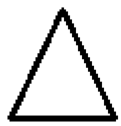
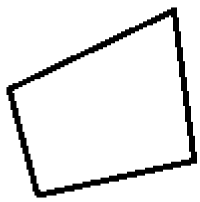
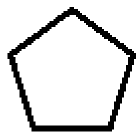
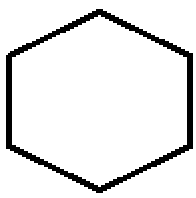
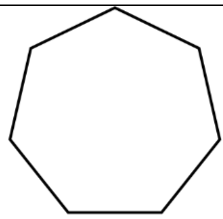
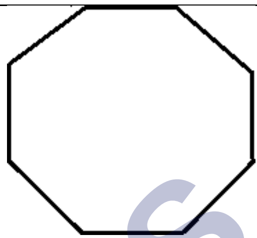
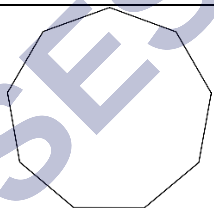
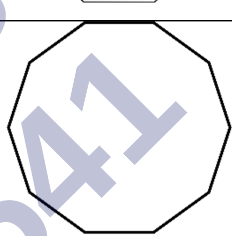
MATHEMATICS

Chapter 3: Understanding Quadrilaterals

Understanding Quadrilaterals

1. A plane figure formed by joining a number of points without lifting a pencil from the paper and without retracing any part of the figure is called a curve.
2. A curve which does not cut itself is called an open curve.
3. A curve which cuts itself is called a closed curve.
4. A simple closed curve is a closed curve which does not pass through one point more than once.
5. A simple closed curve made up of line segments is called a polygon.
6. The line segments that constitute a polygon are known as its sides and their end points are known as the vertices of the polygon.
7. Any two sides with a common end-point (vertex) are called the adjacent sides.
8. The end points of the same side of a polygon are known as the adjacent vertices.
9. The line segment obtained by joining vertices which are not adjacent are called the diagonals of the polygon.
10. Classification of polygons according to the number of sides:

Number of Sides or vertices	Classification	Figure
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	

7	Heptagon	
8	Octagon	
9	Nonagon	
10	Decagon	
N	n-gon	n-gon

11. A polygon having all sides equal and all angles equal is called a regular polygon. Polygons which are not regular are called irregular polygons.

12. A regular polygon is both equiangular and equilateral.

13. A polygon in which at least one angle is more than 180° is called a concave polygon. A polygon in which each angle is less than 180° is called a convex polygon.

14. A polygon having all sides equal and all angles equal is called a regular polygon. Polygons which are not regular are called irregular polygons.

15. For a regular polygon of n sides:

i. each exterior angle = $\left(\frac{360^{\circ}}{n}\right)$

ii. each interior angle = 180° – (each exterior angle).

16. For a convex polygon of n sides:

i. Sum of all exterior angles = 4 right angles.

ii. Sum of all interior angles = (2n – 4) right angles.

17. Number of diagonals in a polygon of n sides = = $\frac{n(n-3)}{2}$.

18. A quadrilateral is a four sided polygon.

19. The sum of all the angles of a quadrilateral is 360°.

20. If the line containing any side of the quadrilateral has the remaining vertices on the same side of it, then the quadrilateral is called a convex quadrilateral.

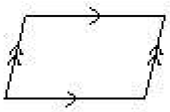
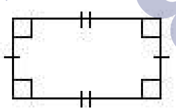
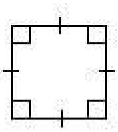
21. In a convex quadrilateral the measure of each angle is less than 180°.

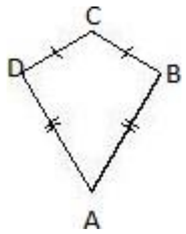
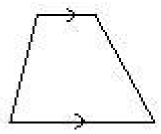
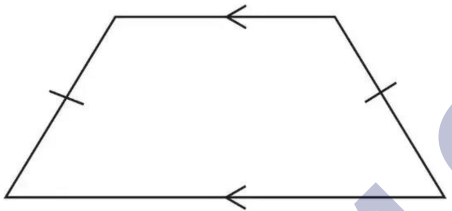
22. The sum of the interior angles of a pentagon is 540°.

23. The sum of the measures of the external angles of any polygon is 360°.

24. Each exterior angle of a regular polygon of n sides is equal to $\left(\frac{360}{n}\right)^{o}$

25. Types of quadrilaterals and their properties:

Name of quadrilateral	Properties
Parallelogram: A quadrilateral with each pair of opposite sides parallel. 	1. Opposite sides are equal. 2. Opposite angles are equal. 3. Adjacent angles are supplementary. 4. Diagonals bisect one another.
Rhombus: A parallelogram with sides o equal length. 	1. All properties of a parallelogram. 2. Diagonals are perpendicular to each other.
Rectangle: A parallelogram with a right angle. 	1. All the properties of a parallelogram. 2. Each of the angles is a right angle. 3. Diagonals are equal.
Square: A rectangle with sides of equal length. 	All the properties of a parallelogram, a rhombus and a rectangle.

