# MATHEMATICS

## Chapter 7: Cubes and Cube Roots

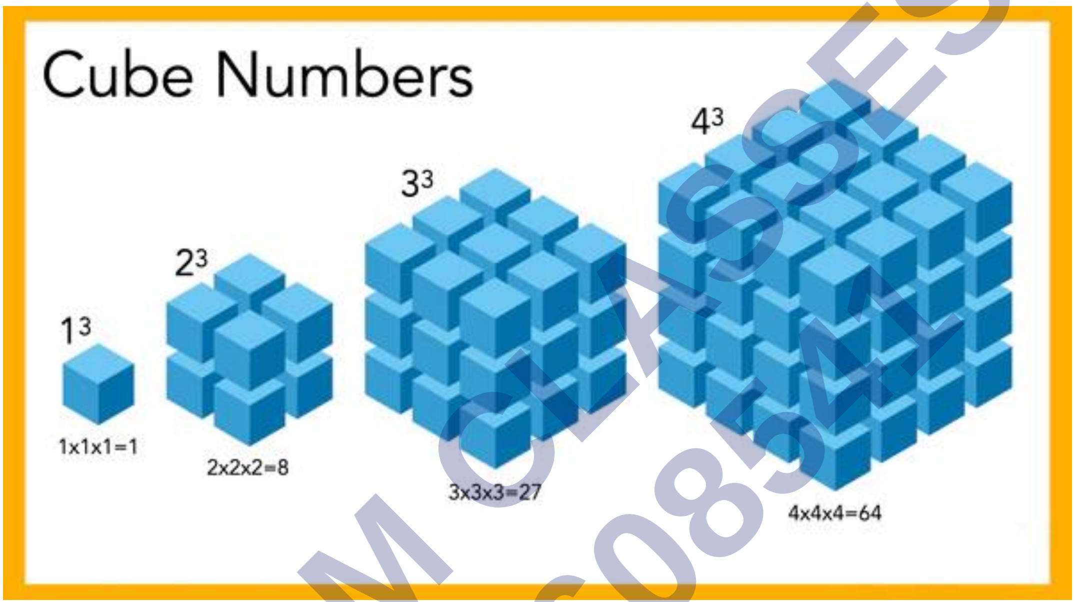
# Cubes and Cube Roots

## Cube Numbers

If a natural number m can be expressed as n3, where n is also a natural number, then m is called the cube number of n.

Numbers like 1, 8, 27 are cube number of the numbers 1, 2, and 3 respectively.

All perfect cube numbers are obtained by multiplying a number by itself three times.






**Cubes Relation with Cube Numbers**

In geometry, a cube is a solid figure where all edges are equal and are perpendicular to each other.

For example, take a cube of unit side. If we arrange these cubes to form a bigger cube of side 3 units, we find that there are a total of 27 such unit cubes that make up a cube of 3 units. Similarly, a cube of 4 units will have 64 such unit cubes.

**Units Digits in Cube Numbers**

Depending on whether a number is odd or even, its cube number is also odd or even respectively.

This is determined by the nature of the cube numbers' unit digit.

- If a number is odd, its cube numbers' unit digit is also odd.
- If a number is even, its cube numbers' unit digit is also even.

The table below shows the units digit of a number and the units digit of the cube of that number:

| Units digit of number | Units digit of its cube |
|---|---|
| 1 | 1 |
| 2 | 8 |
| 3 | 7 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |
| 7 | 3 |
| 8 | 2 |
| 9 | 9 |

## Inside Cube Numbers

**Adding Consecutive Odd Numbers**

$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = 27 = 3^3$

$13 + 15 + 17 + 19 = 64 = 4^3$

$21 + 23 + 25 + 27 + 29 = 125 = 5^3$

We can see from the above pattern, if we need to find the $n^3$, n consecutive odd numbers will be needed, such that their sum is equal to $n^3$.

This pattern holds true for all natural numbers.

Also, if we need to find n3 then we should add n consecutive natural numbers starting from $\left(\frac{\frac{(n-1)(n)}{2}}{1}\right)^{th}$ odd natural number.

## Prime Factorization Method to Find a Cube

In the prime factorization of any number, if each prime factor appears three times, then the number is a perfect cube.

Consider, the number 216. By prime factorization,

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3 = 6^3$

Hence, 216 is a perfect cube.

