

MATHEMATICS

Chapter 13: Exponents and Powers



Exponents and Powers

Powers and Exponents

- Repeated multiplication of the same number can be expressed in the form of exponents.
- Example: $625 = 5 \times 5 \times 5 \times 5$ or 5^4 .
Here '5' is the base raised to the power of 4, where 4 is the exponent and 5^4 is the exponential form of 625.



Exponent

An exponent of a number, represents the number of times the number is multiplied to itself. If 8 is multiplied by itself for n times, then, it is represented as:

$$8 \times 8 \times 8 \times 8 \times \dots n \text{ times} = 8^n$$

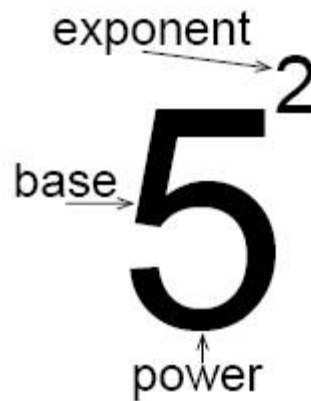
The above expression, 8^n , is said as 8 raised to the power n. Therefore, exponents are also called power or sometimes indices.

Examples:

$$2 \times 2 \times 2 \times 2 = 2^4$$

$$5 \times 5 \times 5 = 5^3$$

$$10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$$



General Form of Exponents

The exponent is a simple but powerful tool. It tells us how many times a number should be multiplied by itself to get the desired result. Thus any number 'a' raised to power 'n' can be expressed as:

$$a^n = \underbrace{a \times a \times a \times \dots \times a}_{n\text{-times}}$$

- Here a is any number and n is a natural number.
- a^n is also called the nth power of a.
- 'a' is the base and 'n' is the exponent or index or power.
- 'a' is multiplied 'n' times, and thereby exponentiation is the shorthand method of repeated multiplication.

Laws of Exponents

Powers with like bases

- $a^n \times a^m = a^{n+m}$.
Example: $3^2 \times 3^4 = 3^6 = 729$
- $\frac{a^n}{a^m} = a^{n-m}$.
Example: $2^5 \div 2^3 = \frac{32}{8} = 4 = 2^2$
- $a^m \times a^{-m} = a^m \times \frac{1}{a^m} = 1$

Power of a Power

- $(a^n)^m = a^{nm}$

Exponent Zero

$$\begin{aligned} \bullet a^m \times \frac{1}{a^m} &= 1 \\ \Rightarrow \frac{a^m}{a^m} &= a^{m-m} = a^0 = 1 \end{aligned}$$

Powers with unlike bases and same exponent

$$\bullet a^n \times b^n = (ab)^n$$

Example: $2^2 \times 3^2 = 4 \times 9 = 36$ which is $= (2 \times 3)^2 = 6^2$

$$\bullet \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

Example: $\left(\frac{3^3}{4^3} = \frac{3}{4}\right)^3$

$$L. H. S. \frac{3^3}{4^3} = \frac{27}{64} = 0.42$$

$$R. H. S. \left(\frac{3}{4}\right)^3 = 0.75^3 = 0.42$$

$\therefore L. H. S. = R. H. S.$

The laws of exponents are demonstrated based on the powers they carry.

- Bases – multiplying the like ones – add the exponents and keep the base same. (Multiplication Law)
- Bases – raise it with power to another – multiply the exponents and keep the base same.
- Bases – dividing the like ones – ‘Numerator Exponent – Denominator Exponent’ and keep the base same. (Division Law)

Let ‘a’ is any number or integer (positive or negative) and ‘m’, ‘n’ are positive integers, denoting the power to the bases, then;

Multiplication Law

As per the multiplication law of exponents, the product of two exponents with the same base and different powers equals to base raised to the sum of the two powers or integers.

$$a^m \times a^n = a^{m+n}$$

Division Law

When two exponents having same bases and different powers are divided, then it results in base raised to the difference between the two powers.

