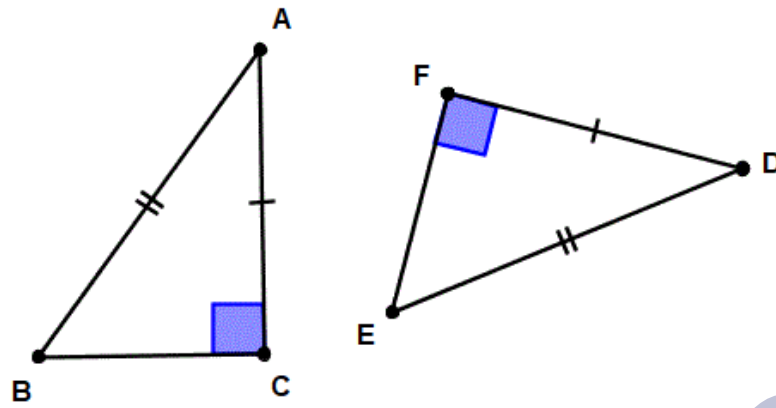
If two triangles have two congruent sides and a congruent non included angle, then triangles are not necessarily congruent. Therefore, SSA congruency is not valid.



RHS Criteria for Congruency

If under a correspondence, the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right-angled triangle, then the triangles are congruent.

Theorem: In two right-angled triangles, if the length of the hypotenuse and one side of one triangle, is equal to the length of the hypotenuse and corresponding side of the other triangle, then the two triangles are congruent.



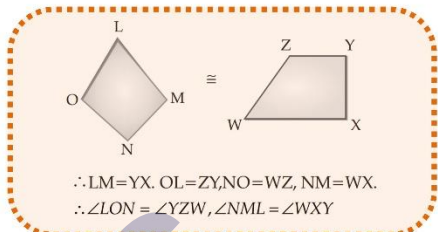
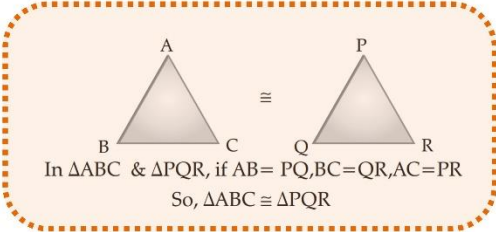
Criteria for Congruency

Criteria for Congruency of two triangles are:

- (i) SSS Rule
- (ii) SAS Rule
- (iii) ASA Rule
- (iv) RHS Rule

SHIVOM CLASSES
8696608541

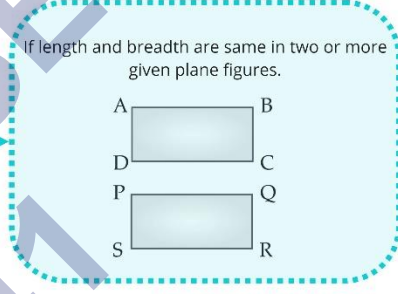
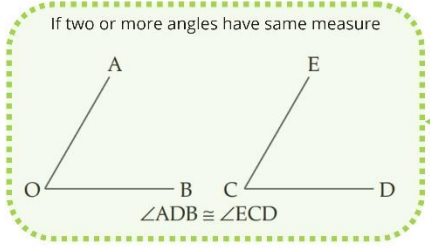
Class : 7th mathematics
Chapter-7 : Congruence of Triangles



Same shape

Same Size

Congruent figures

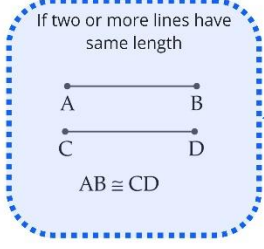


Congruence of plane figures

Congruence of Angles

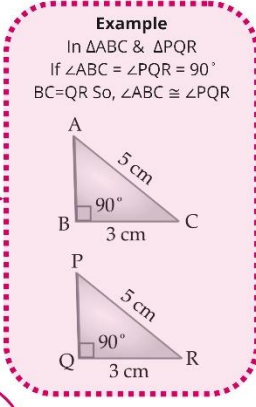
Congruence of Triangles

Congruence of line Segments



R.H.S. criterion (Right Angle hypotenuse Angle)

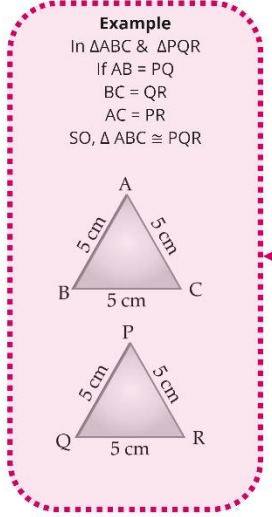
Meaning
If a right-angle, hypotenuse, and a side of two triangles are equal.



