MATHEMATICS

Chapter 9: Some Application of Trigonometry

Some Application of Trigonometry

1. **Trigonometric Ratios**

Ratio of the sides of a right triangle with respect to the acute angles is called the **trigonometric ratios** of the angle.

Trigonometric ratios of acute angle A in right triangle ABC are given below:

i. $\sin\angle A = \frac{\text{side opposite to}\angle\text{A}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{p}{h}$

ii. $\cos\angle A = \frac{\text{side adjacent to}\angle\text{A}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{b}{h}$

iii. $\tan\angle A = \frac{\text{side opposite to}\angle\text{A}}{\text{side adjacent to}\angle\text{A}} = \frac{BC}{AB} = \frac{p}{b}$

iv. $\text{cosec}\,\angle A = \frac{\text{hypotenuse}}{\text{side opposite to}\angle\text{A}} = \frac{AC}{BC} = \frac{h}{p}$

v. $\sec\angle A = \frac{\text{hypotenuse}}{\text{side adjacent to}\angle\text{A}} = \frac{AC}{AB} = \frac{h}{b}$

vi. $\cot\angle A = \frac{\text{side adjacent to}\angle\text{A}}{\text{side opposite to}\angle\text{A}} = \frac{AB}{BC} = \frac{b}{p}$

The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

2. **Relation between trigonometric ratios**

The ratios cosec A, sec A and cot A are the reciprocals of the ratios sin A, cos A and tan A respectively as given:

i. $\text{cosec}\theta = \frac{1}{\sin\theta}$

ii. $\sec\theta = \frac{1}{\cos\theta}$

iii. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

iv. $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

3. **Values of Trigonometric ratios** of some specific angles:

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

4. **Trigonometric ratios of complementary angles**

Two angles are said to complementary angles if their sum is equal to 90°. Based on this relation, the trigonometric ratios of complementary angles are given as follows:

i. sin (90° – *A*) = cos *A*

ii. cos (90° – *A*) = sin *A*

iii. tan (90° – *A)* = cot *A*

iv. cot (90° – *A*) = tan *A*

v. sec (90° – *A*) = cosec *A*

vi. cosec (90° – *A*) = sec *A*

Note: tan 0° = 0 = cot 90°, sec 0° = 1 = cosec 90°, sec 90°, cosec 0°, tan 90° and cot 0° are not defined.

5. **Basic trigonometric identities**:

i. $\sin^2\theta + \cos^2\theta = 1$

ii. $1 + \tan^2\theta = \sec^2\theta;\ 0 \le \theta < 90°$

iii. $1 + \cot^2\theta = \text{cosec}^2\theta;\ 0 \le \theta < 90°$

6. The height or length of an object or the distance between two distant objects can be determined by the help of **trigonometric ratios**.

7. **Line of sight**

The **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

8. **Pythagoras theorem**

It states that "In a right triangle, square of the hypotenuse is equal to the sum of the square of the other two sides".

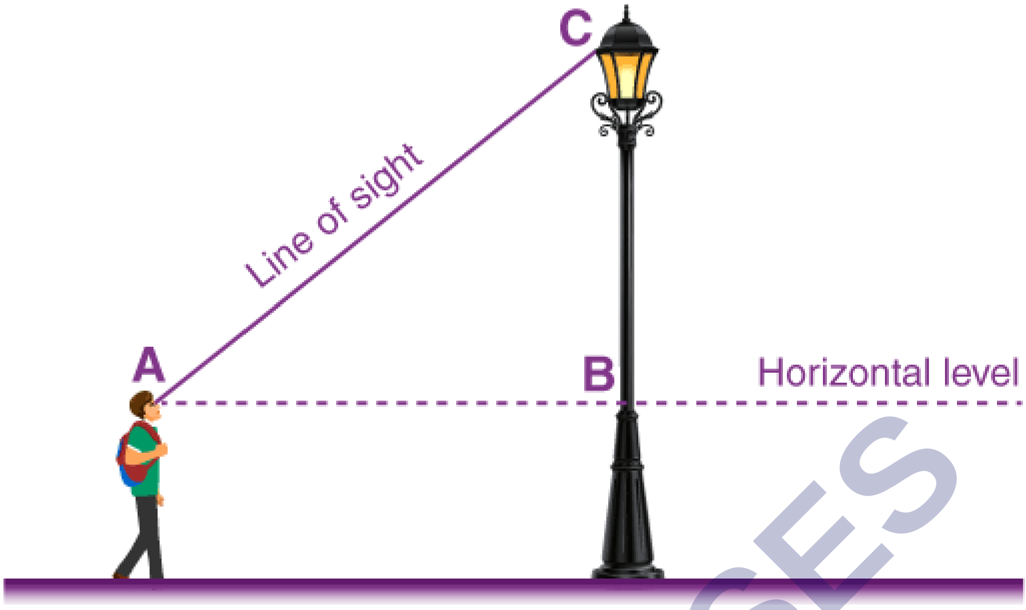
When any two sides of a right triangle are given, its third side can be obtained by using Pythagoras theorem.

9. **Reflection from the water surface**

In case of reflection from the water surface, the two heights above and below the ground level are equal in length.

10. **Heights and Distances**

Horizontal Level and Line of Sight



Line of sight and horizontal level

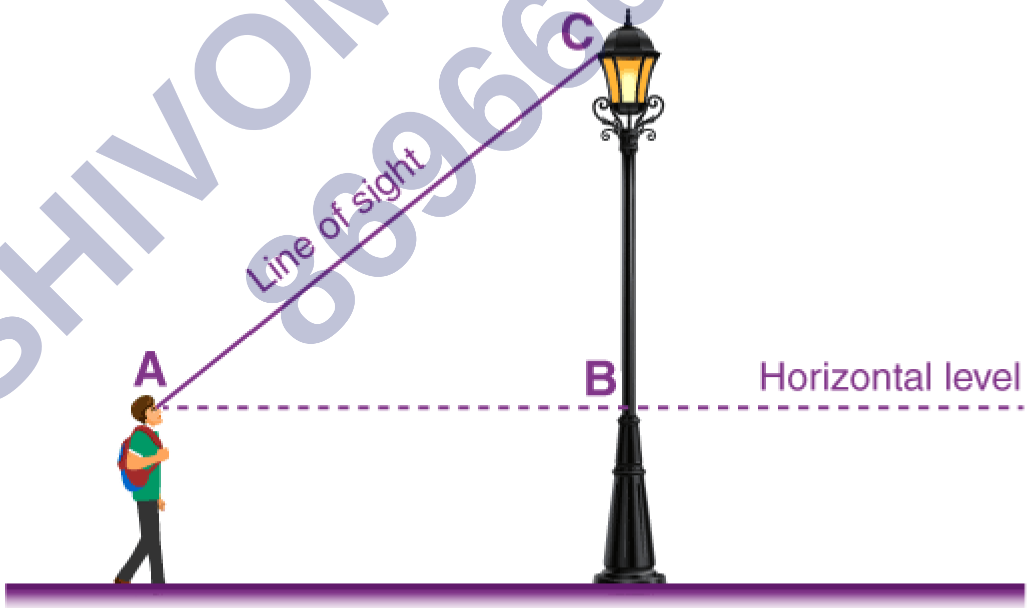
Line of sight is the line drawn from the eye of the observer to the point on the object viewed by the observer.

Horizontal level is the horizontal line through the eye of the observer.

Angle of elevation

The angle of elevation is relevant for objects above horizontal level.

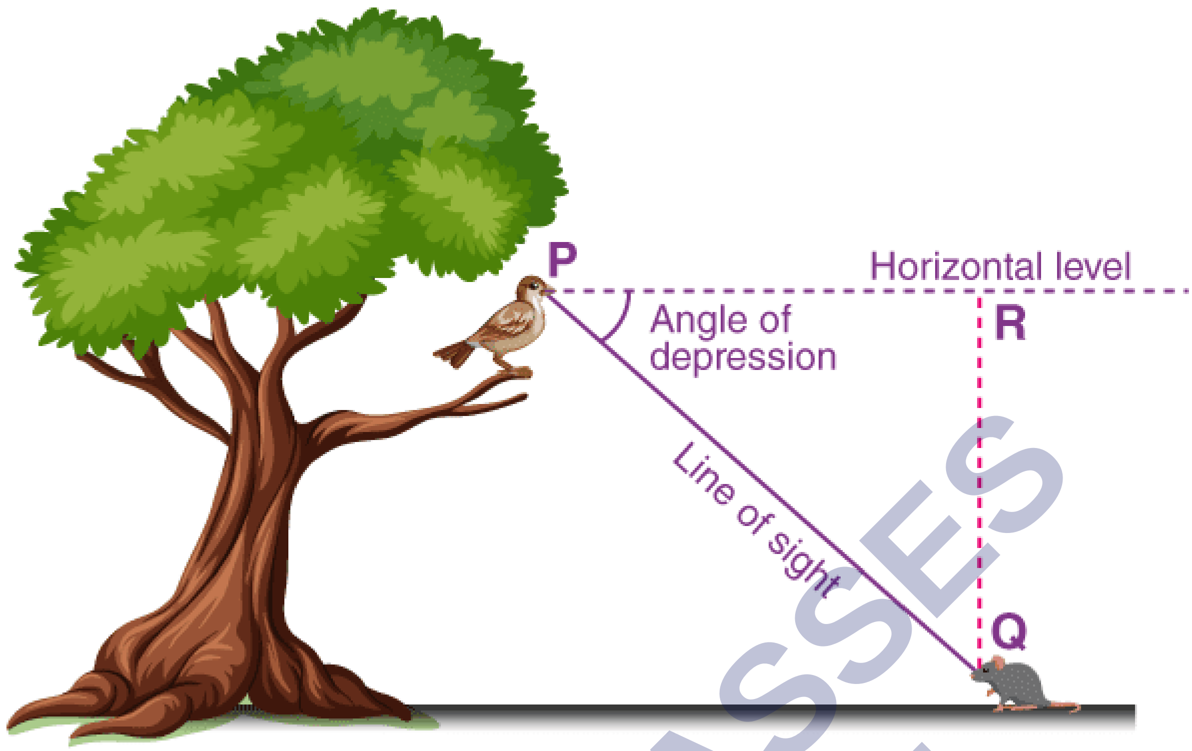
It is the angle formed by the line of sight with the horizontal level.