$$a^m \div a^n = a^m / a^n = a^{m-n}$$

Negative Exponent Law

Any base if has a negative power, then it results in reciprocal but with positive power or integer to the base.

$$a^{-m} = 1/a^m$$

Product With the Same Bases

As per this law, for any non-zero term a,

$$a^m \times a^n = a^{m+n}$$

Example 1: What is the simplification of $5^5 \times 5^1$?

$$\text{Solution: } 5^5 \times 5^1 = 5^{5+1} = 5^6$$

Example 2: What is the simplification of $(-6)^{-4} \times (-6)^{-7}$?

$$\text{Solution: } (-6)^{-4} \times (-6)^{-7} = (-6)^{-4-7} = (-6)^{-11}$$

Note: We can state that the law is applicable for negative terms also. Therefore the term m and n can be any integer.

Quotient with Same Bases

As per this rule,

$$a^m/a^n = a^{m-n}$$

where a is a non-zero term and m and n are integers.

Power Raised to a Power

According to this law, if 'a' is the base, then the power raised to the power of base 'a' gives the product of the powers raised to the base 'a', such as;

$$(a^m)^n = a^{mn}$$

Zero Power

According to this rule, when the power of any integer is zero, then its value is equal to 1, such as;

$$a^0 = 1$$

where 'a' is any non-zero term.

Example: What is the value of $5^0 + 2^2 + 4^0 + 7^1 - 3^1$?

$$\text{Solution: } 5^0 + 2^2 + 4^0 + 7^1 - 3^1 = 1 + 4 + 1 + 7 - 3 = 10$$

Rules of Exponents

The rules of exponents are followed by the laws. Let us have a look at them with a brief

explanation.

Suppose 'a' & 'b' are the integers and 'm' & 'n' are the values for powers, then the rules for exponents and powers are given by:

- $a^0 = 1$

As per this rule, if the power of any integer is zero, then the resulted output will be unity or one.

Example: $5^0 = 1$

- $(a^m)^n = a^{(mn)}$

'a' raised to the power 'm' raised to the power 'n' is equal to 'a' raised to the power product of 'm' and 'n'.

Example: $(5^2)^3 = 5^{2 \times 3}$

- $a^m \times b^m = (ab)^m$

The product of 'a' raised to the power of 'm' and 'b' raised to the power 'm' is equal to the product of 'a' and 'b' whole raised to the power 'm'.

Example: $5^2 \times 6^2 = (5 \times 6)^2$

- $a^m/b^m = (a/b)^m$

The division of 'a' raised to the power 'm' and 'b' raised to the power 'm' is equal to the division of 'a' by 'b' whole raised to the power 'm'.

Example: $5^2/6^2 = (5/6)^2$

Exponents and Powers Applications

Scientific notation uses the power of ten expressed as exponents, so we need a little background before we can jump in. In this concept, we round out your knowledge of exponents, which we studied in previous classes.

The distance between the Sun and the Earth is 149,600,000 kilometres. The mass of the Sun is 1,989,000,000,000,000,000,000,000,000 kilograms. The age of the Earth is 4,550,000,000 years. These numbers are way too large or small to memorize in this way. With the help of exponents and powers, these huge numbers can be reduced to a very compact form and can be easily expressed in powers of 10.

Now, coming back to the examples we mentioned above, we can express the distance between the Sun and the Earth with the help of exponents and powers as following:

Distance between the Sun and the Earth $149,600,000 = 1.496 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1.496 \times 10^8$ kilometers.

Mass of the Sun: 1,989,000,000,000,000,000,000,000,000 kilograms = 1.989×10^{30} kilograms.