Types of triangle by considering angles

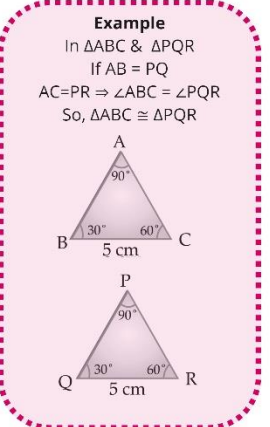
S.S.S. criterion (side side side)

Meaning
If three sides of two triangles are equal.



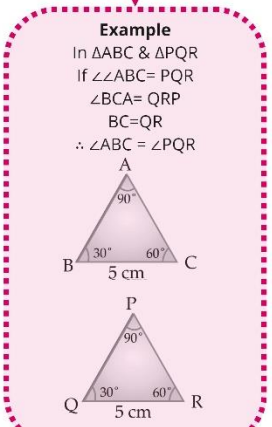
S.A.S. criterion (side Angle side)

Meaning
If one angle and two sides of two triangles are equal.



S.A.S. criterion (Angle side Angle)

Meaning
If two angles and one side of two triangles are equal.



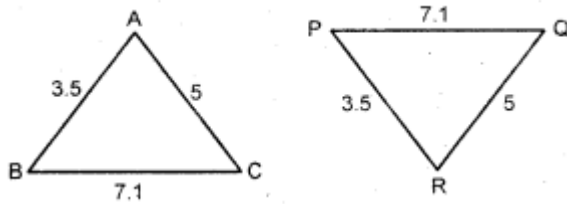
Important Questions

Multiple Choice Questions:

Question 1. An angle is of 50° then its congruent angle is of:

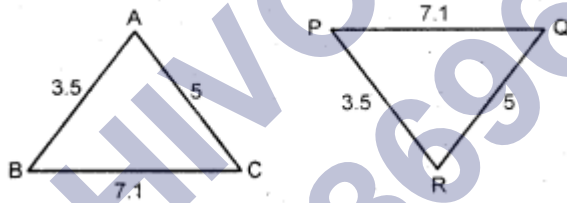
- (a) 40°
- (b) 60°
- (c) 50°
- (d) None of these

Question 2. Given two triangles are congruent then we can write:



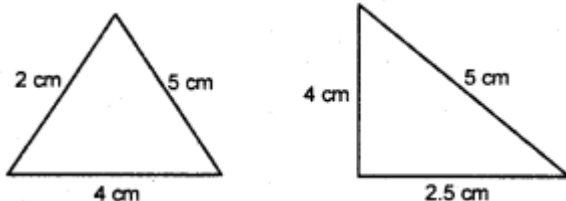
- (a) $\triangle ABC \equiv \triangle PQR$
- (b) $\triangle ABC \equiv \triangle RPQ$
- (c) $\triangle ABC \equiv \triangle QRP$
- (d) none of these

Question 3. In the given figure, lengths of the sides of the triangles are given. Which pair of triangle are congruent?



- (a) $\triangle ABC \equiv \triangle PQR$
- (b) $\triangle BCA \equiv \triangle PQR$
- (c) $\triangle ABC \equiv \triangle QRP$
- (d) none of these

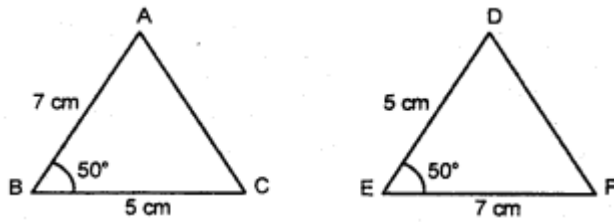
Question 4. Are the following triangles congruent?



- (a) yes
- (b) no

(c) none of these

Question 5. Are the following triangles congruent?