Kite: A quadrilateral with exactly two pairs of equal consecutive sides. 	1. The diagonals are perpendicular to one another. 2. One of the diagonals bisects the other. 3. If ABCD is a kite, then $\angle B = \angle D$ but $\angle A \neq \angle C$.
Trapezium: A quadrilateral with one pair of parallel side is called trapezium. 	1. One pair of parallel sides.
Isosceles Trapezium: A trapezium is said to be an isosceles trapezium, if its non-parallel sides are equal. 	1. One pair of parallel sides. 2. Non-parallel sides are equal.

Introduction to Curves

A curve is a geometrical figure obtained when a number of points are joined without lifting the pencil from the paper and without retracing any portion. It is basically a line which need not be straight.

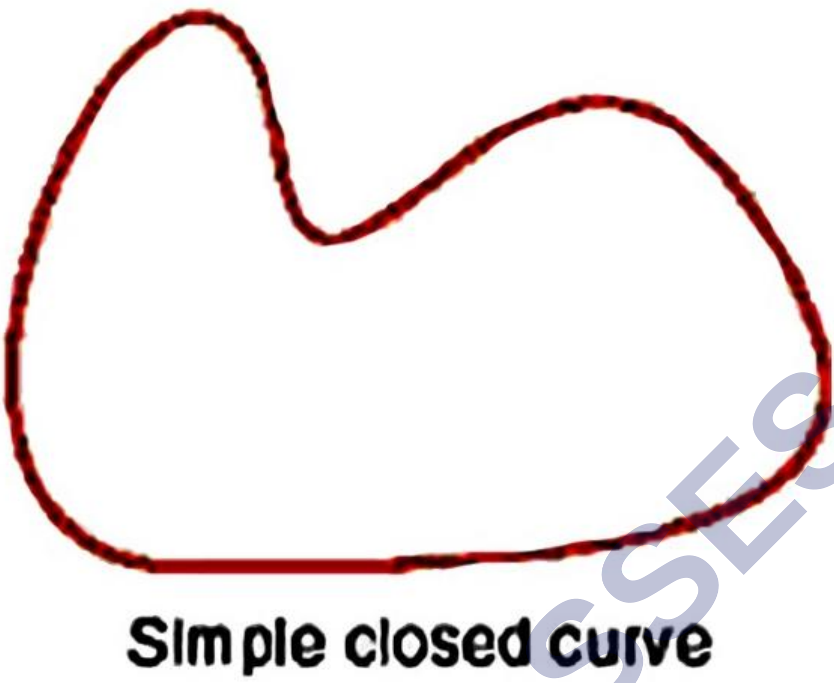
The various types of curves are:

Open curve: An open curve is a curve in which there is no path from any of its point to the same point.

Closed curve: A closed curve is a curve that forms a path from any of its point to the same point.

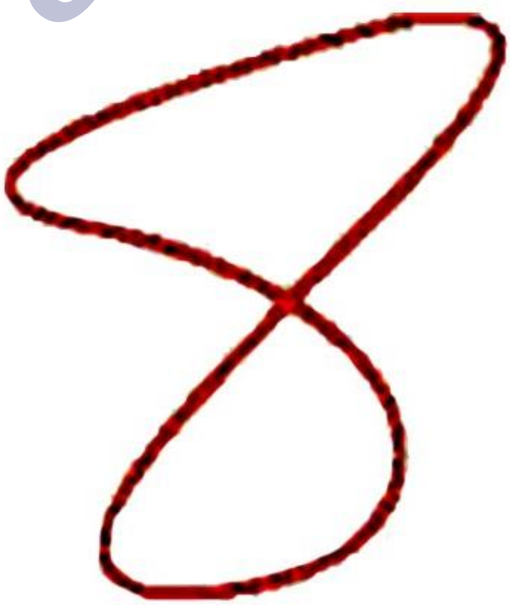
A curve can be:

A closed curve



an open curve

A closed curve which is not simple



Polygons

A simple closed curve made up of only line segments is called a polygon.

Various examples of polygons are Squares, Rectangles, Pentagons etc.

Note:

The sides of a polygon do not cross each other.

For example, the figure given below is not a polygon because its sides cross each other.

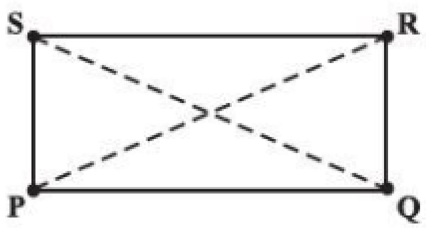
Classification of Polygons on the Basis of Number of Sides / Vertices

Polygons are classified according to the number of sides they have. The following lists the different types of polygons based on the number of sides they have:

- When there are three sides, it is triangle
- When there are four sides, it is quadrilateral
- When there are fives sides, it is pentagon
- When there are six sides, it is hexagon
- When there are seven sides, it is heptagon
- When there are eight sides, it is octagon
- When there are nine sides, it is nonagon
- When there are ten sides, it is decagon

Diagonals

A diagonal is a line segment connecting two non-consecutive vertices of a polygon.

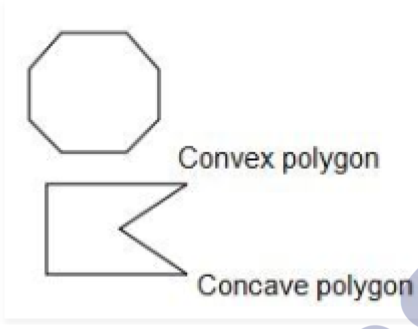


Polygons on the Basis of Shape

Polygons can be classified as concave or convex based on their shape.