Age of the Earth: 4,550,000,000 years = 4.55×10^9 years

Powers with negative exponents

- Numbers can have positive powers which are called positive index. Example $a^n = a \times a \times a \dots n$ times.
- Numbers can also have negative powers such as

$$a^{-m} = \frac{1}{a^m} = \frac{1}{\underbrace{a \times a \times a \dots m \text{ times}}}$$

- Example : $5^{-3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125} = 0.008$

Visualising Exponents

Visualising powers and exponents

- Example 1: 54 can be expressed as product of powers of prime numbers.

$$54 = 2 \times 3 \times 3 \times 3 = 3^3 \times 2^1$$

- Example 2 : We know that $6^4 < 4^6$. This can be visualised as shown below:

$$6^4 = 6 \times 6 \times 6 \times 6 = 1296$$

$$4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096$$

$$\therefore 6^4 < 4^6$$

Uses of Exponents

Expanding a rational number using powers

- Rational Numbers can be expanded using exponents and powers.
- Example 1: 1284 can be written as $1 \times 10^3 + 2 \times 10^2 + 8 \times 10^1 + 4 \times 10^0$.
- Example 2: 0.597 can be written as $5 \times 10^{-1} + 9 \times 10^{-2} + 7 \times 10^{-3}$.

Inter conversion between standard and normal forms

- Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its **standard form**.

- Example:

$$43 = 4.3 \times 10 = 4.3 \times 10^1$$

$$430 = 4.3 \times 100 = 4.3 \times 10^2$$

$$4300 = 4.3 \times 1000 = 4.3 \times 10^3$$

$$43000 = 4.3 \times 10000 = 4.3 \times 10^4$$

Comparison of quantities using exponents

- If two numbers in standard form have the same power of 10, then the number with the larger factor is greater.
E.g : $2.05 \times 10^3 > 1.05 \times 10^3$
- If two numbers in standard form have the same factor, then the number with the larger power of 10 will be greater.
E.g $2.05 \times 10^6 > 2.05 \times 10^3$

Important Questions

Multiple Choice Questions:

Question 1. The exponential form of 10000 is

- (a) 10^3
- (b) 10^4
- (c) 10^5
- (d) none of these

Question 2. The exponential form of 100000 is

- (a) 10^3
- (b) 10^4
- (c) 10^5
- (d) none of these

Question 3. The exponential form of 81 is

- (a) 3^4
- (b) 3^3
- (c) 3^2
- (d) none of these

Question 4. The exponential form of 125 is

- (a) 5^4
- (b) 5^3
- (c) 5^2
- (d) none of these

Question 5. The exponential form of 32 is

- (a) 2^3
- (b) 2^4
- (c) 2^5
- (d) none of these

Question 6. The exponential form of 243 is

- (a) 3^5
- (b) 3^4
- (c) 3^3
- (d) 3^2

Question 7. The exponential form of 64 is

- (a) 2^5
- (b) 2^6
- (c) 2^7
- (d) 2^8

Question 8. The exponential form of 625 is

- (a) 5^2
- (b) 5^3
- (c) 5^4
- (d) 5^5

Question 9. The exponential form of 1000 is

- (a) 10^1
- (b) 10^2
- (c) 10^3
- (d) 10^4

Question 10. The value of $(-2)^3$ is

- (a) 8
- (b) -8
- (c) 16
- (d) -16

Question 11. The value of $(-2)^4$ is

- (a) 8
- (b) -8
- (c) 16
- (d) -16

Question 12. What is the base in 8^2 ?

- (a) 8
- (b) 2
- (c) 6
- (d) 10

Question 13. What is the exponent in 8^2 ?

- (a) 8
- (b) 2

(c) 16

(d) 6

Question 14. (-1) even number =

(a) -1

(b) 1

(c) 0

(d) none of these

Question 15. (-1) odd number =

(a) -1

(b) 1

(c) 0

(d) none of these

Very Short Questions:

- Express 343 as a power of 7.
- Which is greater 3^2 or 2^3 ?
- Express the following number as a powers of prime factors:
 - 144
 - 225
- Find the value of:
 - $(-1)^{1000}$
 - $(1)^{250}$
 - $(-1)^{121}$
 - $(10000)^0$
- Express the following in exponential form:
 - $5 \times 5 \times 5 \times 5 \times 5$
 - $4 \times 4 \times 4 \times 5 \times 5 \times 5$
 - $(-1) \times (-1) \times (-1) \times (-1) \times (-1)$
 - $a \times a \times a \times b \times c \times c \times c \times d \times d$