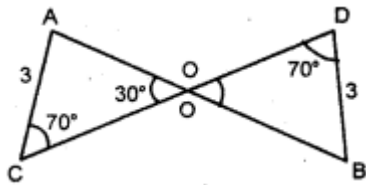


(a) yes

(b) no

(c) none of these

Question 6. In the given figure, say congruency of two triangles.



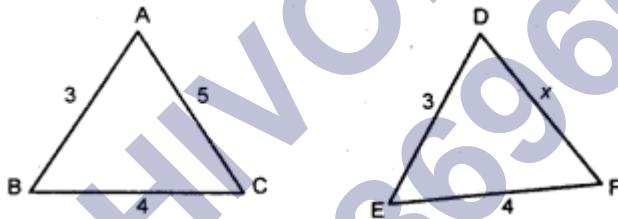
(a) $\triangle AOC \cong \triangle BOD$

(b) $\triangle AOC \neq \triangle BOD$

(c) $\triangle AOC \cong \triangle OBD$

(d) none of these

Question 7. Given triangles are congruent, then what is the measurement of x ?



(a) 3

(b) 4

(c) 5

(d) none of these

Question 8. Given below are measurements of some parts of two triangles. Write the result in symbolic form.

In $\triangle ABC$, $\angle B = 90^\circ$, $AC = 8$ cm $AB = 3$ cm and

$\triangle PQR$, $\angle P = 90^\circ$, $PR = 3$ cm $QR = 8$ cm

(a) $\triangle ABC \cong \triangle RPQ$

(b) $\triangle ABC \cong \triangle PQR$

(c) $\triangle ABC \cong \triangle RPQ$

(d) none of these

Question 9. Given below are measurements of some parts of two triangles. Write the result in symbolic form if they are congruent.

In $\triangle ABC$,

$\angle A = 90^\circ$, $AC = 5$ cm, $BC = 9$ cm

In $\triangle PQR$,

$\angle P = 90^\circ$, $PR = 3$ cm $QR = 8$ cm

(a) are congruent

(b) are not congruent

Question 10. $\triangle ABC$ and $\triangle PQR$ are congruent under the correspondence: $ABC \leftrightarrow RPQ$, then the part of $\triangle ABC$ that correspond to PQ is

(a) AC

(b) AB

(c) BC

(d) None of These

Question 11. $\triangle ABC$ is right triangle in which $\angle A = 90^\circ$ and $AB = AC$. The values of $\angle B$ and $\angle C$ will be

(a) $\angle B = \angle C = 30^\circ$

(b) $\angle B = \angle C = 50^\circ$

(c) $\angle B = \angle C = 45^\circ$

(d) $\angle B = \angle C = 60^\circ$

Question 12. Two students drew a line segment each. What is the condition for them to be congruent?

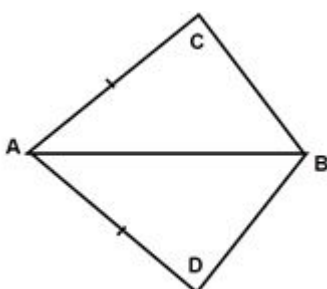
(a) They should be drawn with a scale.

(b) They should be drawn on the same sheet of paper.

(c) They should have different lengths.

(d) They should have the same length.

Question 13. In the quadrilateral $ABCD$, $AC = AD$ and AB bisect $\angle A$ and $\triangle ABC \cong \triangle ABD$. The relation between BC and BD is



- (a) $BC < BD$
 (b) $BC > BD$
 (c) $BC = BD$
 (d) None of these

Question 14. A triangle in which all three sides are of equal lengths is called _____.

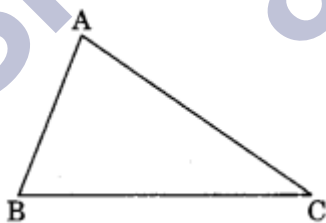
- (a) Isosceles
 (b) Equilateral
 (c) Scalene
 (d) None of these

Question 15. In $\triangle ABC$ and $\triangle PQR$, $AB = 4$ cm, $BC = 5$ cm, $AC = 6$ cm and $PQ = 4$ cm, $QR = 5$ cm, $PR = 6$ cm. then which of the following is true?