Consider, the number 500. By prime factorization,

$500 = 2 \times 2 \times 5 \times 5 \times 5 = 2^2 \times 5^3$

In the above prime factorization 2 appears twice.

Hence, 500 is not a perfect cube.

**Cube Roots**

Finding the cube root is the inverse operation of finding the cube.

We know that $3^3 = 27$. We can also write the same equation as $\sqrt[3]{27}=3$. The symbol ‘3√‘ denotes ‘cube root‘.

**Cube Root of 2**

Let us consider another example of number 2. Since 2 is not a perfect cube number. It is not easy to find the cube root of 2. With the help of the long division method, it is possible to find the cube roots for non-perfect cube numbers. The approximate value of the $\sqrt[3]{2}$ is 1.260.

We can estimate the $\sqrt[3]{2}$ by using the trick here.

Since, 2 = 1 x 1 x 2

Cube root of 2 is approximately equal to (1 + 1+2)/3 = 4/3 = 1.333..

**Cube root of 4**

Again 4 is a number, which is not a perfect cube. If we factorise it, we get:

4 = 2 x 2 x 1

Hence, we can see, we cannot find the cube root by simple factorisation here.

Again, if we use the shortcut method, we get:

$\sqrt[3]{4}$ is equal to (2+2+1)/3 = 1.67

The actual value of $\sqrt[3]{4}$ is 1.587, which is approximately equal to 1.67.

Cubes and Cube Roots List of 1 to 15

| Number | Cube($a^3$) | Cube root $\sqrt[3]{a}$ |
|---|---|---|
| 1 | 1 | 1.000 |
| 2 | 8 | 1.260 |
| 3 | 27 | 1.442 |
| 4 | 64 | 1.587 |
| 5 | 125 | 1.710 |
| 6 | 216 | 1.817 |
| 7 | 343 | 1.913 |
| 8 | 512 | 2.000 |
| 9 | 729 | 2.080 |
| 10 | 1000 | 2.154 |
| 11 | 1331 | 2.224 |
| 12 | 1728 | 2.289 |
| 13 | 2197 | 2.351 |
| 14 | 2744 | 2.410 |
| 15 | 3375 | 2.466 |



**Cube Root of 64**

Since 64 is a perfect cube of 4, therefore, it is easy to find its cube-root by the prime factorisation method.

64 = 2 x 2 x 2 x 2 x 2 x 2

$\sqrt[3]{64} = \sqrt[3]{(2 \times 2 \times 2 \times 2 \times 2 \times 2)}$

= 2 x 2

= 4

**Cube Root of 216**

Since, 216 is perfect cube of 6, hence we can find the cube root of 216 by factorisation.

216 = 2 x 2 x 2 x 3 x 3 x 3

$\sqrt[3]{216} = \sqrt[3]{(2 \times 2 \times 2 \times 3 \times 3 \times 3)}$

$\sqrt[3]{216} = 2 \times 3$

$\sqrt[3]{216} = 6$

**Cube Root of 343**

Let us find the cube root of 343 with the help of the the prime factorisation method.

Dividing 343 by smallest prime factor, till we get the remainder as 1. Follow the below steps;

| | |
|---|---|
| 7 | 343 |
| 7 | 49 |
| 7 | 7 |
| | 1 |

Therefore, 343 = 7 × 7 × 7

And, $\sqrt[3]{343} = 7$

**Cube Root of 512**

To find the cube root of 512 we have to factorise it first.

The prime factorisation of 512 can be written as:

512 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

Taking the cube roots both the sides, we get;

$\sqrt[3]{512} = \sqrt[3]{(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)}$

$\sqrt[3]{512} = 2 \times 2 \times 2$

$\sqrt[3]{512} = 8$

**Cube Root of 729**

Now, let's find the cubic root of 729.

| | |
|---|---|
| 3 | 729 |
| 3 | 243 |
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

Cube root of 729

729 = 3 × 3 × 3 × 3 × 3 × 3 = 9 × 9 × 9

Therefore, the cube root of 729 i.e. $\sqrt[3]{729} = 9$

## Cube root using prime factorization

We can find the cube root of a number by prime factorization method by the following steps:

resolve the number into its prime factors. Consider the number 5832. 5832 = (2 × 2 × 2) × (3 × 3 × 3) × (3 × 3 × 3).

make groups of three same prime factors.

take one prime factor from each group and multiply them. Their product is the required cube root.