Short Questions:

- Express each of the following as product of powers of their prime factors:
 - 405

- (ii) 504
 (iii) 500
2. Simplify the following and write in exponential form:

- (i) $(5^2)^3$
 (ii) $(2^3)^3$
 (iii) $(a^b)^c$
 (iv) $[(5)^2]^2$

3. Verify the following:

$$(i) \left(-\frac{3}{4}\right)^3 = -\frac{27}{64} \quad (ii) \left(-\frac{2}{3}\right)^6 = \frac{64}{729}$$

4. Simplify:

$$(i) \frac{2^2 \times 3^4 \times 2^5}{2^4 \times 9} \quad (ii) 2^3 \times k^3 \times 5k^4$$

5. Simplify and write in exponential form:

$$(i) \left(\frac{3^5}{3^2}\right) \times 3^{10} \quad (ii) 8^2 \div 2^3$$

6. Express each of the following as a product of prime factors in the exponential form:

- (i) 729×125
 (ii) 384×147

7. Simplify the following:

- (i) $10^3 \times 9^0 + 3^3 \times 2 + 7^0$
 (ii) $6^3 \times 7^0 + (-3)^4 - 9^0$

8. Write the following in expanded form:

- (i) 70,824
 (ii) 1,69,835

Long Questions:

1. Find the number from each of the expanded form:

- (i) $7 \times 10^8 + 3 \times 10^5 + 7 \times 10^2 + 6 \times 10^1 + 9$
 (ii) $4 \times 10^7 + 6 \times 10^3 + 5$

2. Find the value of k in each of the following:

$$(i) \left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^6 = \left(\frac{4}{9}\right)^{2k-3}$$

$$(ii) \left(-\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5 = \left(\frac{4}{5}\right)^{6k+1}$$

3. Find the value of

(a) $3^0 \div 4^0$

(b) $(8^0 - 2^0) \div (8^0 + 2^0)$

(c) $(2^0 + 3^0 + 4^0) - (4^0 - 3^0 - 2^0)$

4. Express the following in standard form:

(i) 8,19,00,000

(ii) 5,94,00,00,00,000

(iii) 6892.25

5. Evaluate:

$$(i) \frac{5^4 \times 7^5 \times 2^9}{8 \times 49 \times 5^2} \quad (ii) \frac{15^4 \times 18^3}{3^3 \times 5^2 \times 12^2}$$

6. Find the value of x, if

$$\frac{2^{2x} \times 4 \times 2^x - 8^x}{(2^5)^3 \times 9} = \frac{1}{24}$$

7.

If $\frac{x}{y} = \left(\frac{3}{2}\right)^2 + \left(\frac{5}{7}\right)^0$, find the value of $\left(\frac{y}{x}\right)^3$

Answer Key-

Multiple Choice Questions:

1. (b) 10^4

2. (c) 10^5

3. (a) 3^4

4. (b) 5^3

5. (c) 2^5

6. (a) 3^5

7. (b) 2^6

8. (c) 5^4
9. (c) 10^3
10. (b) -8
11. (c) 16
12. (a) 8
13. (b) 2
14. (b) 1
15. (a) -1

Very Short Answer:

1. We have $343 = 7 \times 7 \times 7 = 7^3$

Thus, $343 = 7^3$

$$\begin{array}{r|l}
 7 & 343 \\
 \hline
 7 & 49 \\
 \hline
 7 & 7 \\
 \hline
 & 1
 \end{array}$$

2. We have $3^2 = 3 \times 3 = 9$

$$2^3 = 2 \times 2 \times 2 = 8$$

Since $9 > 8$

Thus, $3^2 > 2^3$

3. (i) We have

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$$

Thus, $144 = 2^4 \times 3^2$

$$\begin{array}{r|l}
 2 & 144 \\
 \hline
 2 & 72 \\
 \hline
 2 & 36 \\
 \hline
 2 & 18 \\
 \hline
 3 & 9 \\
 \hline
 3 & 3 \\
 \hline
 & 1
 \end{array}$$