A concave polygon is a polygon in which at least one of its interior angles is greater than 90°. Polygons that are concave have at least some portions of their diagonals in their exterior.

A convex polygon is a polygon with all its interior angle less than 180°. Polygons that are convex have no portions of their diagonals in their exterior.



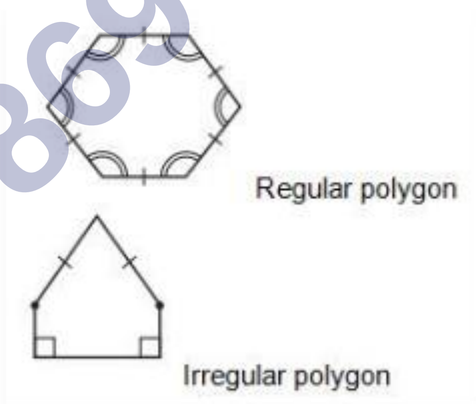
Classification of Polygons based on their shape.

Polygons on the Basis of Regularity

Polygons can also be classified as regular polygons and irregular polygons on the basis of regularity.

When a polygon is both equilateral and equiangular it is called as a regular polygon. In a regular polygon, all the sides and all the angles are equal. Example: Square

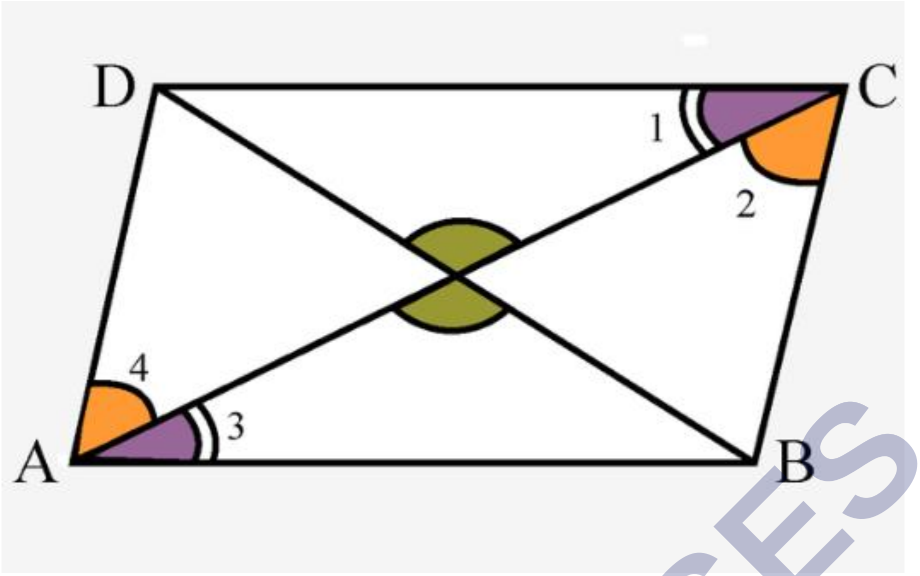
A polygon which is not regular i.e. it is not equilateral and equiangular, is an irregular polygon. Example: Rectangle



Introduction to Quadrilaterals

Angle Sum Property of a Polygon

According to the angle sum property of a polygon, the sum of all the interior angles of a polygon is equal to $(n-2) \times 180^\circ$, where n is the number of sides of the polygon.

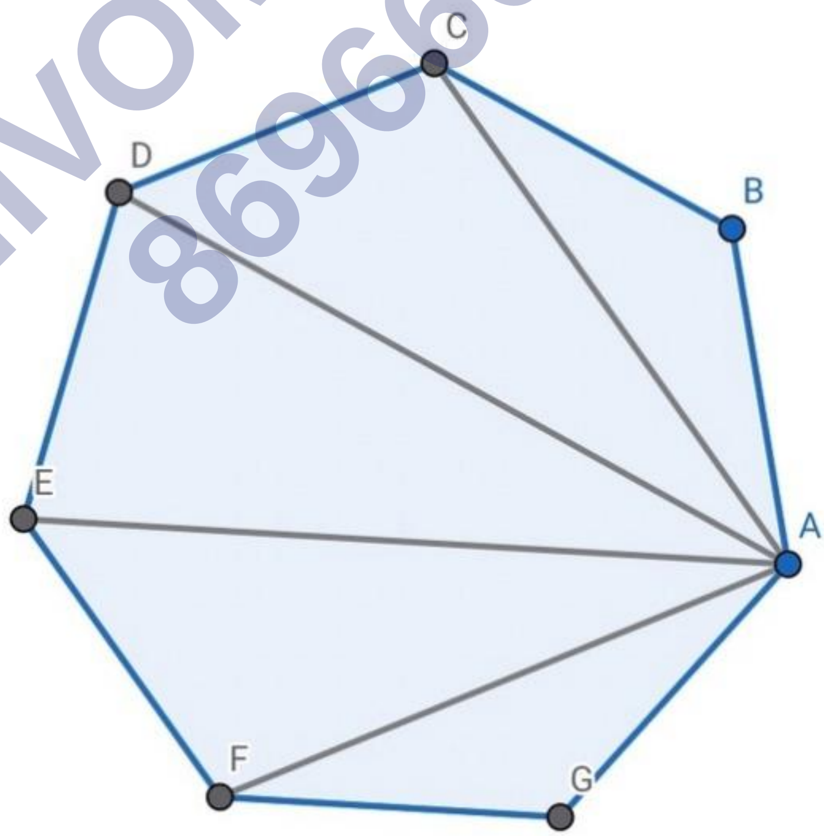


Division of a quadrilateral into two triangles.