Angle of depression

The angle of depression is relevant for objects below horizontal level.

It is the angle formed by the line of sight with the horizontal level.



11. Calculating Heights and Distances

To, calculate heights and distances, we can make use of trigonometric ratios.

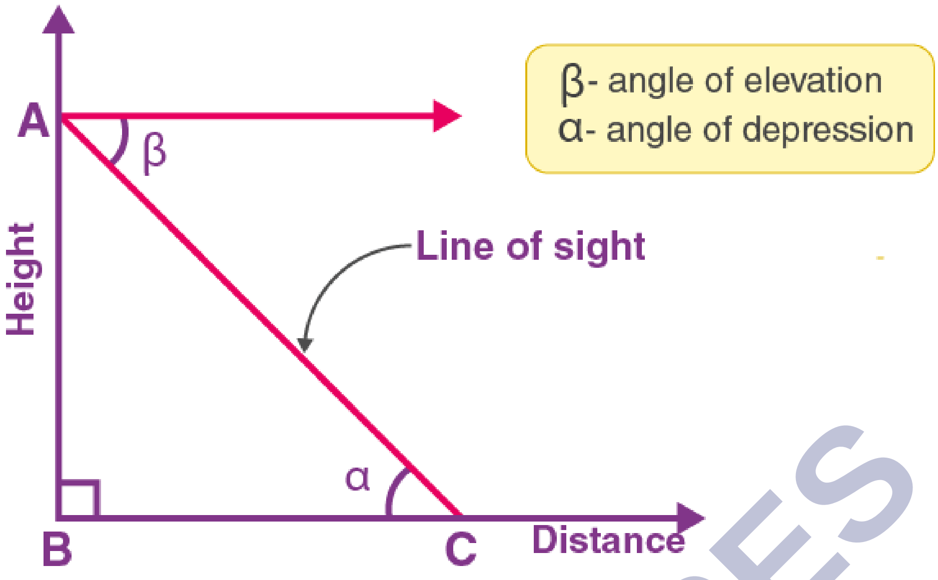
Step 1: Draw a line diagram corresponding to the problem.

Step 2: Mark all known heights, distances and angles and denote unknown lengths by variables.

Step 3: Use the values of various trigonometric ratios of the angles to obtain the unknown lengths from the known lengths.

Height and Distance in Trigonometry

The measurement of an object facing vertically is the height. Distance is defined as the measurement of an object from a point in a horizontal direction. If an imaginary line is drawn from the observation point to the top edge of the object, a triangle is formed by the vertical, horizontal and imaginary line.



From the figure, the point of observation is C. AB denotes the object's height. BC gives the distance between the object and the observer. The line of sight is given by AC. Angles alpha and beta represent the angle of elevation and depression respectively. If any of the two quantities are provided [a side or an angle], the remaining can be found using them. The law of alternate angles states that the magnitude of the angle of elevation and angle of depression are equal in magnitude. tan α = height / distance

12. Measuring the distances of Celestial bodies with the help of trigonometry

Large distances can be measured by the parallax method. The parallax angle is half the angle between two line of sights when an object is viewed from two different positions. Knowing the parallax angle and the distance between the two positions, large distances can be measured.

Solved Examples

Example 1: A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

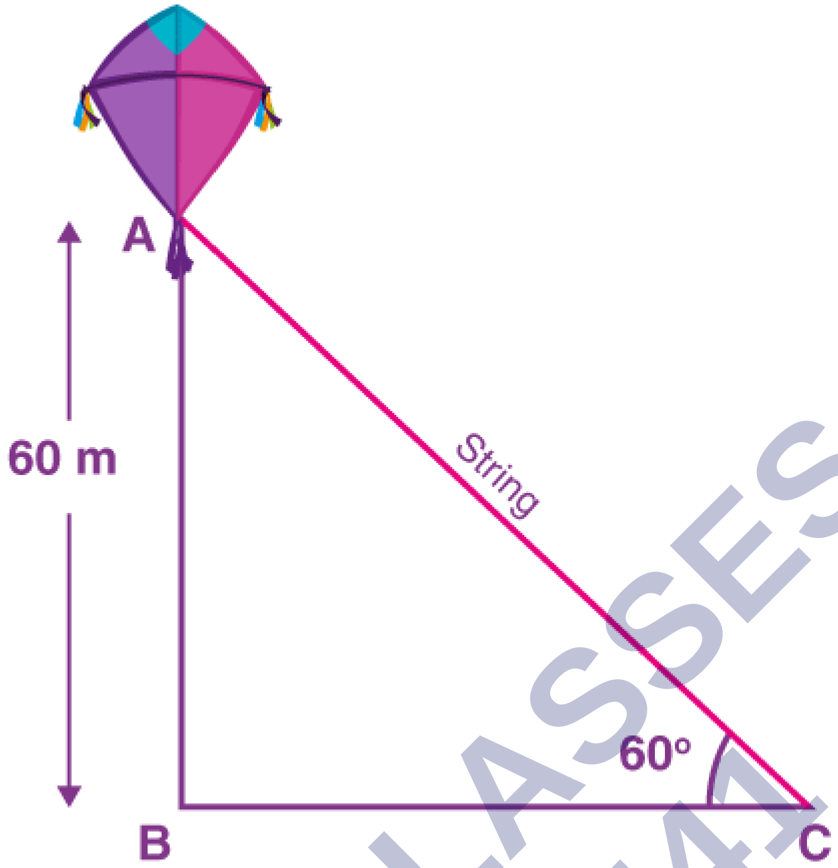
Solution:

Let A be the position of a kite at a height of 60 m above the ground.

Thus, AB = 60 m

Also, AC is the length of the string.

Angle of inclination = ∠ACB = 60



In right triangle ABC,

sin 60° = AB/AC

√3/2 = 60/AC

AC = (60 × 2)√/3

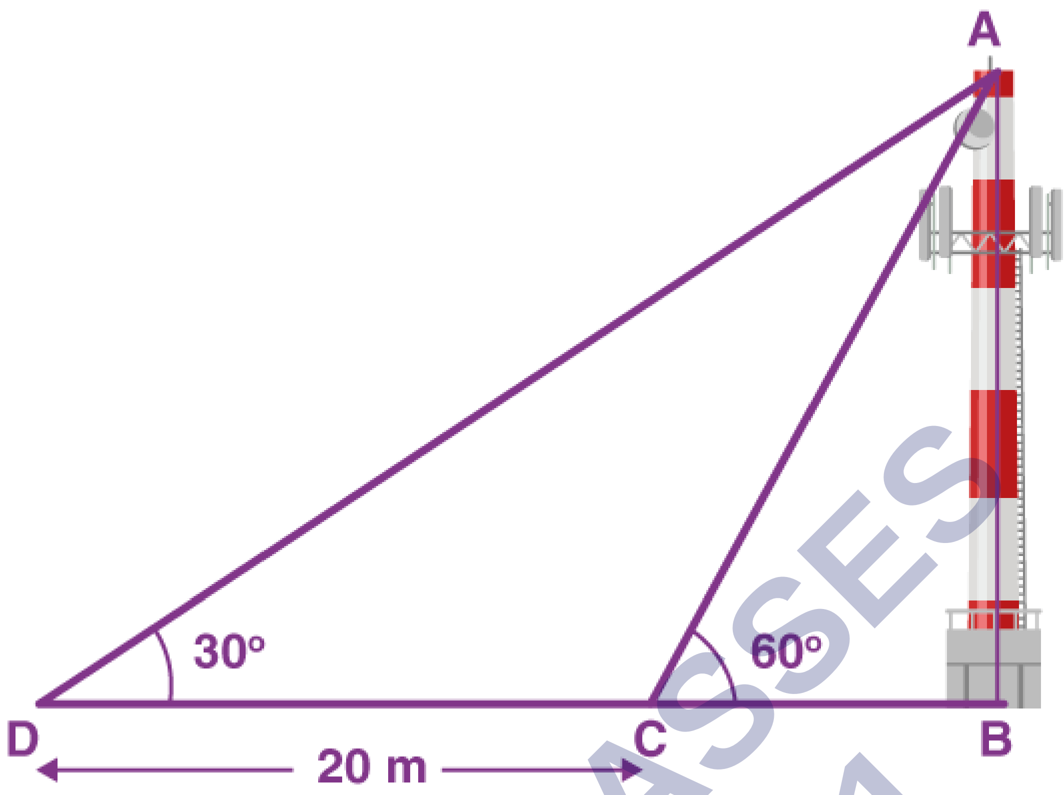
= (120 × √3)/(√3 × √3)

= (120√3)/3

= 40√3

Therefore, the length of the string is 40√3 m.