- (ii) We have

$$225 = 3 \times 3 \times 5 \times 5 = 3^2 \times 5^2$$

$$\text{Thus, } 225 = 3^2 \times 5^2$$

3	225
3	75
5	25
5	5
	1

4. (i) $(-1)^{1000} = 1$ [$\because (-1)^{\text{even number}} = 1$]
 (ii) $(1)^{250} = 1$ [$\because (1)^{\text{even number}} = 1$]
 (iii) $(-1)^{121} = -1$ [$\because (-1)^{\text{odd number}} = -1$]
 (iv) $(10000)^0 = 1$ [$\because a^0 = 1$]
5. (i) $5 \times 5 \times 5 \times 5 \times 5 = (5)^5$
 (ii) $4 \times 4 \times 4 \times 5 \times 5 \times 5 = 4^3 \times 5^3$
 (iii) $(-1) \times (-1) \times (-1) \times (-1) \times (-1) = (-1)^5$
 (iv) $a \times a \times a \times b \times c \times c \times c \times d \times d = a^3 b^1 c^3 d^2$

Short Answer:

1. (i) We have
 $405 = 3 \times 3 \times 3 \times 3 \times 5 = 3^4 \times 5^1$
 Thus, $405 = 3^4 \times 5^1$

3	405
3	135
3	45
3	15
5	5
	1

- (ii) We have
 $504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 2^3 \times 3^2 \times 7^1$
 Thus, $504 = 2^3 \times 3^2 \times 7^1$

$$\begin{array}{r|l}
 2 & 504 \\
 \hline
 2 & 252 \\
 \hline
 2 & 126 \\
 \hline
 3 & 63 \\
 \hline
 3 & 21 \\
 \hline
 7 & 7 \\
 \hline
 & 1
 \end{array}$$

(iii) We have

$$500 = 2 \times 2 \times 5 \times 5 \times 5 = 2^2 \times 5^3$$

Thus, $500 = 2^2 \times 5^3$

$$\begin{array}{r|l}
 2 & 500 \\
 \hline
 2 & 250 \\
 \hline
 5 & 125 \\
 \hline
 5 & 25 \\
 \hline
 5 & 5 \\
 \hline
 & 1
 \end{array}$$

2. (i) $(5^2)^3 = 5^{2 \times 3} = 5^6$

(ii) $(2^3)^3 = 2^{3 \times 3} = 2^9$

(iii) $(a^b)^c = a^{b \times c} = a^{bc}$

(iv) $[(5)^2]^3 = 5^{2 \times 3} = 5^6$

3.

(i) $\left(-\frac{3}{4}\right)^3 = \left(-\frac{3}{4}\right) \times \left(-\frac{3}{4}\right) \times \left(-\frac{3}{4}\right)$
 $= \frac{3 \times 3 \times 3}{4 \times 4 \times 4} = -\frac{27}{64}$

(ii) $\left(-\frac{2}{3}\right)^6 = \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right)$
 $\times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right)$
 $= \frac{64}{729}$ Hence verified.

4.

(i) $\frac{2^2 \times 3^4 \times 2^5}{2^4 \times 9} = \frac{2^{2+5} \times 3^4}{2^4 \times 3^2} = \frac{2^7 \times 3^4}{2^4 \times 3^2}$
 $= 2^{7-4} \times 3^{4-2} = 2^3 \times 3^2$
 $= 8 \times 9 = 72$

(ii) $2^3 \times k^3 \times 5k^4 = 8 \times 5 \times k^{3+4} = 40k^7$

5.

$$(i) \left(\frac{3^5}{3^2}\right) \times 3^{10} = 3^{5-2} \times 3^{10} = 3^3 \times 3^{10} = 3^{3+10} = 3^{13}$$

$$(ii) 8^2 \div 2^3 = (2^3)^2 \div 2^3 = 2^{3 \times 2} \div 2^3 \\ = 2^6 \div 2^3 = 2^{6-3} = 2^3 = 8$$