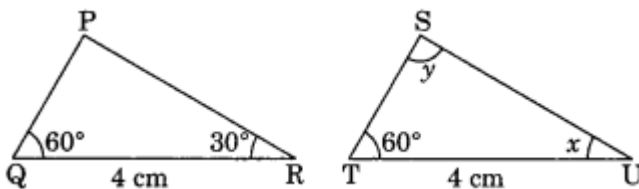
- (a) $\triangle ABC \cong \triangle QRP$
 (b) $\triangle ABC \cong \triangle PQR$
 (c) $\triangle ABC \cong \triangle RQP$
 (d) None of these

Very Short Questions:

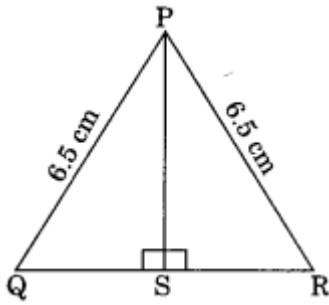
- In the given figure, name
 - the side opposite to vertex A
 - the vertex opposite A to side AB
 - the angle opposite to side AC
 - the angle made by the sides CB and CA



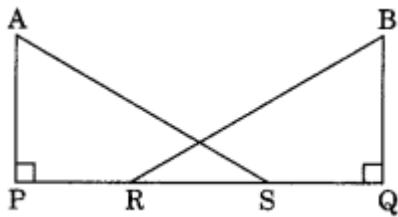
- Examine whether the given triangles are congruent or not.
- In the given congruent triangles under ASA, find the value of x and y , $\triangle PQR = \triangle STU$.



4. In the following figure, show that $\Delta PSQ = \Delta PSR$.



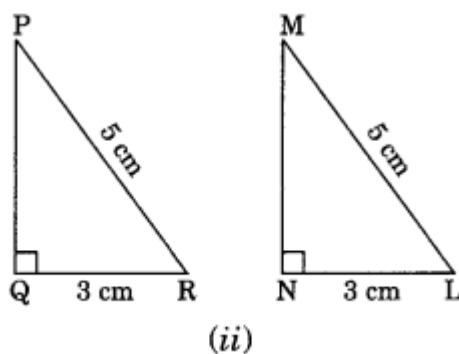
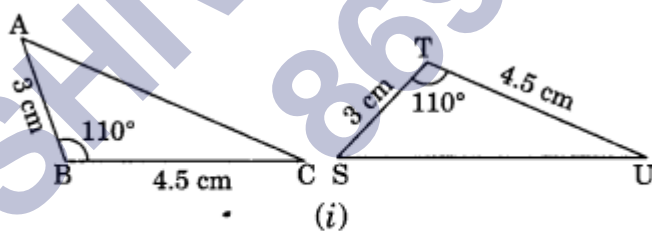
5. Can two equilateral triangles always be congruent? Give reasons.
 6. In the given figure, $AP = BQ$, $PR = QS$. Show that $\Delta APS = \Delta BQR$



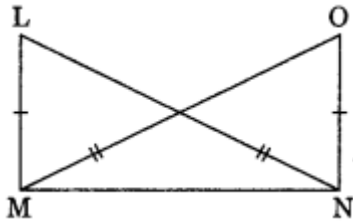
7. Without drawing the figures of the triangles, write all six pairs of equal measures in each of the following pairs of congruent triangles.
 (i) $\Delta ABC = \Delta DEF$
 (ii) $\Delta XYZ = \Delta MLN$
 8. Lengths of two sides of an isosceles triangle are 5 cm and 8 cm, find the perimeter of the triangle.

Short Questions:

1. Write the rule of congruence in the following pairs of congruent triangles.



2. In the given figure, state the rule of congruence followed by congruent triangles LMN and ONM.

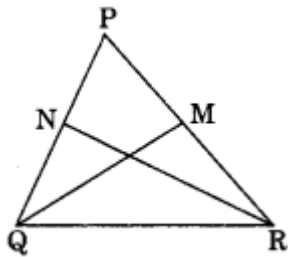


3. In the given figure, PQR is a triangle in which $PQ = PR$. QM and RN are the medians of the triangle. Prove that

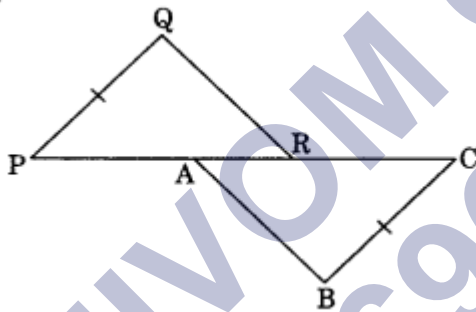
(i) $\Delta NQR = \Delta MRQ$

(ii) $QM = RN$

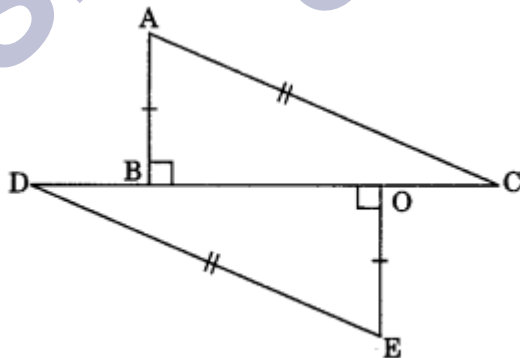
(iii) $\Delta PMQ = \Delta PNR$



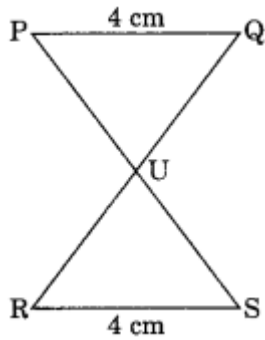
4. In the given figure, $PQ = CB$, $PA = CR$, $\angle P = \angle C$. Is $\Delta QPR = \Delta BCA$? If yes, state the criterion of congruence.



5. In the given figure, state whether $\Delta ABC = \Delta EOD$ or not. If yes, state the criterion of congruence.

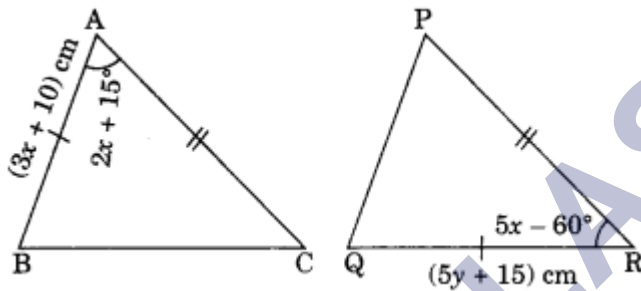


6. In the given figure, $PQ \parallel RS$ and $PQ = RS$. Prove that $\Delta PUQ = \Delta SUR$.

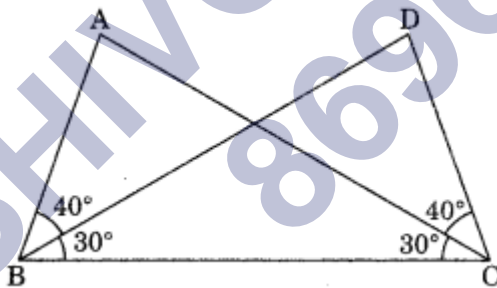


Long Questions:

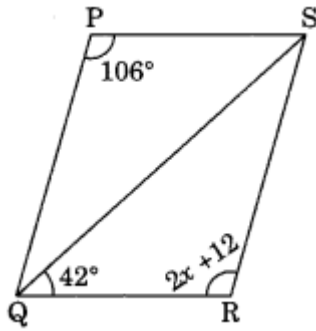
1. In the given figure $\triangle BAC = \triangle QRP$ by SAS criterion of congruence. Find the value of x and y .



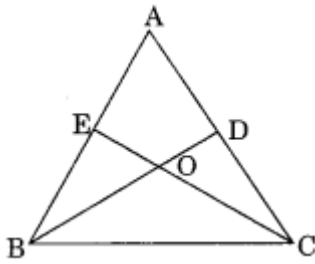
2. Observe the figure and state the three pairs of equal parts in triangles ABC and DCB.
- Is $\triangle ABC = \triangle DCB$? Why?
 - Is $AB = DC$? Why?
 - Is $AC = DB$? Why?



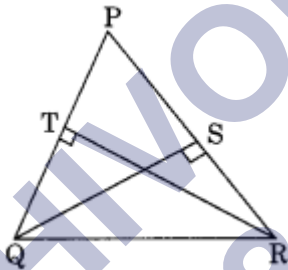
3. In the given figure, $\triangle QPS = \triangle SRQ$. Find each value.
- x
 - $\angle PQS$
 - $\angle PSR$



4. In $\triangle ABC$, medians BD and CE are equal and intersect each other at O . Prove that $\triangle ABC$ is an isosceles triangle.