Example 2: A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° as shown in the figure. Find the height of the tower and the width of the canal.



Solution:

Given,

AB is the height of the tower.

DC = 20 m (given)

In right ΔABD,

tan 30° = AB/BD

1/√3 = AB/(20 + BC)

AB = (20 + BC)/√3....(i)

In right ΔABC,

tan 60° = AB/BC

√3 = AB/BC

AB = √3 BC....(ii)

From (i) and (ii),

√3 BC = (20 + BC)/√3

3 BC = 20 + BC

2 BC = 20

BC = 10

Substituting the value of BC in equation (ii),

AB = $(20 + 10)/\sqrt{3} = 30/\sqrt{3} = 10\sqrt{3}$

Therefore, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

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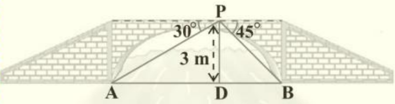
Chapter- 9 : Some Applications of Trigonometry

Some Applications of Trigonometry

Examples

Distance

Determine width AB



From figure, AB = AD + DB

In right ΔAPD ∠A = 30°, ∠D = 90°

$\tan 30° = \frac{PD}{AD}$ *i.e.,* AD = $3\sqrt{3}$ m

In right ΔBPD ∠B = 45°, ∠D = 90°

$\tan 45° = \frac{PD}{BD}$ *i.e.,* BD = 3

∴ AB = $(3\sqrt{3} + 3)$m = $3(\sqrt{3} + 1)$m

Object Height

Determine height of object AB

In ABC ∠B = 90°, ∠C = 60°

Here, $\tan 60° = \frac{AB}{BC}$

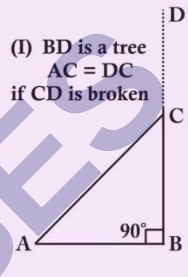
$\sqrt{3} = \frac{AB}{15}$

i.e., AB = $15\sqrt{3}$m

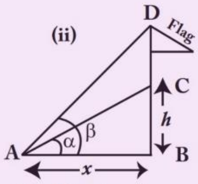
Application Trigonometric Ratio (To Determine)

Height / Length of an object

(I) BD is a tree
AC = DC
if CD is broken



(ii)

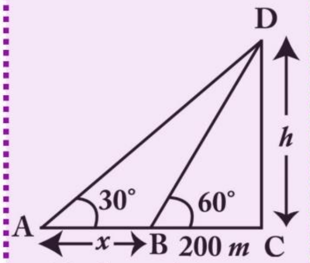


Find flag length

$\tan \alpha = \frac{h}{x}$ ...(i)

$\tan \beta = \frac{h + DC}{x}$ ...(ii)

Distance between two objects



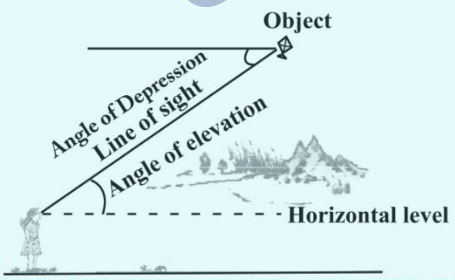
Find x and h

$\tan 60° = \frac{h}{200}$ ...(i)

$\tan 60° = \frac{h}{x + 200}$ ...(ii)

Measuring Angles

Angle of Elevation is equal to Angle of Depression



Important Questions

Multiple Choice questions-

1. The tops of two poles of height 16m and 10m are connected by a wire. If the wire makes an angle of 60° with the horizontal, then the length of the wire is

(a) 10m

(b) 12m

(c) 16m

(d) 18m

2. A 20 m long ladder touches the wall at a height of 10 m. The angle which the ladder makes with the horizontal is

(a) 450

(b) 300

(c) 900

(d) 600

3. If the length of the shadow of a tower is √3 times that of its height, then the angle of elevation of the sun is

(a) 30°

(b) 45°

(c) 60°

(d) 75°

4. If sun's elevation is 60° then a pole of height 6 m will cast a shadow of length

(a) 3√2 m

(b) 6√3 m

(c) 2√3 m

(d) √3 m

5. The angle of elevation of top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°. The length of the tower is

(a) √3 m

(b) $2\sqrt{3}$ m

(c) $5\sqrt{3}$m

(d) $10\sqrt{3}$ m

6. A contractor planned to install a slide for the children to play in a park. If he prefers to have a slide whose top is at a height of 1.5m and is inclined at an angle of 30° to the ground, then the length of the slide would be

(a) 1.5m

(b) $2\sqrt{3}$m

(c) $\sqrt{3}$m

(d) 3m

7. When the length of shadow of a vertical pole is equal to $\sqrt{3}$ times of its height, the angle of elevation of the Sun's altitude is

(a) 30°

(b) 45°

(c) 60°

(d) 15

8. From a point P on the level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100m high, the distance between P and the foot of the tower is

(a) $100\sqrt{3}$m

(b) $200\sqrt{3}$m

(c) $300\sqrt{3}$m

(d) $150\sqrt{3}$m

9. When the sun's altitude changes from 30° to 60°, the length of the shadow of a tower decreases by 70m. What is the height of the tower?

(a) 35 m

(b) 140 m

(c) 60.6 m

(d) 20.2 m

10. The ___________ of an object is the angle formed by the line of sight with the horizontal when the object is below the horizontal level.

(a) line of sight

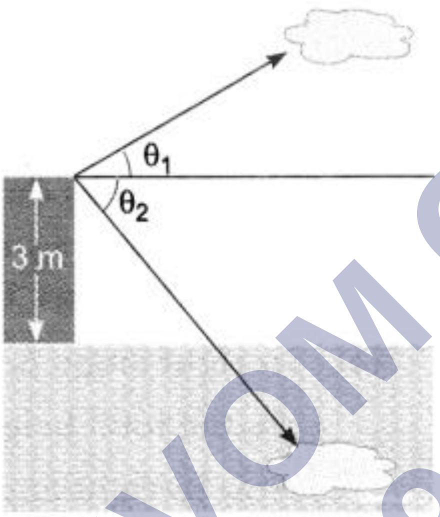
(b) angle of elevation

(c) angle of depression

(d) none of these

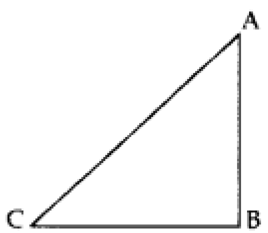
Very Short Questions:

1. If a man standing on a platform, 3 meters above the surface of a lake observes a cloud and its reflection in the lake, then the angle of elevation of the cloud is equal to the angle of depression of its reflection.



2. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then calculate the height of the wall.

3. In the given figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the Sun's altitude.



4. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

5. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun?

6. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find x : y

7. The height of a tower is 12 m. What is the length of its shadow when 10 Sun's altitude is 45°?

8. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°

Short Questions :

1. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

2. A tree breaks due to storm and the broken part bends, so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

3. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

4. Determine the height of a mountain if the elevation of its top at an unknown distance from the base is 30° and at a distance 10 km further off from the mountain, along the same line, the angle of elevation is 15°. (Use tan 15° = 0.27)

5. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.

6. From a point P on the ground, the angle of elevation of the top of a 10m tall building is 30°. A flag is hosted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45o. Find the length of the flagstaff and the distance of the building from the point P. (You may take $\sqrt{3}$ = 1.732).

7. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

8. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string

with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

9. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

10. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.

Long Questions :

1. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60°, respectively. Find the height of the tower.

2. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the bottom of the pedestal is 45°. Find the height of the pedestal.

3. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

OR

From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower. (Use $\sqrt{3} = 1.732$]

4. At a point, the angle of elevation of a tower is such that its tangent is $\frac{5}{12}$ On walking 240 m to the tower, the tangent of the angle of elevation becomes $\frac{3}{4}$. Find the height of the tower.

5. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

6. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

7. In Fig. ABDC is a trapezium in which AB || CD. Line segments RN and LM are drawn parallel to AB such that AJ = JK = KP. If AB = 0.5 m and AP = BQ = 1.8 m, find the lengths of AC, BD, RN and LM.