6. (i) $729 \times 125 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3^6 \times 5^3$ Thus, $729 \times 125 = 3^6 \times 5^3$

3	729
3	243
3	81
3	27
3	9
3	3
	1

5	125
5	25
5	5
	1

(ii) $384 \times 147 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7 = 2^7 \times 3^2 \times 7^2$ Thus, $384 \times 147 = 2^7 \times 3^2 \times 7^2$

2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

3	147
7	49
7	7
	1

7. (i) $10^3 \times 9^0 + 3^3 \times 2 + 7^0$

$$= 1000 + 54 + 1$$

$$= 1055$$

(ii) $6^3 \times 7^0 + (-3)^4 - 9^0$

$$= 216 \times 1 + 81 - 1$$

$$= 216 + 80$$

$$= 296$$

8. (i) 70,824

$$= 7 \times 10000 + 0 \times 1000 + 8 \times 100 + 2 \times 10 + 4 \times 10^0$$

$$= 7 \times 10^4 + 8 \times 10^2 + 2 \times 10^1 + 4 \times 10^0$$

$$(ii) 1,69,835$$

$$= 1 \times 100000 + 6 \times 10000 + 9 \times 1000 + 8 \times 100 + 3 \times 10 + 5 \times 10^0$$

$$= 1 \times 10^5 + 6 \times 10^4 + 9 \times 10^3 + 8 \times 10^2 + 3 \times 10^1 + 5 \times 10^0$$

Long Answer:

$$1. (i) 7 \times 10^8 + 3 \times 10^5 + 7 \times 10^2 + 6 \times 10^1 + 9$$

$$= 7 \times 100000000 + 3 \times 100000 + 7 \times 100 + 6 \times 10 + 9$$

$$= 700000000 + 300000 + 700 + 60 + 9$$

$$= 700300769$$

$$(ii) 4 \times 10^7 + 6 \times 10^3 + 5$$

$$= 4 \times 10000000 + 6 \times 1000 + 5$$

$$= 40000000 + 6000 + 5$$

$$= 40006005$$

2.

$$(i) \left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^6 = \left(\frac{4}{9}\right)^{2k-3}$$

$$\left(\frac{2}{3}\right)^{3+6} = \left[\left(\frac{2}{3}\right)^2\right]^{2k-3}$$

$$\Rightarrow \left(\frac{2}{3}\right)^9 = \left(\frac{2}{3}\right)^{2(2k-3)}$$

$$\Rightarrow \left(\frac{2}{3}\right)^9 = \left(\frac{2}{3}\right)^{4k-9}$$

Comparing the powers of similar base, we have

$$4k - 9 = 9$$

$$4k = 9 + 9$$

$$4k = 18$$

$$\Rightarrow k = \frac{21^{18^9}}{A_2} = \frac{9}{2}$$

$$\begin{aligned}
 \text{(ii)} \quad & \left(-\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5 = \left(\frac{4}{5}\right)^{6k+1} \\
 & \Rightarrow \left(\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5 = \left(\frac{4}{5}\right)^{6k+1} \quad [\because (-a)^2 = a^2] \\
 & \Rightarrow \left(\frac{4}{5}\right)^{2+5} = \left(\frac{4}{5}\right)^{6k+1} \\
 & \Rightarrow \left(\frac{4}{5}\right)^7 = \left(\frac{4}{5}\right)^{6k+1}
 \end{aligned}$$

Comparing the powers of similar base

$$\begin{aligned}
 & 6k + 1 = 7 \\
 \Rightarrow & 6k = 7 - 1 \\
 \Rightarrow & 6k = 6 \\
 \therefore & k = 1
 \end{aligned}$$