5. Prove that the lengths of altitudes drawn to equal sides of an isosceles triangle are also equal.
- $\angle TRQ = \angle SQR$?
 - If $\angle TRQ = 30^\circ$, find the base angles of the $\triangle PQR$.
 - Is $\triangle PQR$ an equilateral triangle?



ANSWER KEY -

Multiple Choice Questions:

- (c) 50°
- (b) $\triangle ABC \cong \triangle RPQ$
- (a) $\triangle ABC \cong \triangle PQR$
- (b) no
- (b) no
- (a) $\triangle AOC \cup \triangle BOD$
- (c) 5
- (a) $\triangle ABC \cong \triangle RPQ$

9. (b) are not congruent
10. (c) BC
11. (c) $\angle B = \angle C = 45^\circ$
12. (d) They should have the same length.
13. (c) $BC = BD$
14. (b) Equilateral
15. (b) $\triangle ABC \cong \triangle PQR$

Very Short Answer:

1. (a) The side opposite to vertex A is BC.
 (b) The vertex opposite to side AB is C.
 (c) The angle opposite to side AB is $\angle ACB$.
 (d) The angle made by the sides CB and CA is $\angle ACB$.

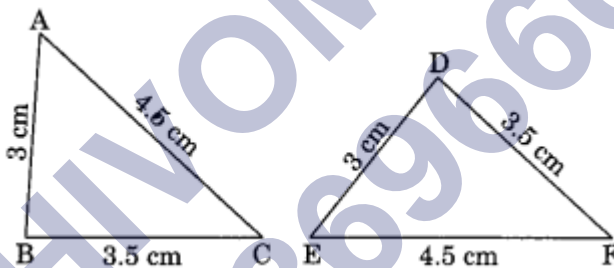
2. Here,

$$AB = DE = 3 \text{ cm}$$

$$BC = DF = 3.5 \text{ cm}$$

$$AC = EF = 4.5 \text{ cm}$$

$$\triangle ABC = \triangle EDF \text{ (By SSS rule)}$$



So, $\triangle ABC$ and $\triangle EDF$ are congruent.

3. Given: $\triangle PQR = \triangle STU$ (By ASA rule)

$$\angle Q = \angle T = 60^\circ \text{ (given)}$$

$$\overline{QR} = \overline{TU} = 4 \text{ cm (given)}$$

$$\angle x = 30^\circ \text{ (for ASA rule)}$$

Now in $\triangle STU$,

$$\angle S + \angle T + \angle U = 180^\circ \text{ (Angle sum property)}$$

$$\angle y + 60^\circ + \angle x = 180^\circ$$

$$\angle y + 60^\circ + 30^\circ = 180^\circ$$

$$\angle y + 90^\circ = 180^\circ$$

$$\angle y = 180^\circ - 90^\circ = 90^\circ$$

Hence, $x = 30^\circ$ and $y = 90^\circ$.

4. In $\triangle PSQ$ and $\triangle PSR$

$$\overline{PQ} = \overline{PR} = 6.5 \text{ cm (Given)}$$

$$\overline{PS} = \overline{PS} \text{ (Common)}$$

$$\angle PSQ = \angle PSR = 90^\circ \text{ (Given)}$$

$$\triangle PSQ = \triangle PSR \text{ (By RHS rule)}$$

5. No, any two equilateral triangles are not always congruent.

Reason: Each angle of an equilateral triangle is 60° but their corresponding sides cannot always be the same.

6. In $\triangle APS$ and $\triangle BQR$

$$AP = BQ \text{ (Given)}$$

$$PR = QS \text{ (Given)}$$

$$PR + RS = QS + RS \text{ (Adding RS to both sides)}$$

$$PS = QR$$

$$\angle APS = \angle BQR = 90^\circ \text{ (Given)}$$

$$\triangle APS = \triangle BQR \text{ (by SAS rule)}$$

7. (i) Given: $\triangle ABC = \triangle DEF$

$$\text{Here } AB = DE$$

$$BC = EF$$

$$AC = DF$$

$$\angle A = \angle D, \angle B = \angle E \text{ and } \angle C = \angle F$$

- (ii) Given $\triangle XYZ = \triangle MLN$

$$\text{Here } XY = ML$$

$$YZ = LN$$

$$XZ = MN$$

$$\angle X = \angle M, \angle Y = \angle L \text{ and } \angle Z = \angle N$$

8. Since the lengths of any two sides of an isosceles triangle are equal, then

Case I: The three sides of the triangle are 5 cm, 5 cm and 8 cm.

