

MATHEMATICS

Chapter 7: Triangles



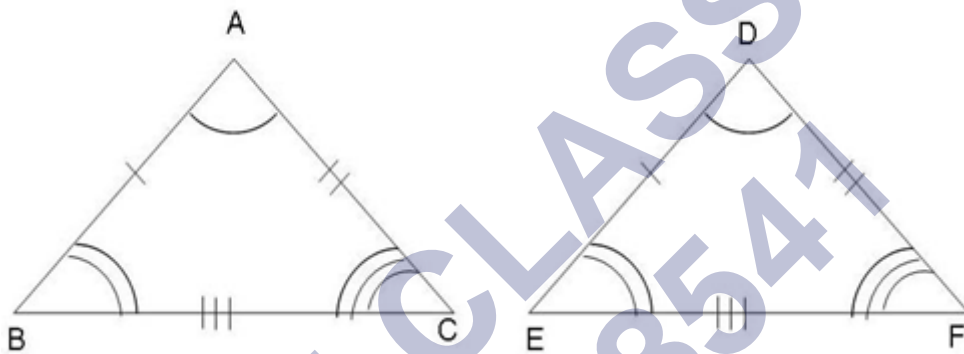
Triangles

What are congruent figures

- Two figures are said to be **congruent** if they are of the same shape and of the same size.
- Two circles of the same radii are congruent.
- Two squares of the same sides are congruent.

Congruent triangles

If two triangles ABC and DEF are congruent under the correspondence $A \leftrightarrow D$, $B \leftrightarrow E$ and $C \leftrightarrow F$, then symbolically, it is expressed as $\triangle ABC \cong \triangle DEF$.

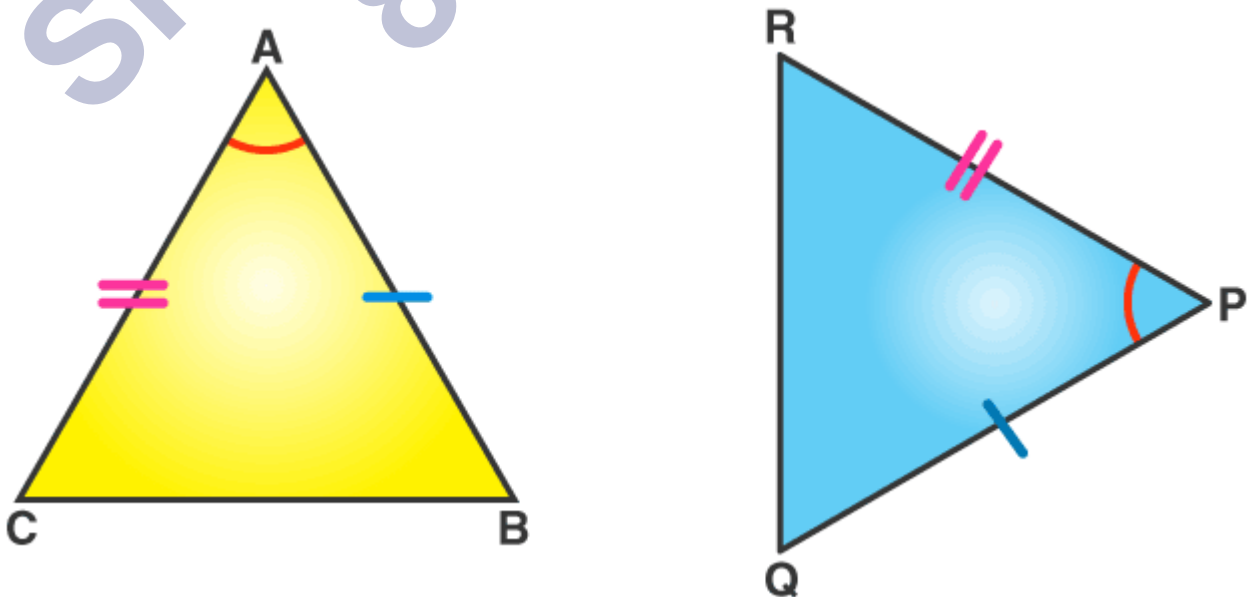


In congruent triangles, **corresponding parts are equal**. We write in short 'CPCT' for corresponding parts of congruent triangles.

SAS (Side – Angle – Side) congruence rule

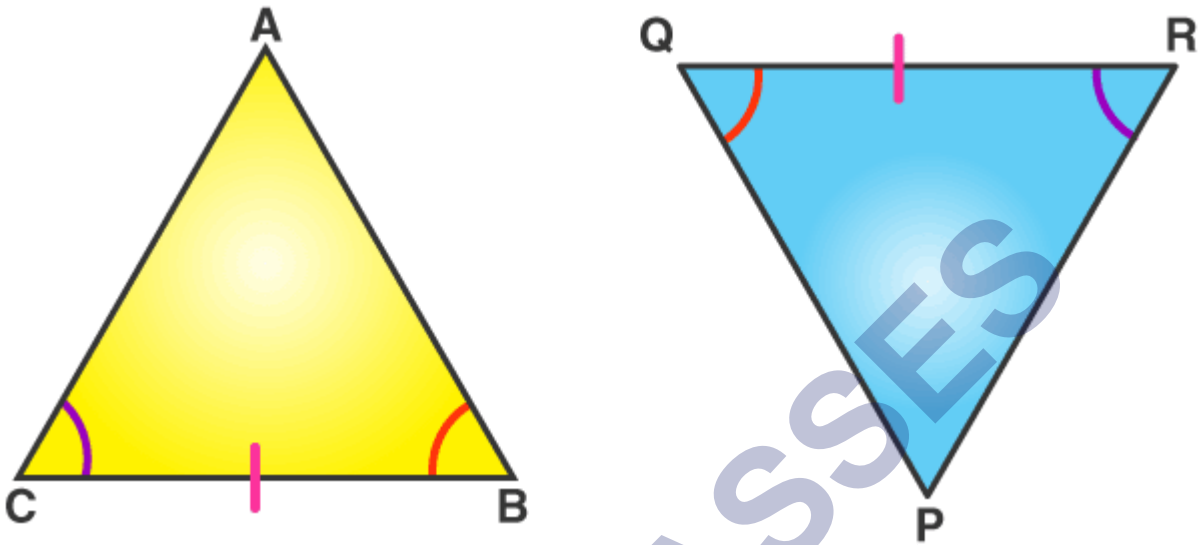
Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.

Note: SAS congruence rule holds but not ASS or SSA rule.



ASA (Angle – Side – Angle) congruence rule

Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.