As we can see for the above quadrilateral, if we join one of the diagonals of the quadrilateral, we get two triangles.

The sum of all the interior angles of the two triangles is equal to the sum of all the interior angles of the quadrilateral, which is equal to $360^\circ = (4 - 2) \times 180^\circ$.

So, if there is a polygon which has n sides, we can make (n – 2) non-overlapping triangles which will perfectly cover that polygon.



The sum of the interior angles of the polygon will be equal to the sum of the interior angles of the triangles = (n − 2) × 180°

Sum of Measures of Exterior Angles of a Polygon

The sum of the measures of the external angles of any polygon is 360°.

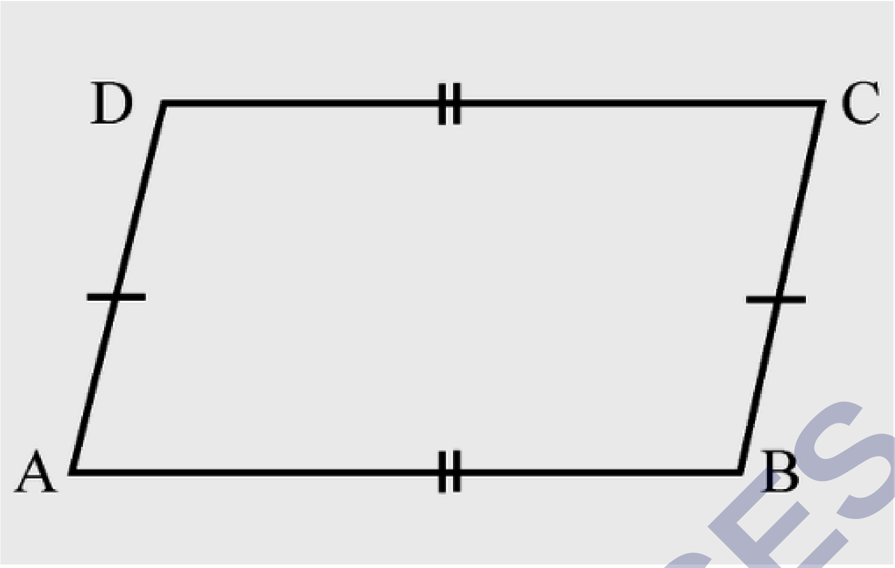
Properties of Parallelograms

Elements of a Parallelogram

- There are four sides and four angles in a parallelogram.
- The opposite sides and opposite angles of a parallelogram are equal.
- In the parallelogram ABCD, the sides $\overline{AB}$ and $\overline{CD}$ are opposite sides and the sides $\overline{AB}$ and $\overline{BC}$ are adjacent sides.
- Similarly, ∠ABC and ∠ADC are opposite angles and ∠ABC and ∠BCD are adjacent angles.

Angles of a Parallelogram

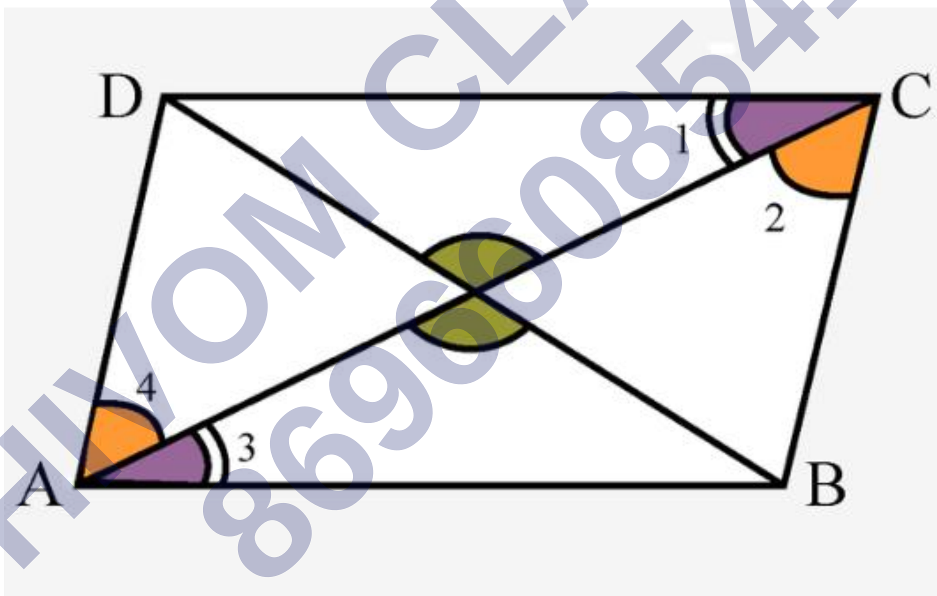
- The opposite angles of a parallelogram are equal.
- In the parallelogram ABCD, ∠ABC = ∠ADC and ∠DAB = ∠BCD.
- The adjacent angles in a parallelogram are supplementary.
- ∴ In the parallelogram ABCD, ∠ABC + ∠BCD = ∠ADC + ∠DAB = 180°



Diagonals of a Parallelogram

The diagonals of a parallelogram bisect each other at the point of intersection.