$$\text{Perimeter of the triangle} = 5 \text{ cm} + 5 \text{ cm} + 8 \text{ cm} = 18 \text{ cm}$$

Case II: The three sides of the triangle are 5 cm, 8 cm and 8 cm.

$$\text{Perimeter of the triangle} = 5 \text{ cm} + 8 \text{ cm} + 8 \text{ cm} = 21 \text{ cm}$$

Hence, the required perimeter is 18 cm or 21 cm.

Short Answer :

1. (i) Here, $AB = ST = 3 \text{ cm}$
 $BC = TU = 4.5 \text{ cm}$
 $\angle ABC = \angle STU = 110^\circ$
 $\triangle ABC = \triangle STU$ (By SAS rule)
- (ii) Here $\angle PQR = \angle MNL = 90^\circ$
 hypot. $PR = \text{hypot. } ML$
 $QR = NL = 3 \text{ cm}$
 $\triangle PQR = \triangle MNL$ (By RHS rule)
2. In $\triangle LMN$ and $\triangle ONM$
 $LM = ON$
 $LN = OM$
 $MN = NM$
 $\triangle LMN = \triangle ONM$
3. $\triangle PQR$ is an isosceles triangle. [$\because PQ = PR$]
 $\Rightarrow \frac{1}{2}PQ = \frac{1}{2}PR$
 $\Rightarrow NQ = MR$ and $PN = PM$
- (i) In $\triangle NQR$ and $\triangle MRQ$
 $NQ = MR$ (Half of equal sides)
 $\angle NQR = \angle MRQ$ (Angles opposite to equal sides)
 $QR = RQ$ (Common)
 $\triangle NQR = \triangle MRQ$ (By SAS rule)
- (ii) $QM = RN$ (Congruent parts of congruent triangles)
- (iii) In $\triangle PMQ$ and $\triangle PNR$
 $PN = PM$ (Half of equal sides)
 $PR = PQ$ (Given)
 $\angle P = \angle P$ (Common)
 $\triangle PMQ = \triangle PNR$ (By SAS rule)
4. $PQ = CB$, $PA = CR$
 and $\angle P = \angle C$
 In $\triangle QPR$ and $\triangle BCA$,
 $PQ = CB$ (Given)
 $\angle QPR = \angle BCA$ (Given)

$$PA = CR \text{ (Given)}$$

$$PA + AR = CR + AR \text{ (Adding AR to both sides)}$$

$$\text{or } PR = CA$$

$$\Delta QPR = \Delta BCA \text{ (By SAS rule)}$$

5. In ΔABC and ΔEOD

$$AB = OE$$

$$\angle ABC = \angle EOD = 90^\circ$$

$$AC = ED$$

$$\Delta ABC = \Delta EOD$$

$$\text{Hence, } \Delta ABC = \Delta EOD$$

RHS is the criterion of congruence.

6. In ΔPUQ and ΔSUR

$$PQ = SR = 4 \text{ cm}$$

$$\angle UPQ = \angle USR \text{ (Alternate interior angles)}$$

$$\angle PQU = \angle SRU \text{ (Alternate interior angles)}$$

$$\Delta PUQ = \Delta SUR \text{ (By ASA rule)}$$

Long Answer:

1. Given: $\Delta BAC = \Delta QRP$ (By SAS rule)

$$\text{So, } BA = QR$$

$$\Rightarrow 3x + 10 = 5y + 15 \text{(i)}$$

$$\angle BAC = \angle QRP$$

$$\Rightarrow 2x + 15^\circ = 5x - 60^\circ \text{(ii)}$$

From eq. (ii), we have

$$2x + 15 = 5x - 60$$

$$\Rightarrow 2x - 5x = -15 - 60$$

$$\Rightarrow -3x = -75$$

$$\Rightarrow x = 25$$

From eq. (i), we have

$$3x + 10 = 5y + 15$$

$$\Rightarrow 3 \times 25 + 10 = 5y + 15$$

$$\Rightarrow 75 + 10 = 5y + 15$$

$$\Rightarrow 85 = 5y + 15$$

$$\Rightarrow 85 - 15 = 5y$$

$$\Rightarrow 70 = 5y$$

$$\Rightarrow y = 14$$

Hence, the required values of x and y are 25 and 14 respectively.