**AAS (Angle – Angle – Side) congruence rule**

Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

AAS congruency can be proved in easy steps. Suppose we have two triangles ABC and DEF, where,

$\angle B = \angle E$ [Corresponding angles] $\angle C = \angle F$ [Corresponding angles] And

$AC = DF$ [Adjacent sides]

By angle sum property of triangle, we know that;

$$\angle A + \angle B + \angle C = 180 \dots\dots\dots(1)$$

$$\angle D + \angle E + \angle F = 180 \dots\dots\dots(2)$$

From equation 1 and 2 we can say;

$$\angle A + \angle B + \angle C = \angle D + \angle E + \angle F$$

$$\angle A + \angle E + \angle F = \angle D + \angle E + \angle F \text{ [Since, } \angle B = \angle E \text{ and } \angle C = \angle F] \angle A = \angle D$$

Hence, in triangle ABC and DEF,

$$\angle A = \angle D$$

$$AC = DF$$

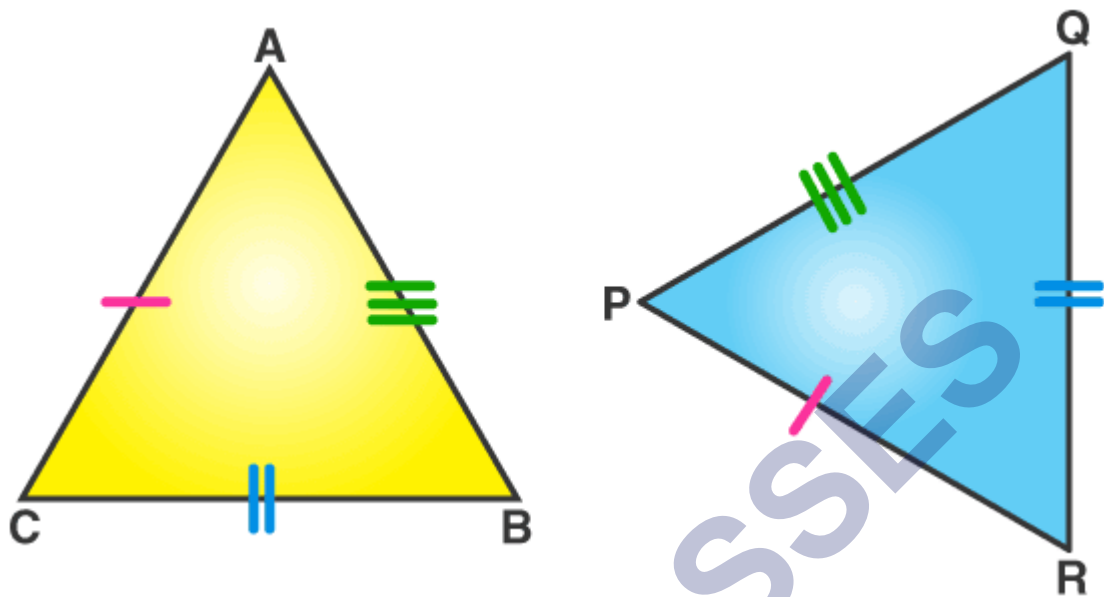
$$\angle C = \angle F$$

Hence, by ASA congruency,

$$\Delta ABC \cong \Delta DEF$$

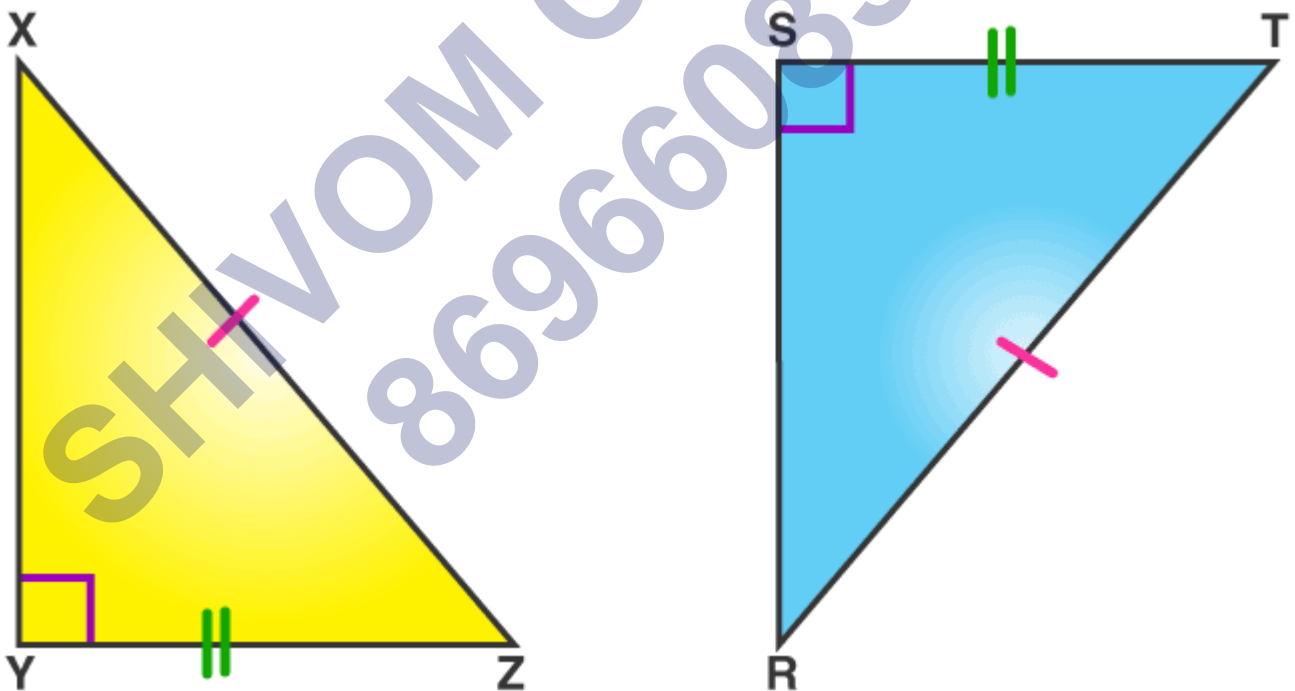
SSS (Side – Side – Side) congruent rule

If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.



RHS (Right Angle – Hypotenuse – Side) congruence rule

If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.



In above figure, hypotenuse $XZ = RT$ and side $YZ = ST$, hence triangle $XYZ \cong$ triangle RST .

Congruence of Triangles

Congruence of triangles: Two triangles are said to be congruent if all three corresponding sides are equal and all the three corresponding angles are equal in measure. These triangles can be slides, rotated, flipped and turned to be looked identical. If repositioned, they coincide with each other. The symbol of congruence is ' \cong '.

The corresponding sides and angles of congruent triangles are equal. There are basically four congruency rules that prove if two triangles are congruent. But it is necessary to find all six dimensions. Hence, the congruence of triangles can be evaluated by knowing only three values out of six. The meaning of congruence in Maths is when two figures are similar to each other based on their shape and size. Also, learn about Congruent Figures here.

Congruence is the term used to define an object and its mirror image. Two objects or shapes are said to be congruent if they superimpose on each other. Their shape and dimensions are the same. In the case of geometric figures, line segments with the same length are congruent and angles with the same measure are congruent.

CPCT is the term, we come across when we learn about the congruent triangle. Let's see the condition for triangles to be congruent with proof.

Congruent meaning in Maths

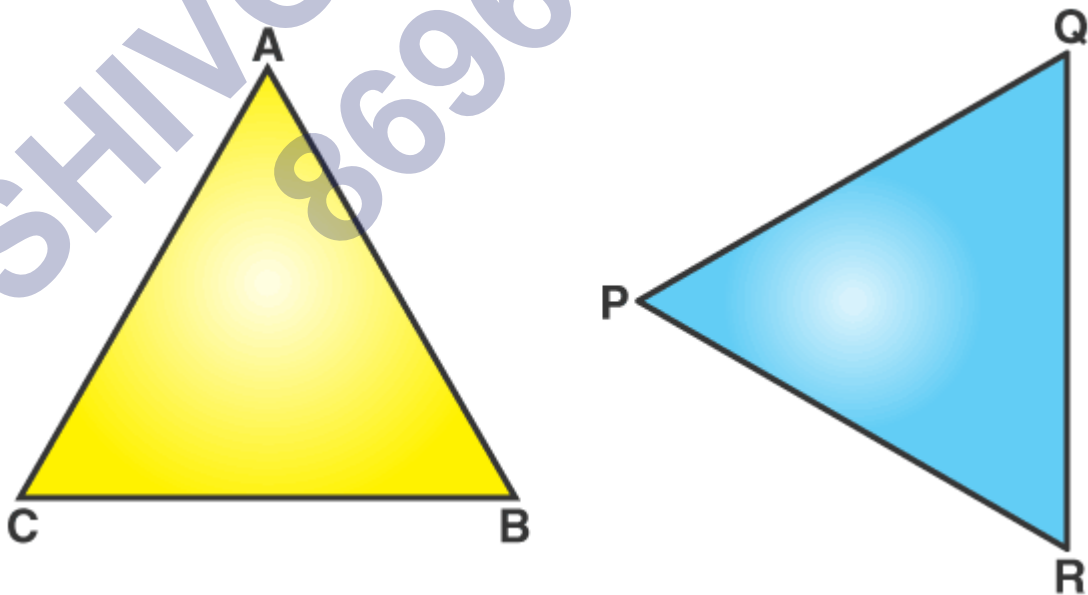
The meaning of congruent in Maths is addressed to those figures and shapes that can be repositioned or flipped to coincide with the other shapes. These shapes can be reflected to coincide with similar shapes.