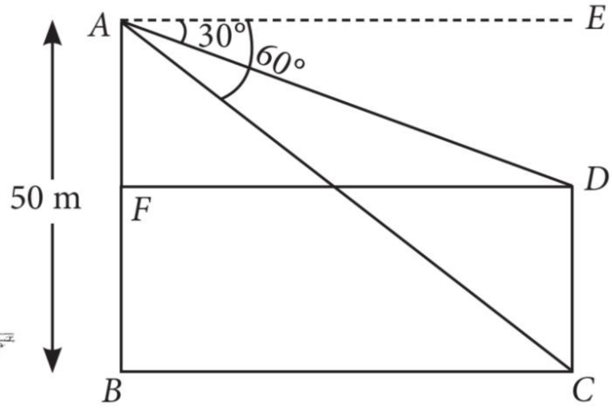
8. Two poles of equal heights are standing opposite to each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

9. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

10. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 metres away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river.

Case Study Answers:

1. There are two temples on each bank of a river. One temple is 50m high. A man, who is standing on the top of 50m high temple, observed from the top that angle of depression of the top and foot of other temple are 30º and 60º respectively. Take $\sqrt{3} = 1.73$



Based on the above information, answer the following questions.

i. Measure of ∠ADF is equal to:

a. 45º

b. 60º
c. 30º
d. 90º

ii. Measure of ∠ACB is equal to:

a. 45º
b. 60º
c. 30º
d. 90º

iii. Width of the river is:

a. 28.90m
b. 26.75m
c. 25m
d. 27m

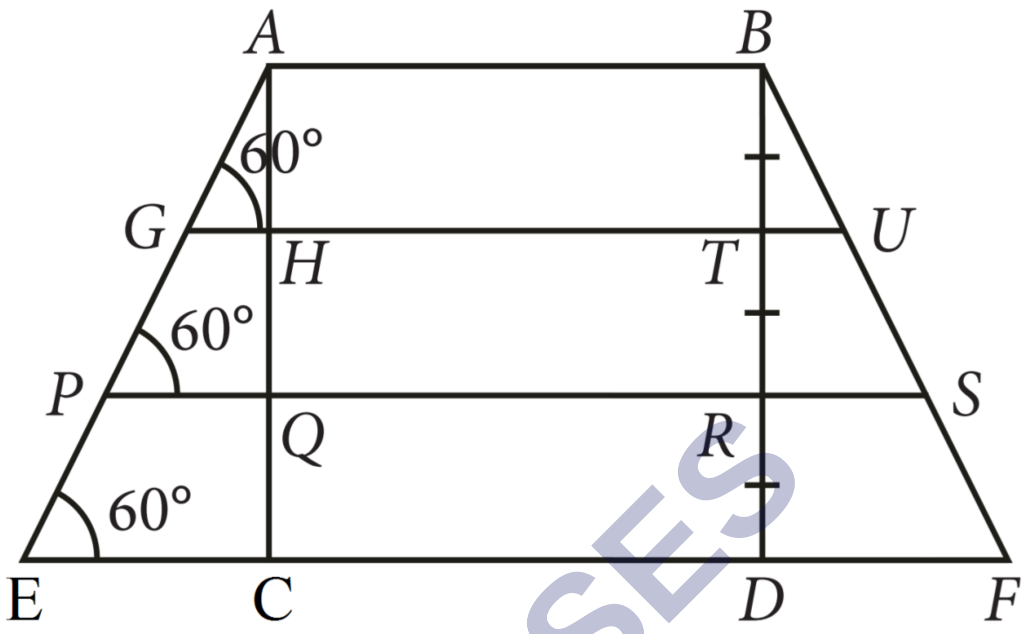
iv. Height of the other temple is:

a. 32.5m
b. 35m
c. 33.33m
d. 40m

v. Angle of depression is always:

a. Reflex angle.
b. Straight.
c. An obtuse angle.
d. An acute angle.

2. Aditi purchase a wooden bar stool for her living room with square A top of side 2m and having height of 6m above the ground. Also each leg is inclined at an angle of 60º to the ground as shown in the figure (not drawn to scale).



Based on the above information, answer the following questions. Take $\sqrt{3} = 1.73$

i. Find the length of the each leg.

  a. 5.9m
  b. 6.93m
  c. 7.3m
  d. 8.2m

ii. Find the length of GH.

  a. 0.53m
  b. 1m
  c. 1.15m
  d. 2.73m

iii. The length of second step is:

  a. 4.3m
  b. 4.99m
  c. 5.68m
  d. 6.78m

iv. The length of PQ =

  a. 1.56m
  b. 2.31m
  c. 3.34m
  d. 5.68m

v. The length of first step is:

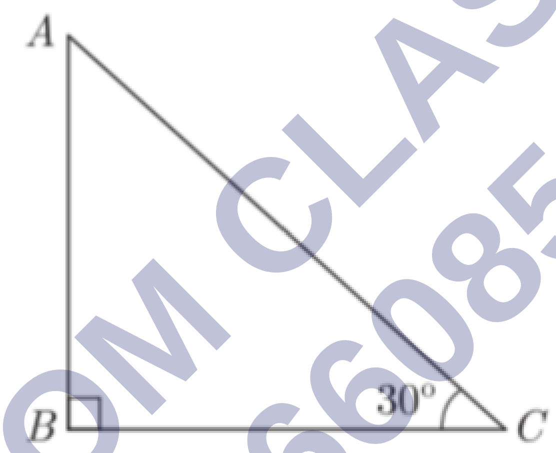
a. 4.78m
b. 5.34m
c. 6.62m
d. 7.82m

Assertion Reason Questions-

1. ***Directions:*** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Assertion: In the figure, if BC = 20 m, then height AB is 11.56 m



Reason : $\tan\theta = \frac{AB}{BC} = \frac{\text{perpendicular}}{\text{base}}$ where θ is the angle $\angle ACB$.

2. ***Directions:*** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Assertion: If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45^0

Reason: According to pythagoras theorem, $h^2 = 1^2 + b^2$, where h = hypotenuse, 1 = length and b = base.

Answer Key-

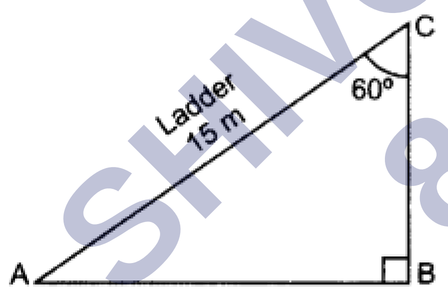
Multiple Choice questions-

1. (b) 12m
2. (b) 300
3. (a) 30°
4. (c) 2√3 m
5. (d) 10√3 m
6. (d) 3m
7. (a) 30°
8. (a) 100√3m
9. (c) 60.6 m
10. (c) angle of depression

Very Short Answer :

1. False, θ1 ≠ θ1

2.



∠BAC = 180° – 90° – 60o = 30°

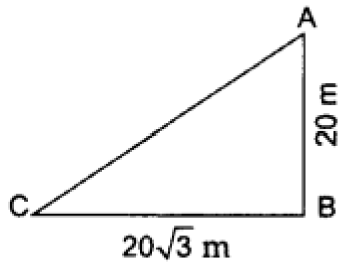
sin 30° = $\frac{BC}{AC}$

$\frac{1}{2}$ = $\frac{BC}{15}$

2BC = 15

BC = $\frac{15}{2}m$

3. AB = 20 m, BC =$20\sqrt{3}$ m,



θ = ?

In ΔABC,

$$\frac{AB}{BC} = \tan\theta$$

$$\frac{20}{20\sqrt{3}} = \tan\theta$$

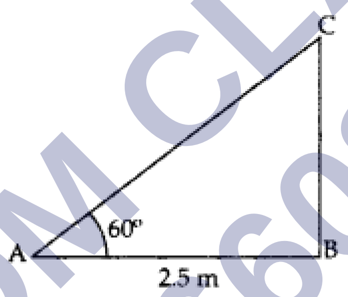
$$\frac{1}{\sqrt{3}} = \tan\theta$$

$$\tan\theta = \tan 30^\circ \quad \Rightarrow \quad \theta = 30^\circ$$

4. Let AC be the ladder

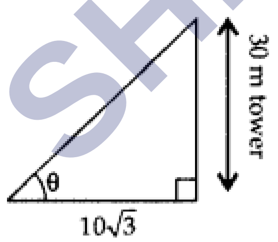
$$\cos 60^\circ = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{2.5}{AC}$$



∴ Length of ladder, AC = 5 m 2.5 m

5. Let required angle be θ.



$\tan\theta = \frac{30}{10\sqrt{3}}$

$\tan\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \tan 60^\circ \therefore \theta = 60^\circ$

6. When base is same for both towers and their heights are given, i.e., x and y respectively Let the base of towers be k.

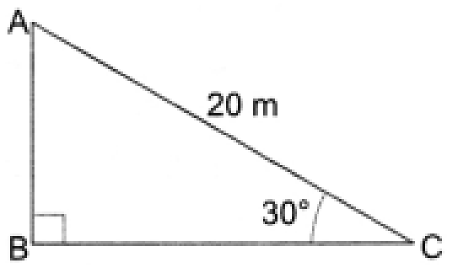
$\tan 30^\circ = \frac{x}{k}$, | $\tan 60^\circ = \frac{y}{k}$

$x = k \tan 30^\circ = \frac{k}{\sqrt{3}}$ ...(i) | $y = k \tan 60^\circ = k\sqrt{3}$ ...(ii)

From equations (i) and (ii),

$$\frac{x}{y} = \frac{\frac{k}{\sqrt{3}}}{k\sqrt{3}} = \frac{k}{\sqrt{3}} \times \frac{1}{k\sqrt{3}} = \frac{1}{3} = \mathbf{1:3}$$

7.