2. (i) In $\triangle ABC$ and $\triangle DCB$

$$\angle ABC = \angle DCB = 70^\circ (40^\circ + 30^\circ = 70^\circ) \text{ (Given)}$$

$$\angle ACB = \angle DCB = 30^\circ \text{ (Given)}$$

$$BC = CB \text{ (Common)}$$

$$\triangle ABC = \triangle DCB \text{ (By ASA rule)}$$

(ii) Yes,

$$AB = DC \text{ (Congruent parts of congruent triangles)}$$

(iii) Yes,

$$AC = DB \text{ (Congruent parts of congruent triangles)}$$

3. (a) $\triangle QPS = \triangle SRQ$

$$\angle QPS = \angle SRQ \text{ (Congruent part of congruent triangles)}$$

$$106 = 2x + 12$$

$$\Rightarrow 106 - 12 = 2x$$

$$\Rightarrow 94 = 2x$$

$$\Rightarrow x = 47$$

$$\angle QRS = 2 \times 47 + 12 = 94 + 12 = 106^\circ$$

So, PQRS is a parallelogram.

$$\angle QSR = 180^\circ - (42^\circ + 106^\circ) = 180^\circ - 148^\circ = 32^\circ$$

(b) $\angle PQS = 32^\circ$ (alternate interior angles)

$$(c) \angle PSQ = 180^\circ - (\angle QPS + \angle PQS) = 180^\circ - (106^\circ + 32^\circ) = 180^\circ - 138^\circ = 42^\circ$$

$$\angle PSR = 32^\circ + 42^\circ = 74^\circ$$

4. We know that the medians of a triangle intersect each other in the ratio 2 : 1.

$$BD = CE \text{ (Given)}$$

$$\frac{2}{3}BD = \frac{2}{3}CE$$

$$\Rightarrow OB = OC$$

$$\frac{1}{3}BD = \frac{1}{3}CE$$

$$\Rightarrow OE = OD$$

In $\triangle BOE$ and $\triangle COD$,

$$OB = OC$$

$$OE = OD$$

$$\angle BOE = \angle COD \text{ (Vertically opposite angles)}$$

$$\triangle BOE = \triangle COD \text{ (By SAS rule)}$$

$$BE = CD \text{ (Congruent parts of congruent triangles)}$$

$$2BE = 2CD$$

$$\Rightarrow AB = AC$$

Hence $\triangle ABC$ is an isosceles triangle.

5. In $\triangle QTR$ and $\triangle RSQ$

$$\angle QTR = \angle RSQ = 90^\circ \text{ (Given)}$$

$$\angle TQR = \angle SRQ \text{ (Base angle of an isosceles triangle)}$$

$$\angle QRT = \angle RQS \text{ (Remaining third angles)}$$

$$QR = QR \text{ (Common)}$$

$$\triangle QTR = \triangle RSQ \text{ (By ASA rule)}$$

$$QS = RT \text{ (Congruent parts of congruent triangles)}$$

Hence proved.

$$(i) \angle TRQ = \angle SQR \text{ (Congruent parts of congruent triangles)}$$

(ii) In $\triangle QTR$,

$$\angle TRQ = 30^\circ \text{ (Given)}$$

$$\angle QTR + \angle TQR + \angle QRT = 180^\circ \text{ (Angle sum property)}$$

$$\Rightarrow 90^\circ + \angle TQR + 30^\circ = 180^\circ$$

$$\Rightarrow 120^\circ + \angle TQR = 180^\circ$$

$$\Rightarrow \angle TQR = 180^\circ - 120^\circ = 60^\circ$$

$$\Rightarrow \angle TQR = \angle SRQ = 60^\circ$$

Each base angle = 60°

(iii) In $\triangle PQR$,

$$\angle P + \angle Q + \angle R = 180^\circ \text{ (Angle sum property)}$$

$$\Rightarrow \angle P + 60^\circ + 60^\circ = 180^\circ \text{ (From ii)}$$

$$\Rightarrow \angle P + 120^\circ = 180^\circ$$

$$\Rightarrow \angle P = 180^\circ - 120^\circ = 60^\circ$$

Hence, $\triangle PQR$ is an equilateral triangle.