In the parallelogram ABCD given below, OA = OC and OB = OD.



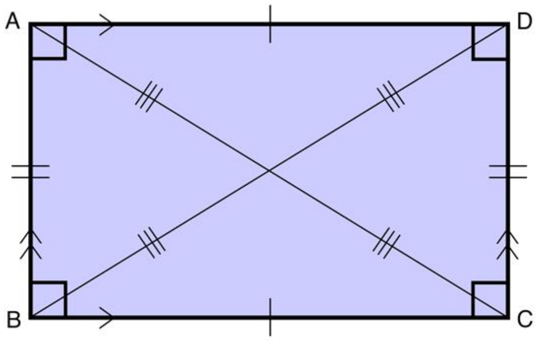
Properties of Special Parallelograms

Rectangle

A rectangle is a parallelogram with equal angles and each angle is equal to 90∘.

Properties:

- Opposite sides of a rectangle are parallel and equal.
- The length of diagonals of a rectangle is equal.
- All the interior angles of a rectangle are equal to 90∘.
- The diagonals of a rectangle bisect each other at the point of intersection.

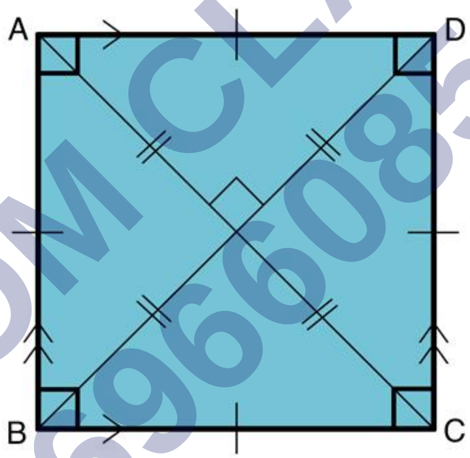


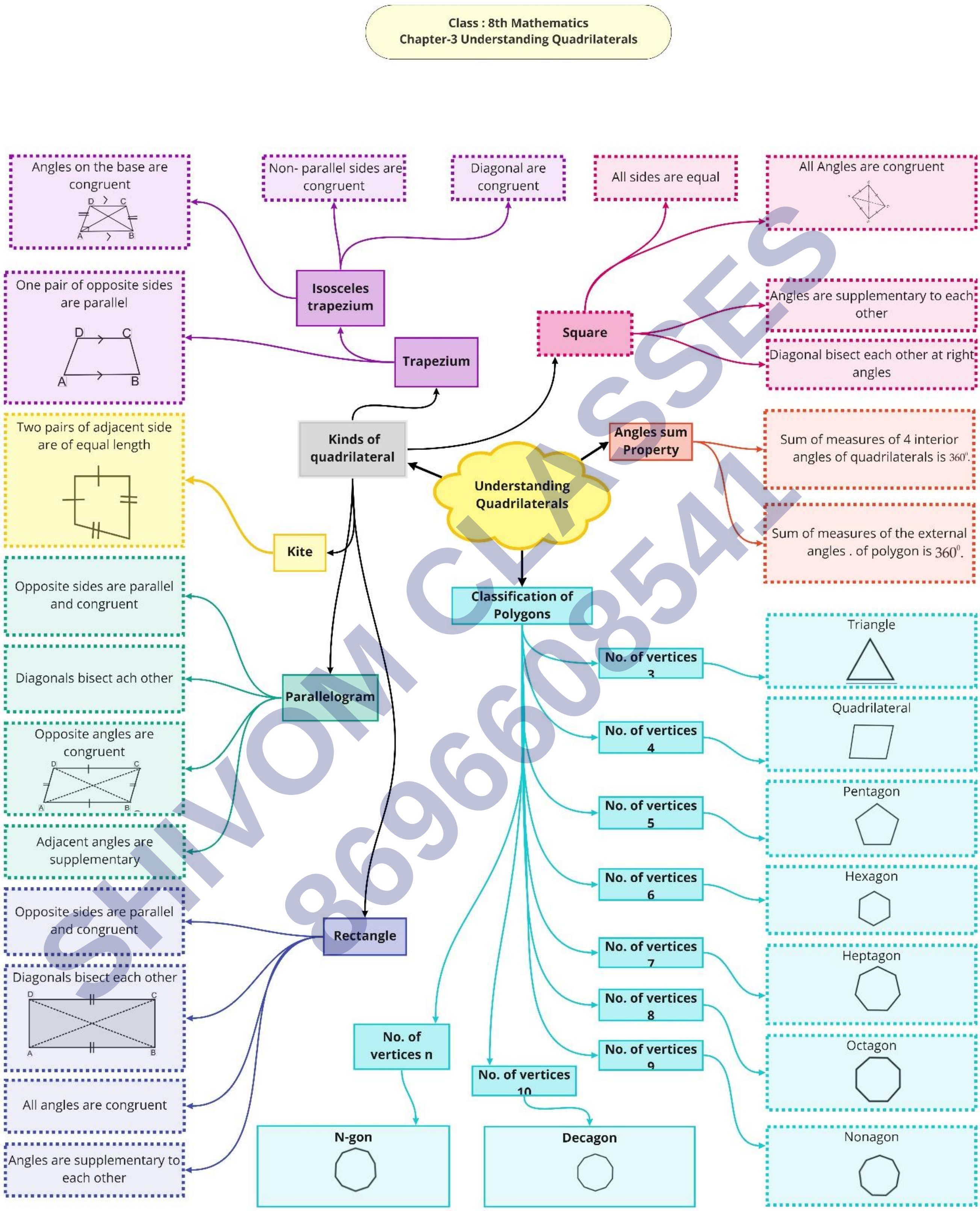
Square

A square is a rectangle with equal sides. All the properties of a rectangle are also true for a square.