Let AB be the tower

Then, ∠C = 45°, AB = 12 m

$$\tan 45^\circ = \frac{AB}{BC} = \frac{12}{BC} \quad \Rightarrow \quad 1 = \frac{12}{BC} \quad \Rightarrow \quad BC = 12 \text{ m}$$

∴ The length of the shadow is 12 m.

8. Let AB be the vertical pole and AC be the long rope tied to point C.

In right ΔABC, we have

$$\sin 30^\circ = \frac{AB}{BC} \quad \Rightarrow \quad \frac{1}{2} = \frac{AB}{20} \quad \Rightarrow \quad \frac{20}{2} = AB \quad \Rightarrow \quad AB = 10 \text{ m}$$

Therefore, height of the pole is 10 m.

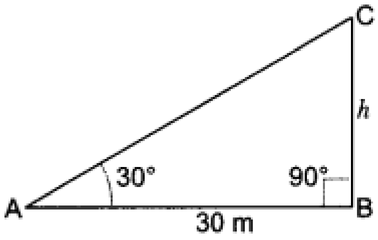
Short Answer :

1. Let BC be the tower whose height is h metres and A be the point at a distance of 30 m from the foot of the tower. The angle of elevation of the top of the tower from point A is given to be 30°.

Now, in right angle ΔCBA we have,

$$\tan 30° = \frac{BC}{AB} = \frac{h}{30} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{30}$$

$$\Rightarrow h = \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}\text{ m}$$



Hence, the height of the tower is 10 √3 m.

2. In right angle ΔABC, AC is the broken part of the tree.

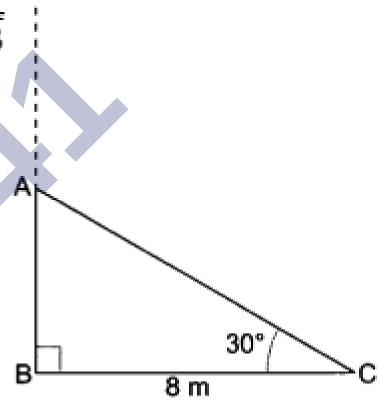
So, the total height of tree = (AB + AC)

Now in right angle ΔABC,

$$\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8} \Rightarrow AB = \frac{8}{\sqrt{3}}$$

Again, $\cos 30° = \frac{BC}{AC}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{AC} \Rightarrow AC = \frac{16}{\sqrt{3}}$$



Hence, the height of the tree = $AB + AC$

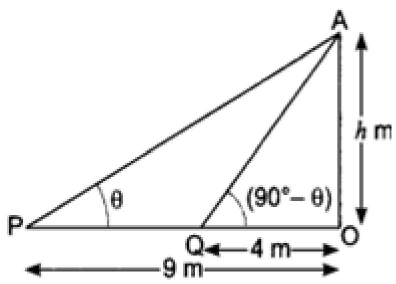
$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}\text{ m}$$

3.

$$\tan\theta = \frac{OA}{OP} = \frac{h}{9} \Rightarrow \tan\theta = \frac{h}{9} \quad \text{... }(i)$$

Again, in ΔAQO we have

$$\tan(90° - \theta) = \frac{OA}{OQ} = \frac{h}{4} \Rightarrow \cot\theta = \frac{h}{4} \quad \text{... }(ii)$$



Multiplying (*i*) and (*ii*), we have

Let OA be the tower of height h meter and P, l be the two points at distance of 9 m and 4 m respectively from the base of the tower.

Now, we have OP = 9 m, OQ = 4 m,

Let ∠APO = θ, ∠AQO = (90° – θ)

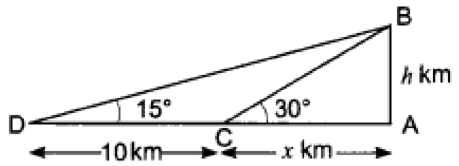
and OA = h meter (Fig. 11.21)

Now, in ΔPOA, we have

$$\tan\theta \times \cot\theta = \frac{h}{9} \times \frac{h}{4} \Rightarrow 1 = \frac{h^2}{36} \Rightarrow h^2 = 36$$

$$h = \pm 6$$

$$\tan 30° = \frac{AB}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$



$$\Rightarrow x = \sqrt{3}h$$

In ΔADB, we have

$$\tan 15° = \frac{AB}{AD} \Rightarrow 0.27 = \frac{h}{x+10}$$

$$\Rightarrow 0.27(x+10) = h \quad ...(i)$$

Height cannot be negative.

Hence, the height of the tower is 6 meter.

4. Let AB be the mountain of height h kilometers. Let C be a point at a distance of x km, from the base of the mountain such that the angle of elevation of the top at C is 30°. Let D be a point at a distance of 10 km from C such that angle of elevation at D is of 15°.

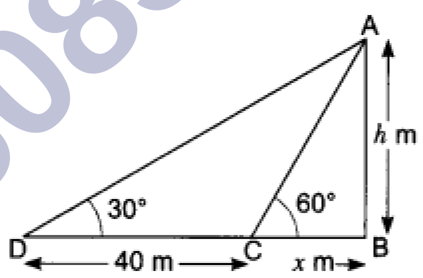
In MBC, we have

In ΔABC, $\tan 60° = \frac{AB}{BC}$ or $\sqrt{3} = \frac{h}{x}$

$$\Rightarrow x\sqrt{3} = h \quad ...(i)$$

In ΔABD, $\tan 30° = \frac{AB}{BD}$

i.e., $$\frac{1}{\sqrt{3}} = \frac{h}{x+40} \quad ...(ii)$$



Substituting x = √3h in equation (i), we get

⇒ 0.27 (√3h + 10) = h

= 0.27 × √3h + 0.27 × 10 = h

⇒ 2.7 = h – 0.27 × √3h

⇒ 27 = h (1 – 0.27 × √3)

⇒ 27 = h (1 – 0.46)

⇒ h = $\frac{2.7}{0.54} = 5$

Hence, the height of the mountain is 5 km

5.

In Fig. AB is the tower and BC is the length of the shadow when the Sun's altitude is 60°, i.e., the angle of elevation of the top of the tower from the tip of the shadow is 60° and DB is the length of the shadow, when the angle of elevation is 30°.

Now, let AB be h m and BC be x m.

According to the question, DB is 40 m longer than BC.

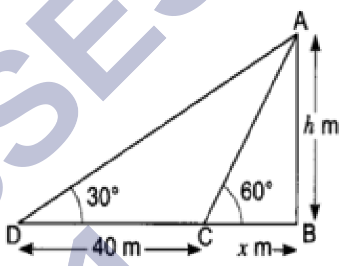
So, BD = (40 + x) m

Now, we have two right triangles ABC and ABD.

Now, we have two right triangles ABC and ABD.

In ΔABC, $\tan 60° = \frac{AB}{BC}$ or $\sqrt{3} = \frac{h}{x}$

$\Rightarrow \quad x\sqrt{3} = h \quad \ldots(i)$



In ΔABD, $\tan 30° = \frac{AB}{BD}$

i.e., $\frac{1}{\sqrt{3}} = \frac{h}{x+40} \quad \ldots(ii)$

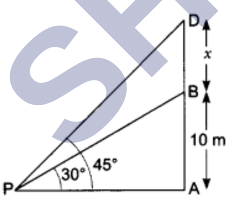
Using (i) in (ii), we get (x √3) √3 = x + 40, i.e., 3x = x + 40

i.e., x = 20

So, h = 20 √3 [From (i)]

Therefore, the height of the tower is 20 √3 m.

6.



In Fig. AB denotes the height of the building, BD the flagstaff and P the given point. Note that there are two right triangles PAB and PAD. We are required to find the length of the flagstaff, i.e., BD and the distance of the building from the point P, i.e., PA.

Since, we know the height of the building AB, we will first consider the right ΔPAB.

We have, $\tan 30° = \frac{AB}{AP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{AP}$

$\Rightarrow \quad AP = 10\sqrt{3}$

i.e., the distance of the building from P is $10\sqrt{3}$ m $= 10 \times 1.732 = 17.32$ m.

Next, let us suppose $DB = x$ m. Then, $AD = (10 + x)$ m.

Now, in right ΔPAD,

$$\tan 45° = \frac{AD}{AP} = \frac{10+x}{10\sqrt{3}} \Rightarrow 1 = \frac{10+x}{10\sqrt{3}} \Rightarrow 10\sqrt{3} = 10 + x$$

i.e., x = 100(√3 – 1) = 7.32

So, the length of the flagstaff is 7.32 m.