Two shapes are congruent if they have the same shape and size. We can also say if two shapes are congruent, then the mirror image of one shape is same as the other.

Congruent Triangles

A polygon made of three line segments forming three angles is known as a Triangle.

Two triangles are said to be congruent if their sides have the same length and angles have same measure. Thus, two triangles can be superimposed side to side and angle to angle.



In the above figure, ΔABC and ΔPQR are congruent triangles. This means,

Vertices: A and P, B and Q, and C and R are the same.

Sides: $AB=PQ$, $QR=BC$ and $AC=PR$;

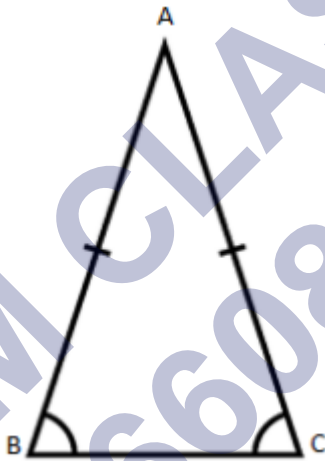
Angles: $\angle A = \angle P$, $\angle B = \angle Q$, and $\angle C = \angle R$.

Congruent triangles are triangles having corresponding sides and angles to be equal. Congruence is denoted by the symbol " \cong ". They have the same area and the same perimeter.

Isosceles triangle and its properties

- A triangle in which two sides are equal is called an **isosceles** triangle.
- Angles opposite to equal sides of an isosceles triangle are equal.
- The sides opposite to equal angles of a triangle are equal.

An isosceles triangle definition states it as a polygon that consists of two equal sides, two equal angles, three edges, three vertices and the sum of internal angles of a triangle equal to 1800. In this section, we will discuss the properties of isosceles triangle along with its definitions and its significance in Maths.



Apart from the isosceles triangle, there is a different classification of triangles depending upon the sides and angles, which have their own individual properties as well. Below is the list of types of triangles;

- Scalene Triangle
- Equilateral Triangle
- Acute angled Triangle
- Right angle Triangle
- Obtuse-angled Triangle

Isosceles triangle basically has two equal sides and angles opposite to these equal sides are also equal. Same like the Isosceles triangle, scalene and equilateral are also classified on the basis of their sides, whereas acute-angled, right-angled and obtuse-angled triangles are defined on the basis of angles. So before, discussing the properties of isosceles triangles, let us discuss first all the types of triangles.

Below are basic definitions of all types of triangles:

Scalene Triangle: A triangle which has all the sides and angles, unequal.

Equilateral Triangle: A triangle whose all the sides are equal and all the three angles are of 60° .

Acute Angled Triangle: A triangle having all its angles less than right angle or 90° .

Right Angled Triangle: A triangle having one of the three angles as right angle or 90° .

Obtuse Angled Triangle: A triangle having one of the three angles as more than right angle or 90° .

Isosceles Triangle Properties

An Isosceles Triangle has the following properties:

- Two sides are congruent to each other.
- The third side of an isosceles triangle which is unequal to the other two sides is called the base of the isosceles triangle.
- The two angles opposite to the equal sides are congruent to each other. That means it has two congruent base angles and this is called an isosceles triangle base angle theorem.
- The angle which is not congruent to the two congruent base angles is called an apex angle.
- The altitude from the apex of an isosceles triangle bisects the base into two equal parts and also bisects its apex angle into two equal angles.
- The altitude from the apex of an isosceles triangle divides the triangle into two congruent right-angled triangles.
- Area of Isosceles triangle = $\frac{1}{2} \times \text{base} \times \text{altitude}$
- Perimeter of Isosceles triangle = sum of all the three sides

Example: If an isosceles triangle has lengths of two equal sides as 5 cm and base as 4 cm and an altitude are drawn from the apex to the base of the triangle. Then find its area and perimeter.

Solution: Given the two equal sides are of 5 cm and base is 4 cm.

We know, the area of Isosceles triangle = $\frac{1}{2} \times \text{base} \times \text{altitude}$

Therefore, we have to first find out the value of altitude here.

The altitude from the apex divides the isosceles triangle into two equal right angles and bisects the base into two equal parts. Thus, by Pythagoras theorem,

$$\text{Hypotenuse}^2 = \text{Base}^2 + \text{Perpendicular}^2$$

$$\text{Or Perpendicular} = \sqrt{\text{Hypotenuse}^2 - \text{Base}^2}$$

$$\therefore \text{Altitude} = \sqrt{5^2 - 2^2}$$

$$= \sqrt{25 - 4}$$

$$= \sqrt{21}$$

So, the area of Isosceles triangle = $\frac{1}{2} \times 4 \times \sqrt{21} = 2\sqrt{21} \text{ cm}^2$

Perimeter of Isosceles triangle = sum of all the sides of the triangle

$$= 5 + 4 + 5 \text{ cm}$$

$$= 14 \text{ cm}$$

Inequalities in a triangle

- If two sides of a triangle are unequal, the **angle opposite to the longer side is greater**.
- In any triangle, the **side opposite to greater (larger) angle is longer**.
- The **sum of any two sides** of a triangle is **greater than the third side**.
- The **difference between any two sides** of a triangle is **less than the third side**.

Relationship between unequal sides of the triangle and the angles opposite to it.

If 2 sides of a triangle are unequal, then the angle opposite to the longer side will be larger than the angle opposite to the shorter side.

Triangle inequality

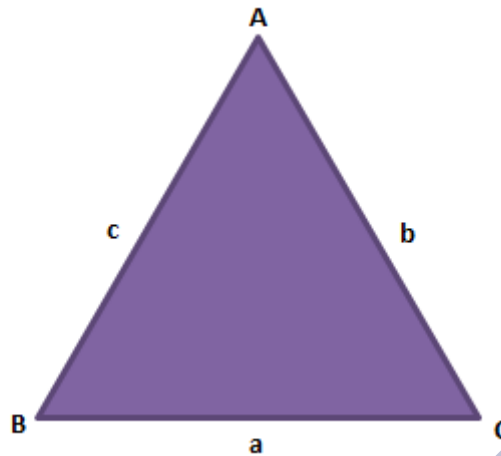
The sum of the lengths of any two sides of a triangle must be greater than the third side.

According to triangle inequality theorem, for any given triangle, the sum of two sides of a triangle is always greater than the third side. A polygon bounded by three line-segments is known as the Triangle. It is the smallest possible polygon. A triangle has three sides, three vertices, and three interior angles. The types of triangles are based on its angle measure and length of the sides. The inequality theorem is applicable for all types triangles such as equilateral, isosceles and scalene. Now let us learn this theorem in details with its proof.