In a square the diagonals:

- bisect one another
- are of equal length
- are perpendicular to one another




Class : 8th Mathematics
Chapter-3 Understanding Quadrilaterals
Understanding Quadrilaterals
Kinds of quadrilateral
Trapezium
Isosceles trapezium
Angles on the base are congruent
Non- parallel sides are congruent
Diagonal are congruent
One pair of opposite sides are parallel
Square
All sides are equal
All Angles are congruent
Angles are supplementary to each other
Diagonal bisect each other at right angles
Angles sum Property
Sum of measures of 4 interior angles of quadrilaterals is 360°.
Sum of measures of the external angles . of polygon is 360°.
Kite
Two pairs of adjacent side are of equal length
Parallelogram
Opposite sides are parallel and congruent
Diagonals bisect ach other
Opposite angles are congruent
Adjacent angles are supplementary
Rectangle
Opposite sides are parallel and congruent
Diagonals bisect each other
All angles are congruent
Angles are supplementary to each other
Classification of Polygons
No. of vertices 3
Triangle
No. of vertices 4
Quadrilateral
No. of vertices 5
Pentagon
No. of vertices 6
Hexagon
No. of vertices 7
Heptagon
No. of vertices 8
Octagon
No. of vertices 9
Nonagon
No. of vertices 10
Decagon
No. of vertices n
N-gon

Important Questions

Multiple Choice Questions:

Question 1. The opposite sides of a parallelogram are of ________ length.

(a) not equal

(b) different

(c) equal

(d) none of these

Question 2. In the quadrilateral ABCD, the diagonals AC and BD are equal and perpendicular to each other. What type of a quadrilateral is ABCD?

(a) A square

(b) A parallelogram

(c) A rhombus

(d) A trapezium

Question 3. If ABCD is an isosceles trapezium, what is the measure of ∠C?

(a) ∠B

(b) ∠A

(c) ∠D

(d) 90°

Question 4. Which of the following is true for the adjacent angles of a parallelogram?

(a) they are equal to each other

(b) they are complementary angles

(c) they are supplementary angles

(d) none of these.

Question 5. State the name of a regular polygon of 6 sides.

(a) pentagon

(b) hexagon

(c) heptagon

(d) none of these

Question 6. The diagonal of a rectangle is 10 cm and its breadth is 6 cm. What is its length?

(a) 6 cm

(b) 5cm

(c) 8cm

(d) 4cm

Question 7. The perimeter of a parallelogram is 180 cm. If one side exceeds the other by 10 cm, what are the sides of the parallelogram?

(a) 40 cm, 50 cm

(b) 45 cm each

(c) 50 cm each

(d) 45 cm, 50 cm

Question 8. A _______ is both 'equiangular' and 'equilateral'.

(a) regular polygon

(b) triangle

(c) quadrilateral

(d) none of these

Question 9. Which of the following quadrilaterals has two pairs of adjacent sides equal and diagonals intersecting at right angles?

(a) square

(b) rhombus

(c) kite

(d) rectangle

Question 10.Which one of the following is a regular quadrilateral?

(a) Square

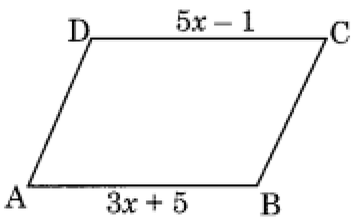
(b) Trapezium

(c) Kite

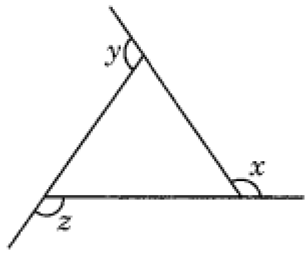
(d) Rectangle

Very Short Questions:

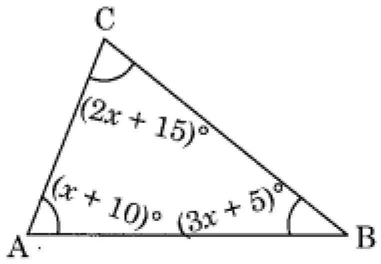
1. In the given figure, ABCD is a parallelogram. Find x.



2. In the given figure find x + y + z.



3. In the given figure, find x.



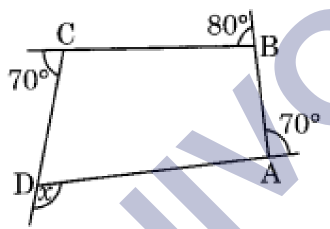
4. The angles of a quadrilateral are in the ratio of 2 : 3 : 5 : 8. Find the measure of each angle.