7. Let AC be a steep slide for elder children and DE be a slide for younger children. Then AB = 3 m and DB = 1.5 m.

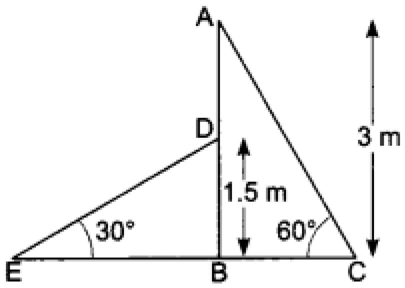
Now, in right angle ΔDBE, we have

$$\sin 30° = \frac{BD}{DE} = \frac{1.5}{DE}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1.5}{DE} \qquad \therefore \quad DE = 2 \times 1.5 = 3 \text{ m}$$

$\therefore$ Length of slide for younger children = 3 m

Again, in right angle ΔABC, we have



$$\sin 60° = \frac{AB}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{AC}$$

$$\Rightarrow AC = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3} \text{ m}$$

So, the length of slide for elder children is 2 √3 m.

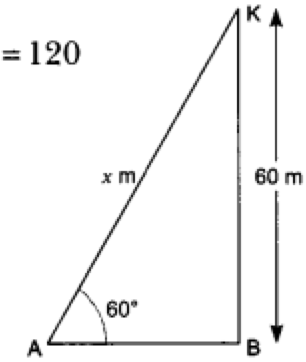
8. Let AB be the horizontal ground and K be the position of the kite and its height from the ground is 60 m and let length of string AK be x m.

∠KAB = 60°

Now, in right angle ΔABK we have

$$\sin 60° = \frac{BK}{AK} = \frac{60}{x} \Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{x} \Rightarrow \sqrt{3}x = 120$$

$$\therefore \quad x = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3} \text{ m}$$



So, the length of string is 40 √3 m.

9. Let AB be the building and PQ be the initial position of the boy (Fig. 11.27) such that

∠APR = 30°

and AB = 30 m

Now, let the new position of the boy be P'Q' at a distance QQ'.

Here, ∠AP'R = 60°

Now, in ΔARP, we have

$$\tan 30° = \frac{AR}{PR} = \frac{AB - RB}{PR}$$

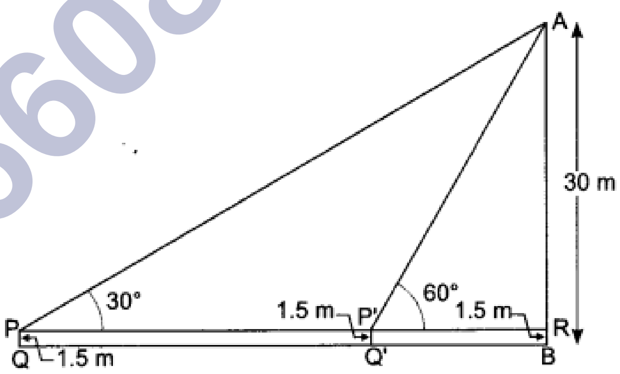
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{30 - 1.5}{PR} = \frac{28.5}{PR}$$

$$PR = 28.5 \times \sqrt{3}$$

Again, in ΔARP' we have

$$\tan 60° = \frac{AR}{P'R} \Rightarrow \sqrt{3} = \frac{28.5}{P'R}$$

$$P'R = \frac{28.5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{28.5\sqrt{3}}{\sqrt{3}} = 9.5\sqrt{3}$$



Therefore, required distance, QQ = PP' = PR – P'R

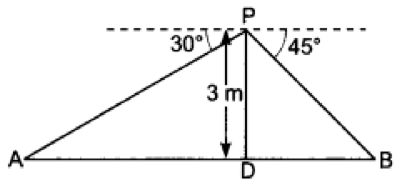
= 28.5 √3 – 9.5 √3 = 19√3

Hence, distance walked by the boy is 19√3 m.

10. In Fig. A and B represent points on the bank on opposite sides of the river, so that AB is the width of the river. P is a point on the bridge at a height of 3 m, i.e., DP = 3m. We are interested to determine the width of the river, which is the length of the side AB of the ΔAPB.

In right ΔADP, ∠A = 30°

So, $\tan 30° = \frac{PD}{AD}$



i.e., $\frac{1}{\sqrt{3}} = \frac{3}{AD}$ or $AD = 3\sqrt{3}$ m

Also, in right ΔPDB,

$$\frac{PD}{DB} = \tan 45° \Rightarrow \frac{3}{DB} = 1$$

∴ DB = 3m

Now, AB = BD + AD = 3 + 3 √3 = 3 (1 + √3) m

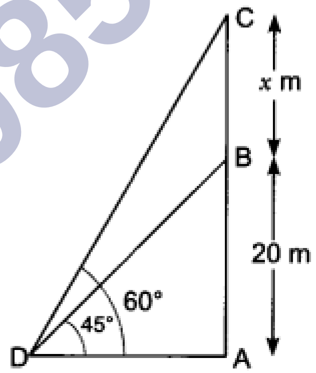
Therefore, the width of the river is 3(√3 + 1) m.

Long Answer :

1. Let AB be a building of height 20 m and BC be the transmission tower of height x m and D be any point on the ground.

Here, ∠BDA = 45° and ∠ADC = 60°

Now, in ΔADC, we have



$$\tan 60° = \frac{AC}{AD} \Rightarrow \sqrt{3} = \frac{x+20}{AD}$$

$$\Rightarrow AD = \frac{x+20}{\sqrt{3}} \quad ...(i)$$

Again, in ΔADB, we have $\tan 45° = \frac{AB}{AD}$

$$\Rightarrow 1 = \frac{20}{AD} \Rightarrow AD = 20 \text{ m} \quad ...(ii)$$

Putting the value of AD in equation (i), we have

$$\Rightarrow 20 = \frac{x+20}{\sqrt{3}} \Rightarrow 20\sqrt{3} = x + 20$$

⇒ x = 20√3 – 20 = 20 (√3 – 1) = 20 (1.732 – 1) = 20 × 0.732 = 14.64 m

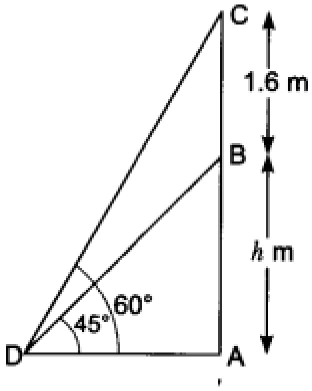
Hence, the height of tower is 14.64 m.

2. Let AB be the pedestal of height h metres and BC be the statue of height 1.6 m.

Let D be any point on the ground such that,

∠BDA = 45° and ∠CDA = 60°

Now, in ΔBDA, we have



$\tan 45° = \frac{AB}{DA} = \frac{h}{DA} \Rightarrow 1 = \frac{h}{DA}$

$\therefore \quad DA = h$ ...(i)

Again in ΔADC, we have

$\tan 60° = \frac{AC}{AD} = \frac{AB + BC}{AD}$

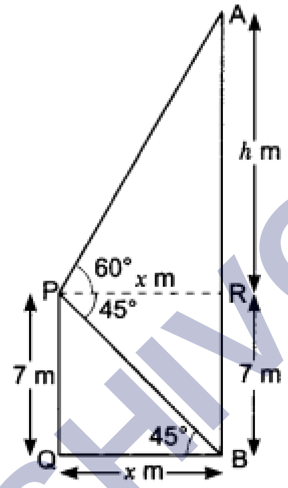
$\Rightarrow \quad \sqrt{3} = \frac{h + 1.6}{h}$ [From equation (i)]

$\Rightarrow \quad \sqrt{3}h = h + 1.6 \quad \Rightarrow \quad (\sqrt{3} - 1)h = 1.6$

$\therefore \quad h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{1.6(\sqrt{3}+1)}{3-1} = \frac{1.6(\sqrt{3}+1)}{2} = 0.8 \times (\sqrt{3}+1)$ m

Hence, height of the pedestal is 0.8 (√3 + 1) m.

3.



Let PQ be the building of height 7 metres and AB be the cable tower. Now it is given that the angle of elevation of the top A of the tower observed from the top P of building is 60° and the angle of depression of the base B of the tower observed from P is 45° (Fig. 11.38).

So, ∠APR = 60° and ∠QBP = 45°

Let QB = x m, AR = h m then, PR = x m

Now, in ΔAPR, we have

$\tan 60° = \frac{AR}{PR}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

⇒ $\sqrt{3}x = h$

⇒ $h = \sqrt{3}x$ ……(i)

Again, in ΔPBQ we have

$\tan 45^\circ = \frac{PQ}{QB}$

⇒ $1 = \frac{7}{x}$

⇒ $x = 7$ ……(ii)

Putting the value of x in equation (i), we have

$h = \sqrt{3} \times 7 = 7\sqrt{3}$

i.e., AR = $7\sqrt{3}$ metres

So, the height of tower = AB = AR + RB = $7\sqrt{3} + 7 = 7(\sqrt{3} + 1)$ m.