Therefore, cube root of 5832 = $\sqrt[3]{5832}$ = 2 × 3 × 3 = 18

## Cube Root of a Cube Number using estimation

If a cube number is given we can find out its cube root using the following steps:

Take any cube number say 117649 and start making groups of three starting from the rightmost digit of the number. So 117649 has two groups, and first group(649) and the second group(117).

The unit's digit of the first group (649) will decide the unit digit of the cube root. Since the number 649 ends with 9, the cube roots unit's digit is 9.

Find the cube of numbers between which the second group lies. The other group is 117. We know that $4^3 = 64$ and $5^3 = 125$. 64 < 117 < 125. Take the smaller number between 4 and 5 as the ten's digit of the cube root. So, 49 is the cube root of 117649.

## Differences of Squares of Triangular Numbers and Converse

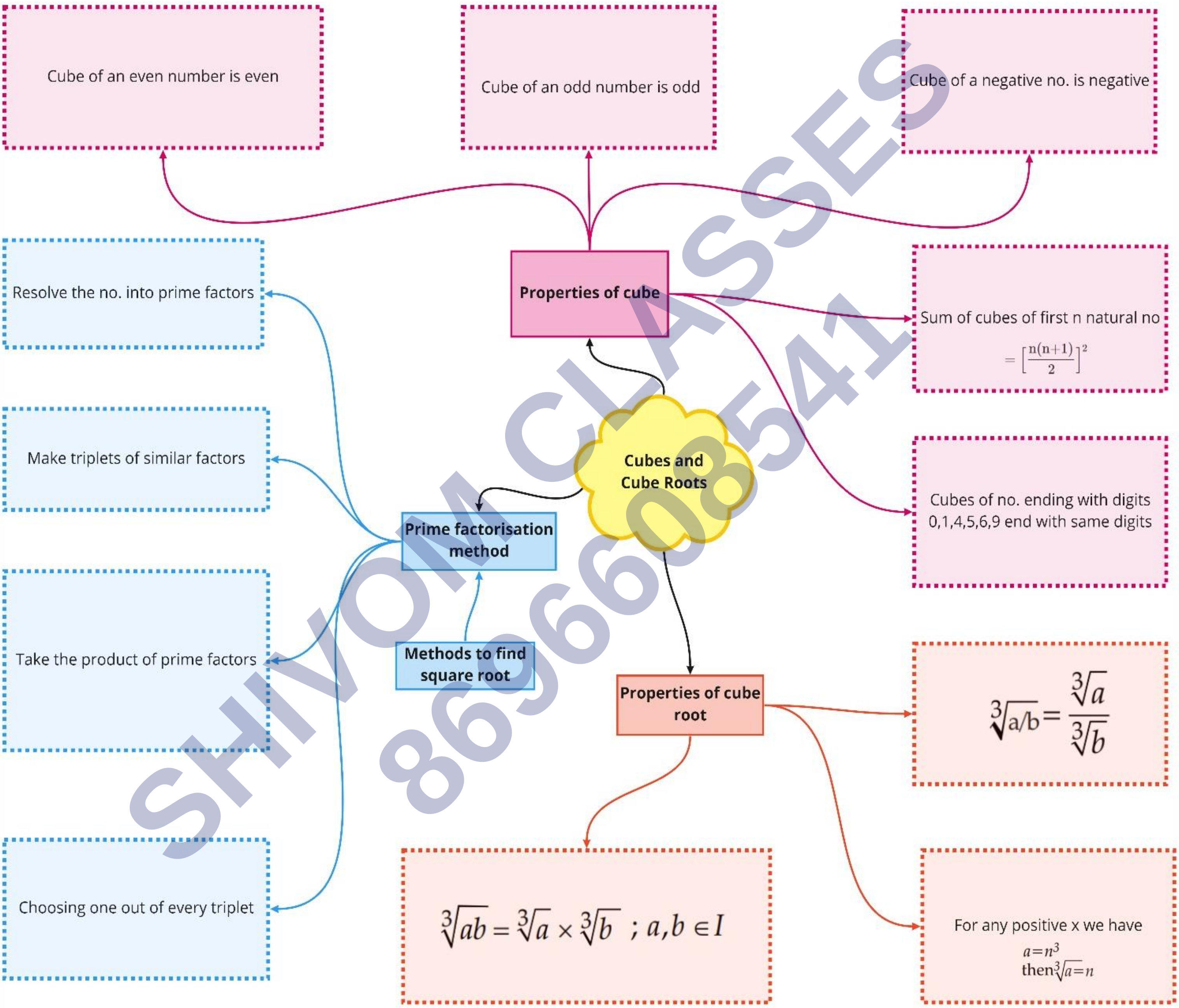
Triangular numbers: It is a sequence of the numbers 1, 3, 6, 10, 15 etc. It is obtained by continued summation of the natural numbers. The dot pattern of a triangular number can be arranged as triangles.