5. Find the measure of an interior angle of a regular polygon of 9 sides.

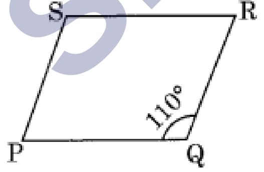
6. Length and breadth of a rectangular wire are 9 cm and 7 cm respectively. If the wire is bent into a square, find the length of its side.

Short Questions :

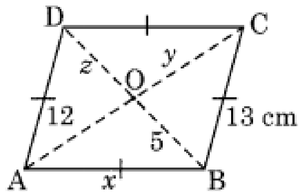
1. In the given figure ABCD, find the value of x.



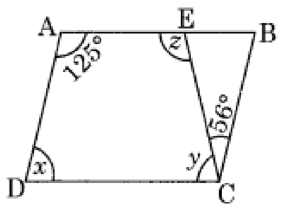
2. In the parallelogram given alongside if m∠Q = 110°, find all the other angles.



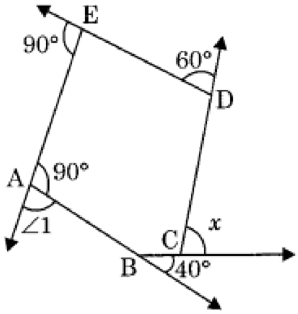
3. In the given figure, ABCD is a rhombus. Find the values of x, y and z.



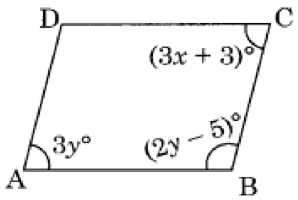
4. In the given figure, ABCD is a parallelogram. Find x, y and z.



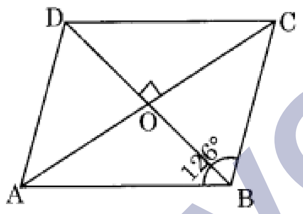
5. Find x in the following figure.



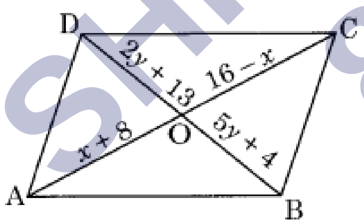
6. In the given parallelogram ABCD, find the value of x and y.



7. ABCD is a rhombus with ∠ABC = 126°, find the measure of ∠ACD.



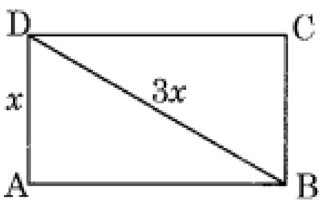
8. Find the values of x and y in the following parallelogram.



Long Questions :

1. The sides AB and CD of a quadrilateral ABCD are extended to points P and Q respectively. Is ∠ADQ + ∠CBP = ∠A + ∠C? Give reason.

2. The diagonal of a rectangle is thrice its smaller side. Find the ratio of its sides.



3. If AM and CN are perpendiculars on the diagonal BD of a parallelogram ABCD, Is ΔAMD = ΔCNB? Give reason.

Answer Key-

Multiple Choice Questions:

1. (c) equal
2. (a) A square
3. (c) ∠D
4. (c) they are supplementary angles
5. (b) hexagon
6. (c) 8cm
7. (a) 40 cm, 50 cm
8. (a) regular polygon
9. (b) rhombus
10. (a) Square

Very Short Answer:

1. AB = DC [Opposite sides of a parallelogram]

$3x + 5 = 5x - 1$

$\Rightarrow 3x - 5x = -1 - 5$

$\Rightarrow -2x = -6$

$\Rightarrow x = 3$

2. We know that the sum of all the exterior angles of a polygon = 360°

$x + y + z = 360°$

3. ∠A + ∠B + ∠C = 180° [Angle sum property]

$(x + 10)° + (3x + 5)° + (2x + 15)° = 180°$

$\Rightarrow x + 10 + 3x + 5 + 2x + 15 = 180$

$\Rightarrow 6x + 30 = 180$

$\Rightarrow 6x = 180 - 30$

$\Rightarrow 6x = 150$

$\Rightarrow x = 25$

4. Sum of all interior angles of a quadrilateral = 360°

Let the angles of the quadrilateral be 2x°, 3x°, 5x° and 8x°.

$2x + 3x + 5x + 8x = 360°$

$\Rightarrow 18x = 360°$

$\Rightarrow x = 20°$

Hence the angles are

$2 \times 20 = 40°$,

$3 \times 20 = 60°$,

$5 \times 20 = 100°$

and $8 \times 20 = 160°$.

5. Measure of an interior angle of a regular polygon

of n sides $= \dfrac{(n-2)\times 180°}{n}$

For $n = 9$, we have

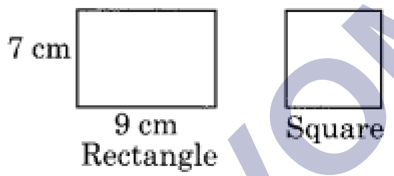
$$\frac{(9-2)\times 180°}{9} = \frac{7\times 180°}{9}$$

$$= 7 \times 20° = 140°$$

Hence, the angle is 140°.

6. Perimeter of the rectangle = 2 [length + breadth]

$= 2[9 + 7] = 2 \times 16 = 32$ cm.



Now perimeter of the square = Perimeter of rectangle = 32 cm.

Side of the square $= \frac{32}{4} = 8$ cm.

Hence, the length of the side of square = 8 cm.