4. In the Fig. let AB be the tower, C and D be the positions of observation from where given that

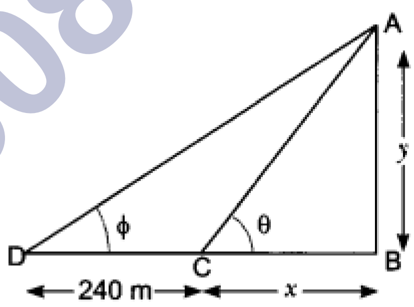
$$\tan \phi = \frac{5}{12} \quad \ldots (i)$$

and $$\tan \theta = \frac{3}{4} \quad \ldots (ii)$$

Let $BC = x$ m, $AB = y$ m

Now in right-angled triangle ABC

$$\tan \theta = \frac{y}{x} \quad \ldots (iii)$$



From (*ii*) and (*iii*), we get $\frac{3}{4} = \frac{y}{x}$

$$\Rightarrow \quad x = \frac{4}{3}y \qquad \ldots(iv)$$

Also in right-angled triangle ABD, we get

$$\tan\phi = \frac{y}{x + 240} \qquad \ldots(v)$$

From (*i*) and (*v*), we get

$$\frac{5}{12} = \frac{y}{x+240} \quad \Rightarrow \quad 12y = 5x + 1200 \qquad \ldots(vi)$$

$$\Rightarrow \quad 12y = 5 \times \frac{4}{3}y + 1200 \qquad \text{(Using } (iv))$$

$$\Rightarrow \quad 12y - \frac{20}{3}y = 1200 \quad \Rightarrow \quad \frac{36y - 20y}{3} = 1200$$

$$\Rightarrow \quad 16y = 3600 \quad \Rightarrow \quad y = \frac{3600}{16} = 225$$

Hence, the height of the tower is 225 metres.

5. Let A and B be two positions of the balloon and G be the point of observation. (eyes of the girl)

Now, we have

AC = BD = BQ – DQ = 88.2 m – 1.2 m = 87 m .

∠AGC = 60°, ∠BGD = 30°

Now, in ΔAGC, we have

$$\tan 60° = \frac{AC}{GC} \quad \Rightarrow \quad \sqrt{3} = \frac{87}{GC}$$

$$\Rightarrow \quad CG = \frac{87}{\sqrt{3}} = \frac{87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{87 \times \sqrt{3}}{3}$$

$$\Rightarrow \quad GC = 29 \times \sqrt{3} \qquad \ldots(i)$$

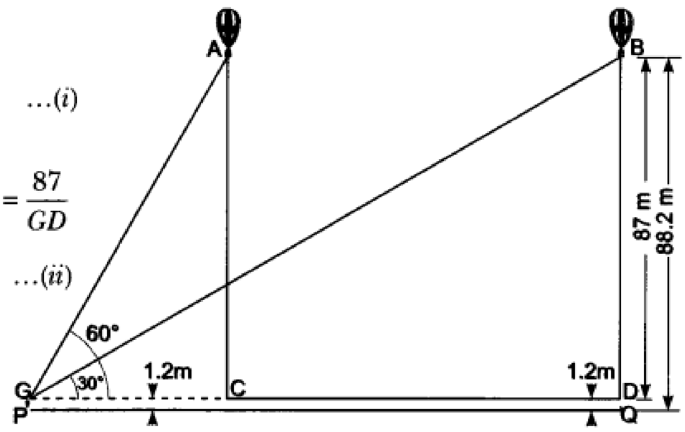
Again, in ΔBGD we have

$$\tan 30° = \frac{BD}{GD} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{87}{GD}$$

$$\Rightarrow \quad GD = 87 \times \sqrt{3} \qquad \ldots(ii)$$

From (*i*) and (*ii*), we have

$$CD = 87 \times \sqrt{3} - 29 \times \sqrt{3}$$
$$= \sqrt{3}\,(87 - 29) = 58\sqrt{3}$$



Hence, the balloon travels 58 √3 metres.

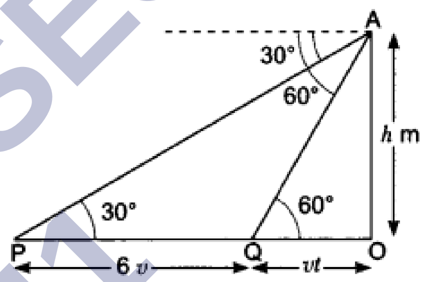
6. Let OA be the tower of height h, and P be the initial position of the car when the angle of depression is 30°.

After 6 seconds, the car reaches to such that the angle of depression at Q is 60°. Let the speed of the car be v metre per second. Then,

PQ = 6υ (∵ Distance = speed × time)

and let the car take t seconds to reach the tower OA from Q (Fig. 11.41). Then, OQ = υt metres.

Now, in ΔAQO, we have



$$\tan 60° = \frac{OA}{QO}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{vt} \quad \Rightarrow \quad h = \sqrt{3}\,vt \quad \text{...(i)}$$

Now, in ΔAPO, we have

$$\tan 30° = \frac{OA}{PO}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{6v + vt} \quad \Rightarrow \quad \sqrt{3}\,h = 6v + vt \quad \text{...(ii)}$$

Now, substituting the value of h from (*i*) into (*ii*), we have

$$\sqrt{3} \times \sqrt{3}\,vt = 6v + vt$$

$$\Rightarrow \quad 3vt = 6v + vt \quad \Rightarrow \quad 2vt = 6v \quad \Rightarrow \quad t = \frac{6v}{2v} = 3$$

Hence, the car will reach the tower from Q in 3 seconds.

7. We have,

AP = 1.8 m

AJ = JK = KP = 0.6 m

AK = 2AJ = 1.2 m

In ΔARJ and ΔBNJ' we have

AJ = BJ, ∠ARJ = ∠BNJ = 60°

and ∠AJR = ∠BJ'N = 90°

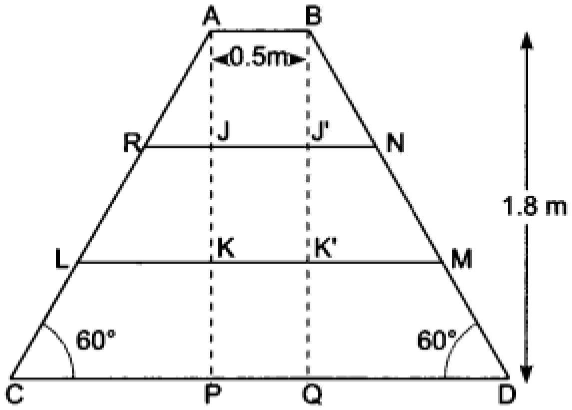
∴ ΔARJ ≅ ΔBNJ

⇒ RJ = NJ (By AAS congruence criterion)

Similarly, ΔALK ≅ ΔBMK"

⇒ LK = MK"

In ΔARJ,



$$\tan 60^\circ = \frac{AJ}{RJ}$$

$$\Rightarrow \quad \sqrt{3} = \frac{0.6}{RJ} \quad \Rightarrow \quad RJ = \frac{0.6}{\sqrt{3}} = \frac{0.6\sqrt{3}}{3} = 0.2 \times 1.732 = 0.3464 \text{ m}$$

In ΔALK,

$$\tan 60^\circ = \frac{AK}{LK} \quad \Rightarrow \quad \sqrt{3} = \frac{1.2}{LK}$$

$$\Rightarrow \quad LK = \frac{1.2}{\sqrt{3}} = \frac{1.2 \times \sqrt{3}}{3} = 0.4 \times 1.732 \text{ m} = 0.6928 \text{ m}$$

Since ΔACP ≅ ΔBDQ

So, BD = AC = 2.0784 m

Now, RN = RJ + JJ + J'N

= 2RJ + AB [∵ RJ = J' N and JJ = AB]

= 2 × 0.3464 +0.5 = 1.1928 m

Length of step LM = LK + KK + KM

= 2LK + AB [∵ LK = K M and KK = AB]

= 2 × 0.6928 + 0.5 = 1.8856 m

Thus, length of each leg = 2.0784 m = 2.1 m

Length of step RN = 1.1928 m = 1.2 m

and, length of step LM = 1.8856 m = 1.9 m

8.

In ΔACP, $\sin 60° = \dfrac{AP}{AC}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{1.8}{AC} \quad \Rightarrow \quad AC = \frac{3.6}{\sqrt{3}} = \frac{3.6\times\sqrt{3}}{3} = 1.2 \times 1.732 = 2.0784 \text{ m}$$

Let AB and CD be two poles of equal height h metre and let P be any point between the poles, such that

∠APB = 60° and ∠DPC = 30°

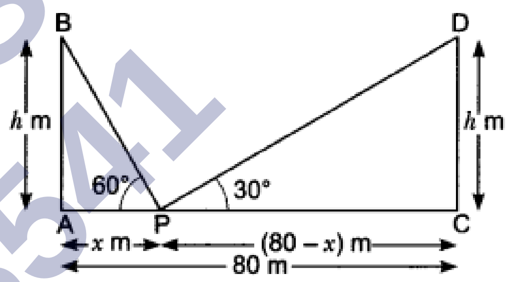
The distance between two poles is 80m.(Given)

Let AP = x m, then PC = (80 – x) m.

h'm Now, in ΔAPB, we have

$$\tan 60° = \frac{AB}{AP} = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad \Rightarrow \quad h = \sqrt{3}x \quad ...(i)$$



Again in ΔCPD, we have

$$\tan 30° = \frac{DC}{PC} = \frac{h}{(80-x)}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{80-x} \quad \Rightarrow \quad h = \frac{80-x}{\sqrt{3}} \quad ...(ii)$$

From (*i*) and (*ii*), we have

$$\sqrt{3}x = \frac{80-x}{\sqrt{3}} \quad \Rightarrow \quad 3x = 80 - x \quad \Rightarrow \quad 4x = 80 \quad \Rightarrow \quad x = \frac{80}{4} = 20 \text{ m}$$

Now, putting the value of x in equation (i), we have

h = √3 × 20 = 20 √3

Hence, the height of the pole is 20 √3 m and the distance of the point from first pole is 20 m and that of the second pole is 60 m.