Triangle Inequality Theorem Proof

The triangle inequality theorem describes the relationship between the three sides of a triangle. According to this theorem, for any triangle, the sum of lengths of two sides is always greater than the third side. In other words, this theorem specifies that the shortest distance between two distinct points is always a straight line.

Consider a ΔABC as shown below, with a, b and c as the side lengths.



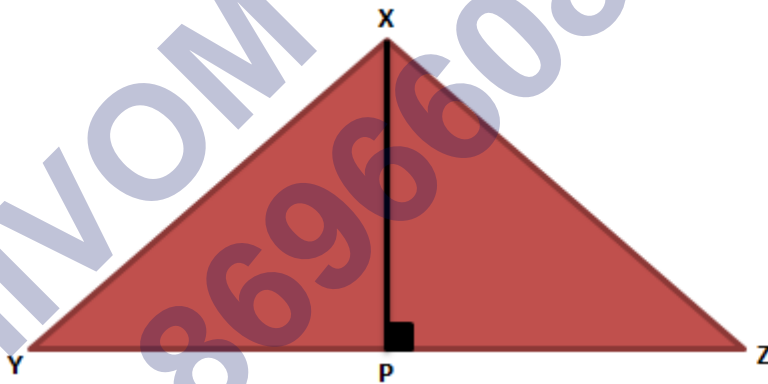
The triangle inequality theorem states that:

$$a < b + c,$$

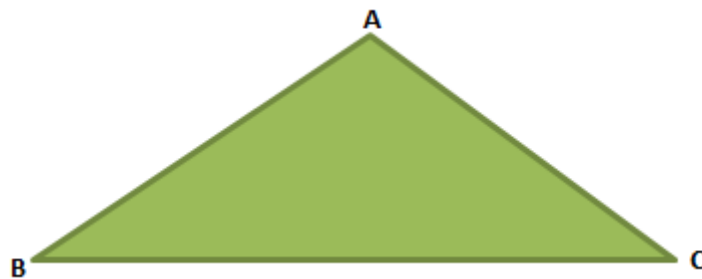
$$b < a + c,$$

$$c < a + b$$

In any triangle, the shortest distance from any vertex to the opposite side is the Perpendicular. In figure below, XP is the shortest line segment from vertex X to side YZ.

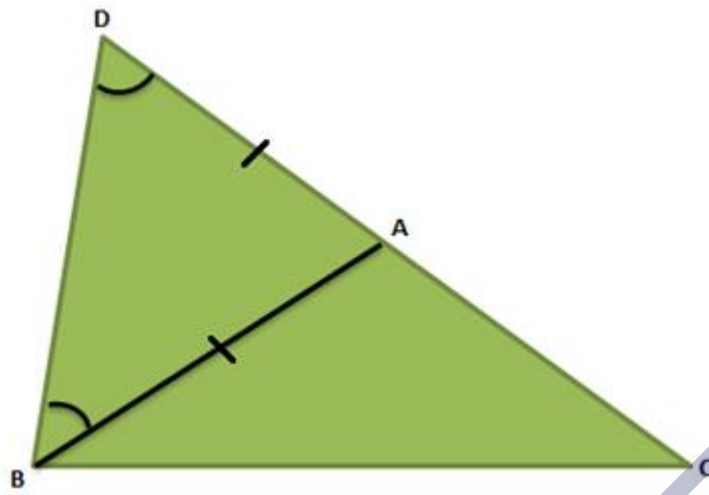


Let us prove the theorem now for a triangle ABC.



To Prove: $|BC| < |AB| + |AC|$

Construction: Consider a ΔABC . Extend the side AC to a point D such that $AD = AB$ as shown in the fig. below.



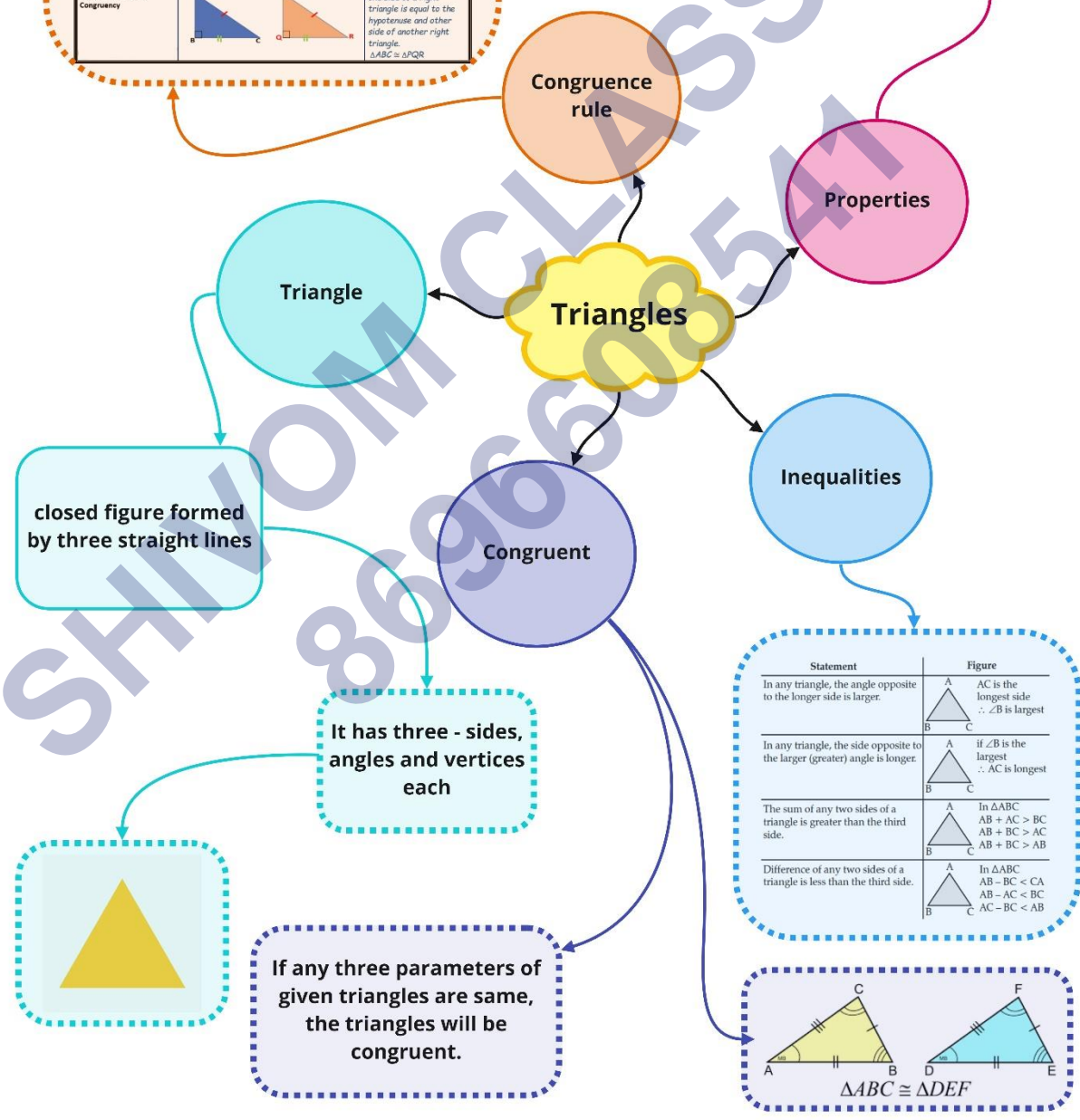
Proof of triangle inequality theorem

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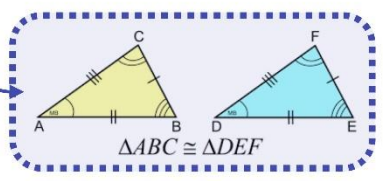
Class : 9th mathematics
Chapter- 7: Triangles