Sum of two consecutive triangular numbers gives us a square number. For example, $1 + 3 = 4 = 2^2$ and $3 + 6 = 9 = 3^2$.

The difference between the squares of two consecutive triangular numbers is a cube number. For example, $3^2 – 1^2 = 9 – 1 = 8 = 2^3$ and $6^2 – 3^2 = 3^6 – 9 = 2^5 = 5^3$.

Also, if the difference between the squares of two numbers is a cube number, then these numbers are consecutive triangular numbers.

Class : 8th Mathematics
Chapter-7 Cubes and Cube Roots



# Important Questions

## Multiple Choice Questions:

Question 1. How many digits will be there in the cube root of 46656?

(a) 2

(b) 1

(c) 3

(d) 4

Question 2. Ones digit of cube of a number depends on the ________ of the number.

(a) tens digit

(b) ones digit

(c) hundred digit

(d) none of these

Question 3. What will be the unit digit of the cube root of a number ends with 8?

(a) 2

(b) 8

(c) 4

(d) 6

Question 4. The symbol for cube root is __________.

(a) √3

(b) $\sqrt[3]{}$

(c) 2√3

(d) 3√3

Question 5. The smallest natural number by which 704 must be divided to obtain a perfect cube is

(a) 22

(b) 12

(c) 11

(d) 13

Question 6. The numbers 1, 8, 27… are ______.

(a) negative numbers

(b) cube numbers

(c) square numbers

(d) none of these

Question 7. If volume of cube is 4913cm³ then length of side of cube is

(a) 16 cm

(b) 17 cm

(c) 18 cm

(d) 19 cm

Question 8. The square of a natural number subtracts from its cube comes 100. The number is ___________.

(a) 2

(b) 3

(c) 5

(d) 1

Question 9. The value of $5^3$ is ___________.