9. Let height of the tower be h metres and width of the canal be x metres, so AB = h m and BC = x m

Now in ΔABC, we have

$\tan 60° = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$ ...(i)

Now, in ΔADB we have

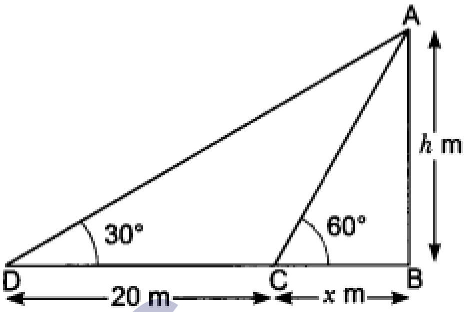
$\tan 30° = \frac{AB}{DB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20 + x}$

$20 + x = \sqrt{3}h$ ...(ii)

From (i) and (ii), we have

$20 + x = \sqrt{3} \times \sqrt{3}x \Rightarrow 20 + x = 3x$

$\Rightarrow 20 = 3x - x = 2x \Rightarrow x = \frac{20}{2} = 10$ m



Now, putting the value of x in equation (i), we have

h = √3 × 10 = 10√3

⇒ h = 10 73 m

Hence, height of the tower is 10 √3 m and width of the canal is 10 m.

10. Let AB be the tree of height metres standing on the bank of a river. Let C be the position of man standing on the opposite bank of the river such that BC = x m. Let D be the new position of the man. It is given that CD = 40 m and the angles of elevation of the top of the tree at C and D are 60° and 30°, respectively, i.e.,

∠ACB = 60° and ∠ADB = 30°.

In ΔACB, we have

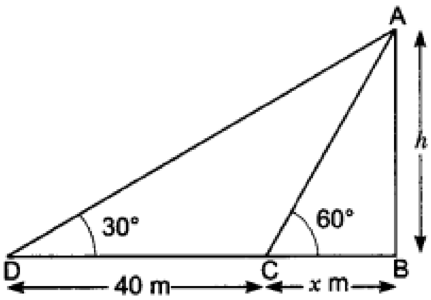
$\tan 60° = \frac{AB}{BC} \Rightarrow \tan 60° = \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(i)

In ΔADB, we have

$\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x + 40}$

$\Rightarrow \sqrt{3}h = x + 40$ ...(ii)



Substituting $x = \frac{h}{\sqrt{3}}$ in equation (*ii*), we get

$$\sqrt{3}h = \frac{h}{\sqrt{3}} + 40 \quad \Rightarrow \quad \sqrt{3}h - \frac{h}{\sqrt{3}} = 40$$

$$\Rightarrow \quad \frac{3h - h}{\sqrt{3}} = 40 \quad \Rightarrow \quad \frac{2h}{\sqrt{3}} = 40$$

$$\Rightarrow \quad h = \frac{40 \times \sqrt{3}}{2} \quad \Rightarrow \quad h = 20\sqrt{3} = 20 \times 1.732 = 34.64 \text{ m}$$

Substituting h in equation (*i*), we get $x = \frac{20\sqrt{3}}{\sqrt{3}}$ metres $=$ 20 metres

Hence, the height of the tree is 34.64 m and width of the river is 20 m.

Case Study Answers:

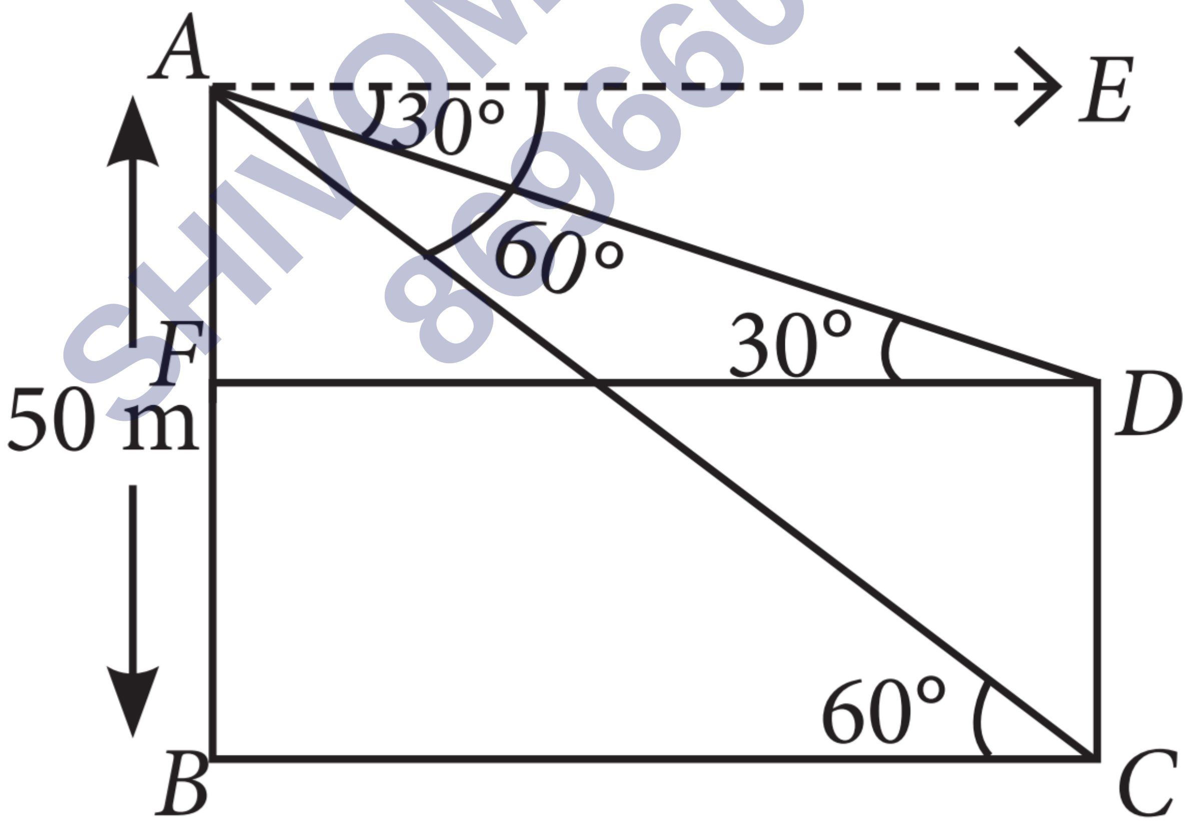
1. Answer :

i. (c) 30°

Solution:

Since, $AE || FD$

$\therefore \angle EAD = \angle ACB = 30^\circ$



ii. (b) 60º

Solution:

Since, $AE||BC$

$\therefore \angle EAC = \angle ACB = 60^\circ$

iii. (a) 28.90m

Solution:

In $\triangle ABC,$

$\tan 60^\circ = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{50}{BC}$

$\Rightarrow BC = \frac{50}{\sqrt{3}} = 28.90m$

iv. (c) 33.33m

Solution:

In $\triangle ADF, \tan 30^\circ = \frac{AF}{FD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB-BF}{FD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{50-CD}{\frac{50}{\sqrt{3}}}$

$\left[\because FD = BC = \frac{50}{\sqrt{3}}\right]$

$\Rightarrow \frac{50}{3} = 50 - CD$

$\Rightarrow CD = 50 - \frac{50}{3} = \frac{100}{3} = 33.33m$

v. (d) An acute angle.

2. Answer :

Given, side of square top = 2m

∴ Given, side of square top = 2m

Also, AC and BD are perpendicular to the ground.Also,

So, AH= HQ = QC (By B.P.T. Theorem)

i. (b) 6.93m

Solution:

In $\triangle AEC$,

$$\sin 60^\circ = \frac{AC}{AE} \Rightarrow \frac{\sqrt{3}}{2} = \frac{6}{AE}$$

$$\Rightarrow AE = 6.93m$$

∴ Length of each leg i.e., AE = BF = 6.93m.

ii. (c) 1.15m

Solution:

In $\triangle AGH, \tan 60^\circ = \frac{AH}{GH} \Rightarrow \sqrt{3} = \frac{2}{GH}$

⇒ GH = 1.15m

iii. (a) 4.3m

Solution:

Length of second step = GH + HT + TU

= 1.15 + 2 + 1.15 = 4.3m

iv. (b) 2.31m

Solution:

In $\triangle APQ$,

$$\tan 60^\circ = \frac{AQ}{PQ} \Rightarrow \sqrt{3} = \frac{4}{PQ}$$

$$\Rightarrow PQ = \frac{4}{\sqrt{3}} m = 2.31m$$

v. (c) 6.62m

Solution:

Length of first step = PQ + QR + RS

= 2.31 + 2 + 2.31 = 6.62m

Assertion Reason Answer-

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.