Congruent Triangles		
SSS (Side – Side – Side) Congruency		When the three sides of a triangle are equal to the other three sides of another triangle. $\Delta ABC \cong \Delta PQR$
SAS (Side – Angle – Side) Congruency		When two sides and the included angle of one triangle are equal to the two sides and included angle of another triangle. $\Delta ABC \cong \Delta PQR$
ASA (Angle – Side – Angle) Congruency		When two angles and the included side of one triangle are equal to two angles and the included side of another triangle. $\Delta ABC \cong \Delta PQR$
AAS (Angle – Angle – Side) Congruency		When two angles and the non-included side of one triangle are equal to two angles and the non-included side of another triangle. $\Delta ABC \cong \Delta PQR$
RHS (Right Angle – Hypotenuse – Side) Congruency		When hypotenuse and one side of a right triangle is equal to the hypotenuse and other side of another right triangle. $\Delta ABC \cong \Delta PQR$

Statement	Figure
Angles opposite to equal side of an isosceles triangle are equal	 AB = AD $\angle B = \angle C$
The sides opposite to equal angles of a triangle are equal	 $\angle BAC = \angle CAD$ $\angle ADB = \angle ADC$ $\Delta ABD \cong \Delta ACD$ (ASA rule) Hence, AB = AC



Statement	Figure
In any triangle, the angle opposite to the longer side is larger.	 AC is the longest side $\therefore \angle B$ is largest
In any triangle, the side opposite to the larger (greater) angle is longer.	 if $\angle B$ is the largest $\therefore AC$ is longest
The sum of any two sides of a triangle is greater than the third side.	 In ΔABC AB + AC > BC AB + BC > AC AB + AC > AB
Difference of any two sides of a triangle is less than the third side.	 In ΔABC AB - BC < CA AB - AC < BC AC - BC < AB



Important Questions

Multiple Choice Questions-

Question 1. $\triangle ABC = \triangle PQR$, then which of the following is true?

- (a) $CB = QP$
- (b) $CA = RP$
- (c) $AC = RQ$
- (d) $AB = RP$

Question 2. In $\triangle ABC$ and $\triangle DEF$, $AB = DE$ and $\angle A = \angle D$. Then two triangles will be congruent by SA axiom if:

- (a) $BC = EF$
- (b) $AC = EF$
- (c) $AC = DE$
- (d) $BC = DE$

Question 3. In a right triangle, the longest side is:

- (a) Perpendicular
- (b) Hypotenuse
- (c) Base
- (d) None of the above

Question 4. In $\triangle ABC$, if $\angle A = 45^\circ$ and $\angle B = 70^\circ$, then the shortest and the longest sides of the triangle are respectively,

- (a) BC, AB
- (b) AB, AC
- (c) AB, BC
- (d) BC, AC

Question 5. If the altitudes from vertices of a triangle to the opposite sides are equal, then the triangle is

- (a) Scalene
- (b) Isosceles
- (c) Equilateral
- (d) Right-angled

Question 6. D is a Point on the Side BC of a $\triangle ABC$ such that AD bisects $\angle BAC$ then:

- (a) $BD = CD$
- (b) $CD > CA$

(c) $BD > BA$

(d) $BA > BD$

Question 7. If $\triangle ABC \cong \triangle PQR$ then which of the following is true:

(a) $CA = RP$

(b) $AB = RP$

(c) $AC = RQ$

(d) $CB = QP$

Question 8. If two triangles ABC and PQR are congruent under the correspondence $A \leftrightarrow P$, $B \leftrightarrow Q$, and $C \leftrightarrow R$, then symbolically, it is expressed as

(a) $\triangle ABC \cong \triangle PQR$

(b) $\triangle ABC = \triangle PQR$

(c) $\triangle ABC$ and $\triangle PQR$ are scalene triangles

(d) $\triangle ABC$ and $\triangle PQR$ are isosceles triangles

Question 9. If the bisector of the angle A of an $\triangle ABC$ is perpendicular to the base BC of the triangle then the triangle ABC is:

(a) Obtuse Angled

(b) Isosceles

(c) Scalene

(d) Equilateral

Question 10.

If $AB = QR$, $BC = RP$ and $CA = QP$, then which of the following holds?

(a) $\triangle BCA \cong \triangle PQR$

(b) $\triangle ABC \cong \triangle PQR$

(c) $\triangle CBA \cong \triangle PQR$

(d) $\triangle CAB \cong \triangle PQR$

Very Short:

1. Find the measure of each exterior angle of an equilateral triangle.

2. If in $\triangle ABC$, $\angle A = \angle B + \angle C$, then write the shape of the given triangle.

3. In $\triangle PQR$, $PQ = QR$ and $\angle R = 50^\circ$, then find the measure of $\angle Q$.

4. If $\triangle SKY \cong \triangle MON$ by SSS congruence rule, then write three equalities of corresponding angles.

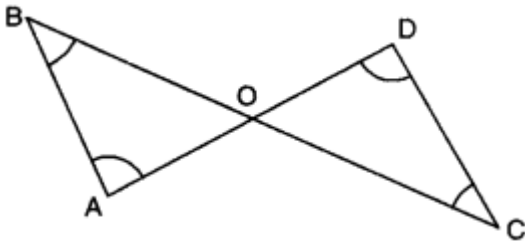
5. Is $\triangle ABC$ possible, if $AB = 6$ cm, $BC = 4$ cm and $AC = 1.5$ cm?

6. In $\triangle MNO$, if $\angle N = 90^\circ$, then write the longest side.

7. In $\triangle ABC$, if $AB = AC$ and $\angle B = 70^\circ$, find $\angle A$.
8. In $\triangle ABC$, if AD is a median, then show that $AB + AC > 2AD$.

Short Questions:

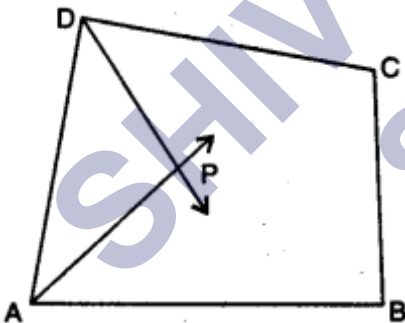
- In the given figure, $AD = BC$ and $BD = AC$, prove that $\angle DAB = \angle CBA$.
- In the given figure, $\triangle ABD$ and $\triangle ABC$ are isosceles triangles on the same base BD . Prove that $\angle ABC = \angle ADC$.
- In the given figure, if $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$, then prove that $BC = CD$.
- In the given figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.



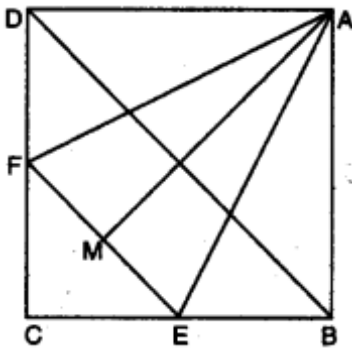
- In the given figure, $AC > AB$ and D is a point on AC such that $AB = AD$. Show that $BC > CD$.
- In a triangle ABC , D is the mid-point of side AC such that $BD = \frac{1}{2} AC$. Show that $\angle ABC$ is a right angle.