(a) 125

(b) 15

(c) 10

(d) 75

Question 10. If $(2744)^{\frac{1}{3}} = 2p + 2$, then the value of P is

(a) 3

(b) 6

(c) 2

(d) 8

## Very Short Questions:

**1.** Find the cubes of the following:

(a) 12

(b) -6

(c) $\frac{2}{3}$

(d) $\frac{-5}{6}$

**2.** Find the cubes of the following:

(a) 0.3

(b) 0.8

(c) .001

(d) 2 – 0.3

3. Is 135 a perfect cube?

4. Find the cube roots of the following:

(a) 1728

(b) 3375

5. Examine if (i) 200 (ii) 864 are perfect cubes.

6. Find the smallest number by which 1323 may be multiplied so that the product is a perfect cube.

## Short Questions:

1. What is the smallest number by which 2916 should be divided so that the quotient is a perfect cube?

2. Check whether 1728 is a perfect cube by using prime factorisation.

3. Using prime factorisation, find the cube root of 5832.

4. Show that $\sqrt[3]{27} \times \sqrt[3]{125} = \sqrt[3]{27 \times 125}$

5. Simplify: $\sqrt[3]{5 - \frac{10}{27}}$

6. Find the cube roots of

(i) $4\frac{12}{125}$

(ii) -0.729



## Long Questions:

1. Express the following numbers as the sum of odd numbers using the given pattern.

$$5^3 - 4^3 = 1 + \frac{5 \times 4}{2} \times 6 = 61$$

$$7^3 - 6^3 = 1 + \frac{7 \times 6}{2} \times 6 = 127$$

(i) $9^3 - 8^3 =$ ________ = ________

(ii) $12^3 - 11^3 =$ ________ = ________

(iii) $51^3 - 50^3 =$ ________ = ________

2. Observe the following pattern and complete the blank spaces.

13 = 1

$2^3 - 1^3 = 1 + \frac{2\times1}{2}\times6 = 7$

$\therefore \quad 2^3 = 1 + 7 = 8$

$3^3 - 2^3 = 1 + \frac{3\times2}{2}\times6 = 19$

$\therefore \quad 3^3 = 2^3 + 19$

$\Rightarrow \quad 3^3 = 1 + 7 + 19$

(i) $4^4$ = ________ (ii) $6^3$ = ________

(iii) $7^3$ = ________ (iv) $9^3$ = ________

(v) $11^3$ = ________

**3.** **Find the cubes of the following numbers: (i) 7, (ii) 12, (iii) 21, (iv) 100, (v) 302**

**4.** **By what number would you multiply 231525 to make it a perfect cube?**

## Answer Key-

## Multiple Choice Questions:

1. (a) 2
2. (b) ones digit
3. (a) 2
4. (b) $\sqrt[3]{}$
5. (c) 11
6. (b) cube numbers
7. (b) 17 cm
8. (c) 5
9. (a) 125
10. (b) 6

## Very Short Answer:

**1.**

(a) $12^3 = 12 \times 12 \times 12 = 1728$

(b) $(-6)^3 = (-6) \times (-6) \times (-6) = -216$

(c) $\left(\frac{2}{3}\right)^3 = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{8}{27}$

(d) $\left(\frac{-5}{6}\right)^3 = \left(\frac{-5}{6}\right)\times\left(\frac{-5}{6}\right)\times\left(\frac{-5}{6}\right) = \frac{-125}{216}$

**2.** (a) $(0.3)^3 = 0.3 \times 0.3 \times 0.3 = 0.027$

(b) $(0.8)^3 = 0.8 \times 0.8 \times 0.8 = 0.512$

(c) $(0.001)^3 = (0.001) \times (0.001) \times (0.001) = 0.000000001$

(d) $(2 - 0.3)^3 = (1.7)^3 = 1.7 \times 1.7 \times 1.7 = 4.913$

**3.** Prime factorisation of 135, is:

$135 = 3 \times 3 \times 3 \times 5$

We find that on making triplet, the number 5 does not make a group of the triplet.

Hence, 135 is not a perfect cube.

| | |
|---|---|
| 3 | 135 |
| 3 | 45 |
| 3 | 15 |
| 5 | 5 |
| | 1 |

**4.**

(a) Prime factorisation of 1728 is:

$1728 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$= 2^3 \times 2^3 \times 3^3$

$\therefore \sqrt[3]{1728} = 2 \times 2 \times 3 = 12$

| | |
|---|---|
| 2 | 1728 |
| 2 | 864 |
| 2 | 432 |
| 2 | 216 |
| 2 | 108 |
| 2 | 54 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

(b) We find the prime factorisation of 3375 as follows:

$3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

$= 3^3 \times 5^3$

$\therefore \sqrt[3]{3375}$

$= 3 \times 5$

$= 15$

| | |
|---|---|
| 3 | 3375 |
| 3 | 1125 |
| 3 | 375 |
| 5 | 125 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

**5.** (i) $200 = 2 \times 2 \times 2 \times 5 \times 5$

If we form triplet of equal factors, the number 2 forms a group of three whereas 5 does not do it.

Therefore, 200 is not a perfect cube.

| | |
|---|---|
| 2 | 200 |
| 2 | 100 |
| 2 | 50 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

(ii) We have 864 = 2 × 2 × 2 × 2 × 2

If we form triplet of equal factors, the number 2 and 3 form a group of three whereas another group of 2's does not do so.

Therefore, 864 is not a perfect cube.

| | |
|---|---|
| 2 | 864 |
| 2 | 432 |
| 2 | 216 |
| 2 | 108 |
| 2 | 54 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

**6.** 1323 = 3 × 3 × 3 × 7 × 7

Since we required one more 7 to make a triplet of 7.

Therefore 7 is the smallest number by which 1323 may be multiplied to make it a perfect cube.

| | |
|---|---|
| 3 | 1323 |
| 3 | 441 |
| 3 | 147 |
| 7 | 49 |
| 7 | 7 |
| | 1 |

## Short Answer:

**1.** Prime factorisation of

2916 = 2 × 2 × 3 × 3 × 3 × 3 × 3 × 3

Since we required one more 2 to make a triplet

Therefore, the required smallest number by which 2916 should be divided to make it a perfect cube is 2 × 2 = 4, i.e., 2916 ÷ 4 = 729 which is a perfect cube.

| | |
|---|---|
| 2 | 2916 |
| 2 | 1458 |
| 3 | 729 |
| 3 | 243 |
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

**2.** Prime factorisation of 1728 is

1728 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3

Since all prime factors can be grouped in triplets.