3. (a) We have $3^0 \div 4^0 = 1 \div 1 = 1$ [$\because a^0 = 1$]
 (b) $(8^0 - 2^0) \div (8^0 + 2^0) = (1 - 1) \div (1 + 1) = 0 \div 2 = 0$
 (c) $(2^0 + 3^0 + 4^0) - (4^0 - 3^0 - 2^0)$
 $= (1 + 1 + 1) - (1 - 1 - 1)$ [$\because a^0 = 1$]
 $= 3 - 1$
 $= 2$
4. (i) $8,19,00,000 = 8.19 \times 10^7$
 (ii) $5,94,00,00,00,000 = 5.94 \times 10^{11}$
 (iii) $6892.25 = 6.89225 \times 10^3$
- 5.

$$\begin{aligned}
 \text{(i)} \quad & \frac{5^4 \times 7^5 \times 2^9}{8 \times 49 \times 5^2} = \frac{5^4 \times 7^5 \times 2^9}{2^3 \times 7^2 \times 5^2} \\
 & = 5^{4-2} \times 7^{5-2} \times 2^{9-3} \\
 & = 5^2 \times 7^3 \times 2^6 \\
 & = 25 \times 323 \times 64 \\
 & = 516800
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad & \frac{15^4 \times 18^3}{3^3 \times 5^2 \times 12^2} \\
 &= \frac{(3 \times 5)^4 \times (2 \times 3 \times 3)^3}{3^3 \times 5^2 \times (2 \times 2 \times 3)^2} = \frac{3^4 \times 5^4 \times (2 \times 3^2)^3}{3^3 \times 5^2 \times (2^2 \times 3)^2} \\
 &= \frac{3^4 \times 5^4 \times 2^3 \times (3^2)^3}{3^3 \times 5^2 \times (2^2)^2 \times 3^2} = \frac{3^4 \times 5^4 \times 2^3 \times 3^6}{3^3 \times 5^2 \times 2^4 \times 3^2} \\
 &= \frac{3^{4+6} \times 5^4 \times 2^3}{3^{3+2} \times 5^2 \times 2^4} = \frac{3^{10} \times 5^4 \times 2^3}{3^5 \times 5^2 \times 2^4} \\
 &= \frac{3^{10-5} \times 5^{4-2}}{2^{4-3}} = \frac{3^5 \times 5^2}{2} \\
 &= \frac{243 \times 25}{2} = \frac{6075}{2} = 3037.50
 \end{aligned}$$

6.

We have

$$\frac{2^{2x} \times 4 \times 2^x - 8^x}{(2^5)^3 \times 9} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{2x} \times 2^2 \times 2^x - (2^3)^x}{2^{5 \times 3} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{2x+2+x} - 2^{3x}}{2^{15} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{3x+2} - 2^{3x}}{2^{15} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{3x} (2^2 - 1)}{2^{15} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{3x} (4 - 1)}{2^{15} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{3x} \times 3}{2^{15} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{3x-15}}{3} = \frac{1}{24}$$

$$\Rightarrow 2^{3x-15} = \frac{3}{24}$$

$$\Rightarrow 2^{3x-15} = \frac{1}{8}$$

$$\Rightarrow 2^{3x-15} = \frac{1}{2^3}$$

$$\Rightarrow 2^{3x-15} = 2^{-3}$$

Comparing the powers of the similar base, we have

$$\begin{aligned} 3x - 15 &= -3 \\ \Rightarrow 3x &= -3 + 15 \\ \Rightarrow 3x &= 12 \\ \Rightarrow x &= \frac{12}{3} = 4 \\ \text{Thus, } x &= 4 \end{aligned}$$

7.

$$\begin{aligned} \frac{x}{y} &= \frac{3^2}{2^2} + \left(\frac{5}{7}\right)^0 \\ \frac{x}{y} &= \frac{9}{4} + 1 \\ \Rightarrow \frac{x}{y} &= \frac{9}{4} \\ \Rightarrow \frac{y}{x} &= \frac{4}{9} \\ \therefore \left(\frac{y}{x}\right)^3 &= \left(\frac{4}{9}\right)^3 = \frac{64}{729} \\ \text{Thus, } \left(\frac{y}{x}\right)^3 &= \frac{64}{729} \end{aligned}$$