Long Questions:

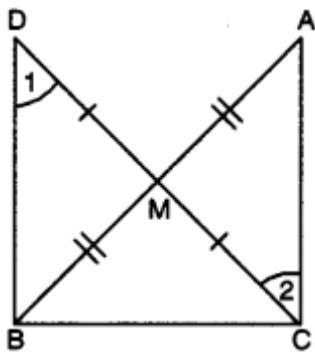
- In the given figure, AP and DP are bisectors of two adjacent angles A and D of quadrilateral $ABCD$. Prove that $2 \angle APD = \angle B + \angle C$.



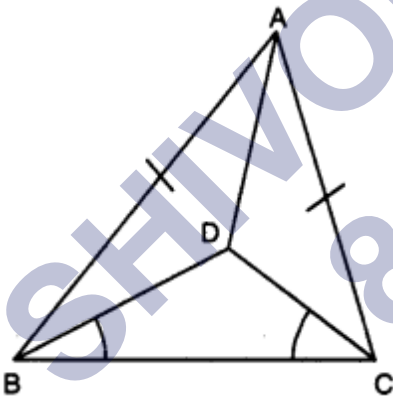
- In figure, $ABCD$ is a square and EF is parallel to diagonal BD and $EM = FM$. Prove that
 - $DF = BE$
 - AM bisects $\angle BAD$.



3. In right triangle ABC, right-angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B (see fig.). Show that : (i) $\triangle AMC \cong \triangle BMD$ (ii) $\angle DBC = 90^\circ$ (iii) $\triangle DBC \cong \triangle ACB$ (iv) $CM = \frac{1}{2} AB$



4. In figure, ABC is an isosceles triangle with $AB = AC$. D is a point in the interior of $\triangle ABC$ such that $\angle BCD = \angle CBD$. Prove that AD bisects $\angle BAC$ of $\triangle ABC$.



5. Prove that two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle.

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

- Assertion and reason both are correct statements and reason is correct explanation for assertion.
- Assertion and reason both are correct statements but reason is not correct explanation for assertion.

- c) Assertion is correct statement but reason is wrong statement.
 d) Assertion is wrong statement but reason is correct statement.

Assertion: If we draw two triangles with angles 30° , 70° and 80° and the length of the sides of one triangle be different than that of the corresponding sides of the other triangle then two triangles are not congruent.

Reason: If two triangles are constructed which have all corresponding angles equal but have unequal corresponding sides, then two triangles cannot be congruent to each other.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

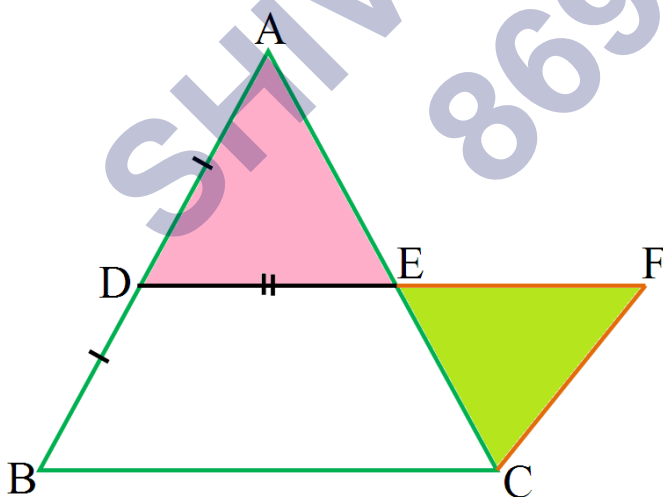
- a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
 b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
 c) Assertion is correct statement but reason is wrong statement.
 d) Assertion is wrong statement but reason is correct statement.

Assertion: If the bisector of the vertical angle of a triangle bisects the base of the triangle, then the triangle is equilateral.

Reason: If three sides of one triangle are equal to three of the other triangle, then the two triangles are congruent.

Case Study Questions-

1. Read the Source/ Text given below and answer these questions:



Hareesh and Deep were trying to prove a theorem. For this they did the following:

- i. Drew a triangle ABC.
- ii. D and E are found as the mid points of AB and AC.
- iii. DE was joined and DE was extended to F so $DE = EF$.

iv. FC was joined.

Answer the following questions:

i. $\triangle ADE$ and $\triangle EFC$ are congruent by which criteria?

- a. SSS
- b. RHS
- c. SAS
- d. ASA

ii. $\angle EFC$ is equal to which angle?

- a. $\angle DAE$
- b. $\angle ADE$
- c. $\angle AED$
- d. $\angle B$

iii. $\angle ECF$ is equal to which angle?

- a. $\angle DAE$
- b. $\angle ADE$
- c. $\angle AED$
- d. $\angle B$

iv. CF is equal to which of the following?

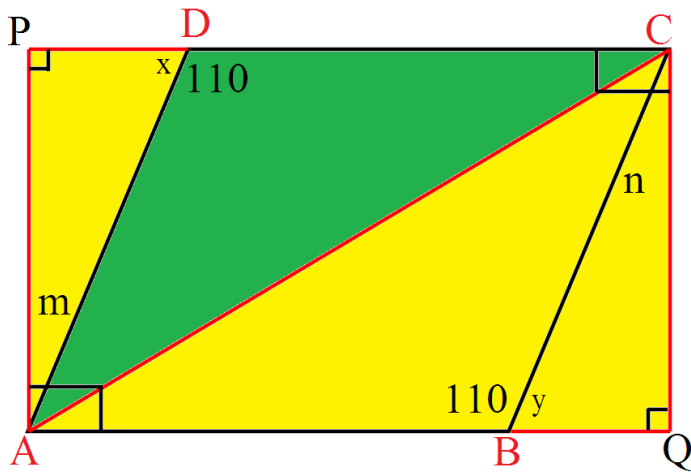
- a. BD
- b. CE
- c. AE
- d. EF

v. CF is parallel to which of the following?

- a. AE
- b. CE
- c. BD
- d. EF

2. Read the Source/ Text given below and answer these questions:

In the middle of the city, there was a park ABCD in the form of a parallelogram form so that $AB = CD$, $AB \parallel CD$ and $AD = BC$, $AD \parallel BC$ Municipality converted this park into a rectangular form by adding land in the form of $\triangle APD$ and $\triangle BCQ$. Both the triangular shape of land were covered by planting flower plants.



Answer the following questions:

- i. What is the value of $\angle x$?
 - a. 110°
 - b. 70°
 - c. 90°
 - d. 100°
- ii. $\triangle APD$ and $\triangle BCQ$ are congruent by which criteria?
 - a. SSS
 - b. SAS
 - c. ASA
 - d. RHS
- iii. PD is equal to which side?
 - a. DC
 - b. AB
 - c. BC
 - d. BQ
- iv. $\triangle ABC$ and $\triangle ACD$ are congruent by which criteria?
 - a. SSS
 - b. SAS
 - c. ASA
 - d. RHS
- v. What is the value of $\angle m$?
 - a. 110°
 - b. 70°
 - c. 90°

d. 20°

Answer Key:

MCQ:

1. (b) $CA = RP$
2. (c) $AC = DE$
3. (b) Hypotenuse
4. (d) BC, AC
5. (b) Isosceles
6. (d) $BA > BD$
7. (a) $CA = RP$
8. (a) $\triangle ABC \cong \triangle PQR$
9. (b) Isosceles
10. (d) $\triangle CAB \cong \triangle PQR$

Very Short Answer:

1. We know that each interior angle of an equilateral triangle is 60° .

$$\therefore \text{Each exterior angle} = 180^\circ - 60^\circ = 120^\circ$$

2. Here, $\angle A = \angle B + \angle C$

And in $\triangle ABC$, by angle sum property, we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle A = 180^\circ$$

$$\Rightarrow 2\angle A = 180^\circ$$

$$\Rightarrow \angle A = 90^\circ$$

Hence, the given triangle is a right triangle.