Therefore, 1728 is a perfect cube.

**3.**

The prime factorisation of 5832 is

$$5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

Therefore,

$$\sqrt[3]{5832} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$$

$$= 2 \times 3 \times 3 = 18$$

| | |
|---|---|
| 2 | 5832 |
| 2 | 2916 |
| 2 | 1458 |
| 3 | 729 |
| 3 | 243 |
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |



**4.**

$$\text{LHS} = \sqrt[3]{27} \times \sqrt[3]{125}$$

$$= \sqrt[3]{3 \times 3 \times 3} \times \sqrt[3]{5 \times 5 \times 5}$$

$$= \sqrt[3]{3^3} \times \sqrt[3]{5^3} = 3 \times 5 = 15$$

$$\text{RHS} = \sqrt[3]{27 \times 125}$$

$$= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5}$$

$$= \sqrt[3]{3^3 \times 5^3} = 3 \times 5 = 15$$

Hence, LHS = RHS

**5.**

$$\sqrt[3]{5-\frac{10}{27}} = \sqrt[3]{\frac{5\times27-10}{27}}$$

$$= \sqrt[3]{\frac{135-10}{27}} = \sqrt[3]{\frac{125}{27}}$$

$$= \sqrt[3]{\frac{5\times5\times5}{3\times3\times3}} = \sqrt[3]{\frac{5^3}{3^3}} = \frac{5}{3}$$

**6.**

$$(i)\ \sqrt[3]{4\frac{12}{125}} = \sqrt[3]{\frac{4\times125+12}{125}} = \sqrt[3]{\frac{500+12}{125}}$$

$$= \sqrt[3]{\frac{512}{125}}$$

$$= \sqrt[3]{\frac{2\times2\times2\times2\times2\times2\times2\times2\times2}{5\times5\times5}}$$

$$= \sqrt[3]{\frac{2^3\times2^3\times2^3}{5^3}}$$

$$= \frac{2\times2\times2}{5} = \frac{8}{5}$$

$$(ii)\ \sqrt[3]{-0.729} = \sqrt[3]{\frac{-729}{1000}}$$

$$= \sqrt[3]{-\frac{3\times3\times3\times3\times3\times3}{10\times10\times10}}$$

$$= \sqrt[3]{-\frac{3^3\times3^3}{10^3}}$$

$$= -\frac{3\times3}{10} = -\frac{9}{10} = -0.9$$



## Long Answer:

**1.**

$$(i)\ 9^3-8^3 = 1+\frac{9\times8}{2}\times6 = 217$$

$$(ii)\ 12^3-11^3 = 1+\frac{12\times11}{2}\times6 = 397$$

$$(iii)\ 51^3-50^3 = 1+\frac{51\times50}{2}\times6 = 7651$$

**2.**

(ii) $6^3 - 5^3 = 1 + \frac{6 \times 5}{2} \times 6 = 91$

$\therefore \quad 6^3 = 5^3 + 91$

$= 1 + 7 + 19 + 37 + 61 + 91$

(iii) $9^3 = 9^3 - 8^3 = 1 + \frac{9 \times 8}{2} \times 6 = 217$

$\therefore \quad 9^3 = 8^3 + 217$

$= 1 + 7 + 19 + 37 + 61 + 91$

$+ 127 + 169 + 217$

(iv) $11^3 - 10^3 = 1 + \frac{11 \times 10}{2} \times 6 = 331$

$\therefore 11^3 = 10^3 + 331$

$= 1 + 7 + 19 + 37 + 61$

$+ 91 + 127 + 169 + 217$

$+ 271 + 331$

**3.**

(i) $(7)^3 = 7 \times 7 \times 7 = 343$

(ii) $(12)^3 = 12 \times 12 \times 12 = 1728$

(iii) $(21)^3 = 21 \times 21 \times 21 = 9621$

(iv) $(100)^3 = 100 \times 100 \times 100 = 1000000$

(v) $(302)^3 = 302 \times 302 \times 302 = 27543608$

**4.** The prime factorisation of 231525 is 5 × 5 × 3 × 3 × 3 × 7 × 7 × 7.

The number that must be multiplied in order that the above product is a perfect cube is 5.

Therefore, Cube root of 231525 × 5 is 5 × 3 × 7 = 105.