3. Here, in $\triangle PQR$, $PQ = QR$

$$\Rightarrow \angle R = \angle P = 50^\circ \text{ (given)}$$

$$\text{Now, } \angle P + \angle Q + \angle R = 180^\circ$$

$$50^\circ + \angle Q + 50^\circ = 180^\circ$$

$$\Rightarrow \angle Q = 180^\circ - 50^\circ - 50^\circ$$

$$= 80^\circ$$

4. Since $\triangle SKY \cong \triangle MON$ by SSS congruence rule, then three equalities of corresponding angles

are $\angle S = \angle M$, $\angle K = \angle O$ and $\angle Y = \angle N$.

5. Since $4 + 1.5 = 5.5 \neq 6$

Thus, triangle is not possible.

6. We know that, side opposite to the largest angle is longest.

\therefore Longest side = MO.

7. Here, in $\triangle ABC$ $AB = AC$ $\angle C = \angle B$ [\angle s opp. to equal sides of a \triangle]

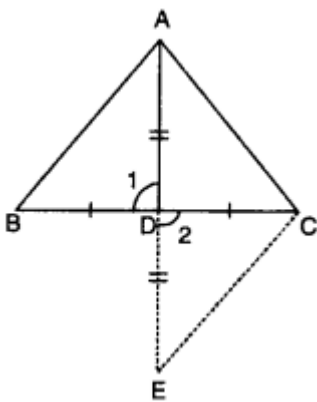
Now, $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle A + 70^\circ + 70^\circ = 180^\circ$

[$\because \angle B = 70^\circ$]

$\Rightarrow \angle A = 180^\circ - 70^\circ - 70^\circ = 40^\circ$

8.



Produce AD to E, such that $AD = DE$.

In $\triangle ADB$ and $\triangle EDC$, we have

$BD = CD$, $AD = DE$ and $\angle 1 = \angle 2$

$\triangle ADB \cong \triangle EDC$

$AB = CE$

Now, in $\triangle AEC$, we have

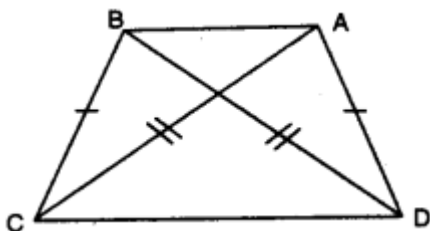
$AC + CE > AE$

$AC + AB > AD + DE$

$AB + AC > 2AD$ [$\because AD = DE$]

Short Answer:

Ans: 1.



In $\triangle DAB$ and $\triangle CBA$, we have

$$AD = BC \text{ [given]}$$

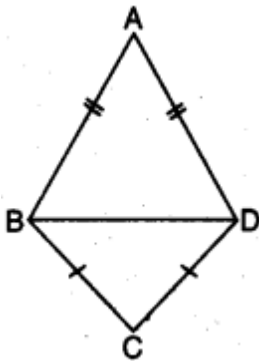
$$BD = AC \text{ [given]}$$

$$AB = AB \text{ [common]}$$

$$\therefore \triangle DAB \cong \triangle CBA \text{ [by SSS congruence axiom]}$$

Thus, $\angle DAB = \angle CBA$ [c.p.c.t.]

Ans: 2.



In $\triangle ABD$, we have

$$AB = AD \text{ (given)}$$

$$\angle ABD = \angle ADB \text{ [angles opposite to equal sides are equal] ... (i)}$$

In $\triangle BCD$, we have

$$CB = CD$$

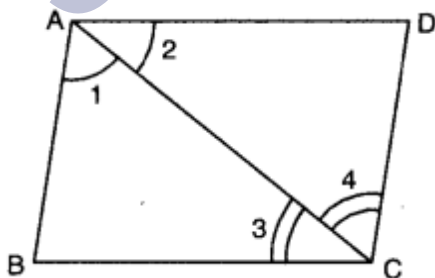
$$\Rightarrow \angle CBD = \angle CDB \text{ [angles opposite to equal sides are equal] ... (ii)}$$

Adding (i) and (ii), we have

$$\angle ABD + \angle CBD = \angle ADB + \angle CDB$$

$$\Rightarrow \angle ABC = \angle ADC$$

Ans: 3.



In $\triangle ABC$ and $\triangle CDA$, we have

$$\angle 1 = \angle 2 \text{ (given)}$$

$$AC = AC \text{ [common]}$$

$$\angle 3 = \angle 4 \text{ [given]}$$

So, by using ASA congruence axiom

$$\triangle ABC \cong \triangle CDA$$

Since corresponding parts of congruent triangles are equal

$$\therefore BC = CD$$

Ans: 4. In $\triangle ABC$ and $\triangle CDA$, we have

$$\angle 1 = \angle 2 \text{ (given)}$$

$$AC = AC \text{ [common]}$$

$$\angle 3 = \angle 4 \text{ [given]}$$

So, by using ASA congruence axiom

$$\triangle ABC \cong \triangle CDA$$

Since corresponding parts of congruent triangles are equal

$$\therefore BC = CD$$

Ans: 5. Here, in $\triangle ABD$, $AB = AD$

$$\angle ABD = \angle ADB$$

[\angle s opp. to equal sides of a \triangle]

In $\triangle BAD$

$$\text{ext. } \angle BDC = \angle BAD + \angle ABD$$

$$\Rightarrow \angle BDC > \angle ABD \dots \text{(ii)}$$

Also, in $\triangle BDC$.

$$\text{ext. } \angle ADB > \angle CBD \dots \text{(iii)}$$

From (ii) and (iii), we have

$$\angle BDC > \angle CDB \text{ [}\because \text{ sides opp. to greater angle is larger]}$$

Ans: 6. Here, in $\triangle ABC$, D is the mid-point of AC.

$$\Rightarrow AD = CD = \frac{1}{2} AC \dots \text{(i)}$$

$$\text{Also, } BD = \frac{1}{2} AC \dots \text{(ii) [Given]}$$

From (i) and (ii), we obtain

$$AD = BD \text{ and } CD = BD$$

$$\Rightarrow \angle 2 = \angle 4 \text{ and } \angle 1 = \angle 3 \dots \text{(iii)}$$

In $\triangle ABC$, we have

$$\angle ABC + \angle ACB + \angle CAB = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 + \angle 1 + \angle 2 = 180^\circ \text{ [using (iii)]}$$

$$\Rightarrow 2(\angle 1 + \angle 2) = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 90^\circ$$

$$\text{Hence, } \angle ABC = 90^\circ$$

Long Answer:

Ans: 1. Here, AP and DP are angle bisectors of $\angle A$ and $\angle D$

$$\therefore \angle DAP = \frac{1}{2} \angle DAB \text{ and } \angle ADP = \frac{1}{2} \angle ADC \dots\dots(i)$$

$$\text{In } \triangle APD, \angle APD + \angle DAP + \angle ADP = 180^\circ$$

$$\Rightarrow \angle APD + \frac{1}{2} \angle DAB + \frac{1}{2} \angle ADC = 180^\circ$$

$$\Rightarrow \angle APD = 180^\circ - \frac{1}{2} (\angle DAB + \angle ADC)$$

$$\Rightarrow 2\angle APD = 360^\circ - (\angle DAB + \angle ADC) \dots\dots(ii)$$

$$\text{Also, } \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\angle B + 2\angle C = 360^\circ - (\angle A + \angle D)$$

$$\angle B + \angle C = 360^\circ - (\angle DAB + \angle ADC) \dots\dots(iii)$$

From (ii) and (iii), we obtain

$$2\angle APD = \angle B + \angle C$$

Ans: 2. (i) $EF \parallel BD$ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [corresponding \angle s]

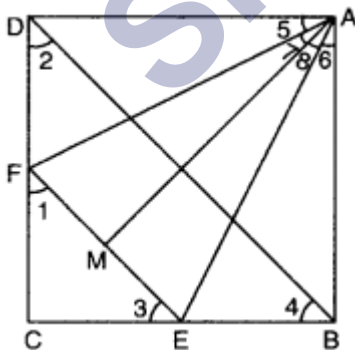
$$\text{Also, } \angle 2 = \angle 4$$

$$\Rightarrow \angle 1 = \angle 3$$

$$\Rightarrow CE = CF \text{ (sides opp. to equal } \angle\text{s of a } \triangle\text{)}$$

$$\therefore DF = BE$$

$$[\because BC - CE = CD - CF]$$



(ii) In $\triangle ADF$ and $\triangle ABE$, we have

$$AD = AB \text{ [sides of a square]}$$

$$DF = BE \text{ [proved above]}$$

$$\angle D = \angle B = 90^\circ$$

$\Rightarrow \triangle ADF \cong \triangle ABE$ [by SAS congruence axiom]

$\Rightarrow AF = AE$ and $\angle 5 = \angle 6 \dots$ (i) [c.p.c.t.]

In $\triangle AMF$ and $\triangle AME$

$AF = AE$ [proved above]

$AM = AM$ [common]

$FM = EM$ (given)

$\therefore \triangle AMF \cong \triangle AME$ [by SSS congruence axiom]

$\therefore \angle 7 = \angle 8 \dots$ (ii) [c.p.c.t.]

Adding (i) and (ii), we have

$$\angle 5 + \angle 7 = \angle 6 + \angle 8$$

$$\angle DAM = \angle BAM$$

$\therefore AM$ bisects $\angle BAD$.

Ans: 3. Given: $\triangle ACB$ in which $\angle C = 90^\circ$ and M is the mid-point of AB .

To Prove:

(i) $\triangle AMC \cong \triangle BMC$

(ii) $\angle DBC = 90^\circ$

(iii) $\triangle DBC \cong \triangle ACB$

(iv) $CM = \frac{1}{2} AB$

Proof: Consider $\triangle AMC$ and $\triangle BMC$,

we have $AM = BM$ [given]

$CM = CM$ [by construction]

$\angle AMC = \angle BMC$ [vertically opposite angles]

$\therefore \triangle AMC \cong \triangle BMC$ [by SAS congruence axiom]

$\Rightarrow AC = BC \dots$ (i) [by c.p.c.t.]

and $\angle 1 = \angle 2$ [by c.p.c.t.]

But $\angle 1$ and $\angle 2$ are alternate angles.

$\Rightarrow BD \parallel CA$

Now, $BD \parallel CA$ and BC is transversal.

$\therefore \angle ACB + \angle CBD = 180^\circ$

$\Rightarrow 90^\circ + \angle CBD = 180^\circ$

$\Rightarrow \angle CBD = 90^\circ$

In $\triangle DBC$ and $\triangle ACB$,

we have $CB = BC$ [common]

$DB = AC$ [using (i)]

$\angle CBD = \angle BCA$

$\therefore \triangle DBC \cong \triangle ACB$

$\Rightarrow DC = AB$

$\Rightarrow \frac{1}{2} AB = \frac{1}{2} DC$

$\Rightarrow \frac{1}{2} AB = CM$ Or $CM = \frac{1}{2} AB$ ($\because CM = \frac{1}{2} DC$)

Ans: 4. In $\triangle BDC$, we have $\angle DBC = \angle DCB$ (given).

$\Rightarrow CD = BD$ (sides opp. to equal \angle s of $\triangle BDC$)

Now, in $\triangle ABD$ and $\triangle ACD$,

we have $AB = AC$ [given]

$BD = CD$ [proved above]

$AD = AD$ [common]

\therefore By using SSS congruence axiom, we obtain

$\triangle ABD \cong \triangle ACD$

$\Rightarrow \angle BAD = \angle CAD$ [c.p.c.t.]

Hence, AD bisects $\angle BAC$ of $\triangle ABC$.

Ans: 5.



Given: Two \triangle s ABC and DEF in which

$\angle B = \angle E$,

$\angle C = \angle F$ and $BC = EF$

To Prove: $\triangle ABC = \triangle DEF$

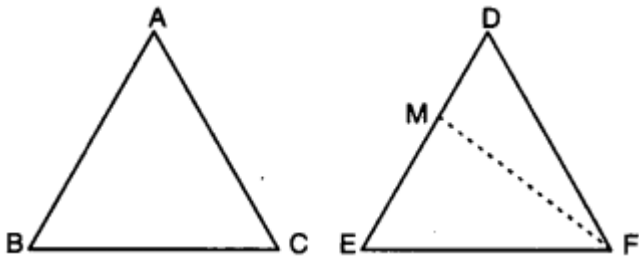
Proof: We have three possibilities

Case I. If $AB = DE$,

we have $AB = DE$,

$\angle B = \angle E$ and $BC = EF$.

So, by SAS congruence axiom, we have $\triangle ABC \cong \triangle DEF$



Case II. If $AB < ED$, then take a point M on ED such that $EM = AB$.

Join MF .

Now, in $\triangle ABC$ and $\triangle MEF$,

we have

$AB = ME$, $\angle B = \angle E$ and $BC = EF$.

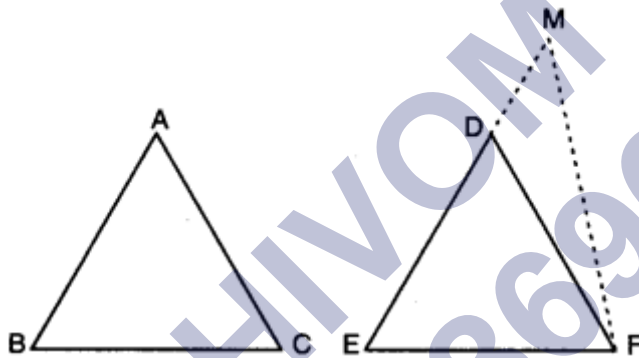
So, by SAS congruence axiom,

we have $\triangle ABC \cong \triangle MEF$

$\Rightarrow \angle ACB = \angle MFE$

But $\angle ACB = \angle DFE$

$\therefore \angle MFE = \angle DFE$



Which is possible only when FM coincides with DF i.e., M coincides with D .

Thus, $AB = DE$

\therefore In $\triangle ABC$ and $\triangle DEF$, we have

$AB = DE$,

$\angle B = \angle E$ and $BC = EF$

So, by SAS congruence axiom, we have

$\triangle ABC \cong \triangle DEF$

Case III. When $AB > ED$

Take a point M on ED produced

such that $EM = AB$.

Join MF

Proceeding as in Case II, we can prove that

$$\triangle ABC = \triangle DEF$$

Hence, in all cases, we have

$$\triangle ABC = \triangle DEF.$$

Assertion and Reason Answers-

1. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
2. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Case Study Answers-

1.

(i)	(c)	SAS
(ii)	(b)	$\angle ADE \angle ADE$
(iii)	(a)	$\angle DAE \angle DAE$
(iv)	(a)	BD
(v)	(c)	BD

2.

(i)	(b)	70°
(ii)	(c)	ASA
(iii)	(d)	BQ
(iv)	(a)	SSS
(v)	(d)	20°