Short Answer:

1. Sum of all the exterior angles of a polygon = 360°

$x + 70° + 80° + 70° = 360°$

$\Rightarrow x + 220° = 360°$

$\Rightarrow x = 360° - 220° = 140°$

2. Given $m\angle Q = 110°$

Then $m\angle S = 110°$ (Opposite angles are equal)

Since $\angle P$ and $\angle Q$ are supplementary.

Then $m\angle P + m\angle Q = 180°$

$\Rightarrow m\angle P + 110° = 180°$

$\Rightarrow m\angle P = 180° - 110° = 70°$

$\Rightarrow m\angle P = m\angle R = 70°$ (Opposite angles)

Hence $m\angle P = 70$, $m\angle R = 70°$

and $m\angle S = 110°$

3. AB = BC (Sides of a rhombus)

x = 13 cm.

Since the diagonals of a rhombus bisect each other

z = 5 and y = 12

Hence, x = 13 cm, y = 12 cm and z = 5 cm.

4. $\angle A + \angle D = 180°$ (Adjacent angles)

$\Rightarrow 125° + \angle D = 180°$

$\Rightarrow \angle D = 180° - 125°$

$x = 55°$

$\angle A = \angle C$ [Opposite angles of a parallelogram]

$\Rightarrow 125° = y + 56°$

$\Rightarrow y = 125° - 56°$

$\Rightarrow y = 69°$

$\angle z + \angle y = 180°$ (Adjacent angles)

$\Rightarrow \angle z + 69° = 180°$

$\Rightarrow \angle z = 180° - 69° = 111°$

Hence the angles x = 55°, y = 69° and z = 111°

5. In the given figure $\angle 1 + 90° = 180°$ (linear pair)

$\angle 1 = 90°$

Now, sum of exterior angles of a polygon is 360°, therefore,

$x + 60° + 90° + 90° + 40° = 360°$

$\Rightarrow x + 280° = 360°$

$\Rightarrow x = 80°$

6. $\angle A + \angle B = 180°$

$3y + 2y - 5 = 180°$

$\Rightarrow 5y - 5 = 180°$

$\Rightarrow 5y = 180 + 5°$

$\Rightarrow 5y = 185°$

⇒ y = 37°

Now ∠A = ∠C [Opposite angles of a parallelogram]

3y = 3x + 3

⇒ 3 × 37 = 3x + 3

⇒ 111 = 3x + 3

⇒ 111 – 3 = 3x

⇒ 108 = 3x

⇒ x = 36°

Hence, x = 36° and y – 37°.

7. ∠ABC = ∠ADC (Opposite angles of a rhombus)

∠ADC = 126°

∠ODC = 12 × ∠ADC (Diagonal of rhombus bisects the respective angles)

⇒ ∠ODC = 12 × 126° = 63°

⇒ ∠DOC = 90° (Diagonals of a rhombus bisect each other at 90°)

In ΔOCD,

∠OCD + ∠ODC + ∠DOC = 180° (Angle sum property)

⇒ ∠OCD + 63° + 90° = 180°

⇒ ∠OCD + 153° = 180°

⇒ ∠OCD = 180° – 153° = 27°

Hence ∠OCD or ∠ACD = 27°

8. Since, the diagonals of a parallelogram bisect each other.

OA = OC

x + 8 = 16 – x

⇒ x + x = 16 – 8

⇒ 2x = 8

x = 4

Similarly, OB = OD

5y + 4 = 2y + 13

⇒ 3y = 9

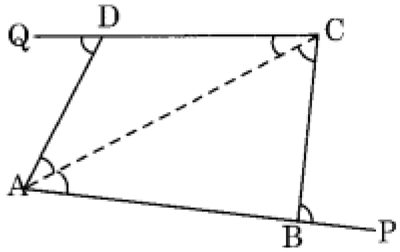
⇒ y = 3

Hence, x = 4 and y = 3

Long Answer:

1. Join AC, then

∠CBP = ∠BCA + ∠BAC and ∠ADQ = ∠ACD + ∠DAC (Exterior angles of triangles)



Therefore,

∠CBP + ∠ADQ = ∠BCA + ∠BAC + ∠ACD + ∠DAC

= (∠BCA + ∠ACD) + (∠BAC + ∠DAC)

= ∠C + ∠A

2. Let AD = x cm

diagonal BD = 3x cm

In right-angled triangle DAB,

$AD^2 + AB^2 = BD^2$ (Using Pythagoras Theorem)

$x^2 + AB^2 = (3x)^2$

$\Rightarrow x^2 + AB^2 = 9x^2$

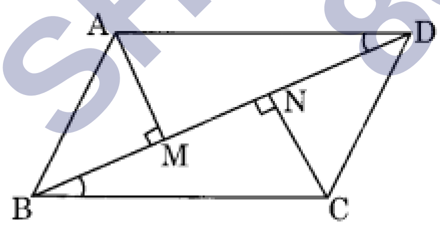
$\Rightarrow AB^2 = 9x^2 - x^2$

$\Rightarrow AB^2 = 8x^2$

$\Rightarrow AB = \sqrt{8}x = 2\sqrt{2}x$

Required ratio of AB : AD = $2\sqrt{2}x : x = 2\sqrt{2} : 1$

3.



In triangles AMD and CNB,

AD = BC (opposite sides of parallelogram)

∠AMB = ∠CNB = 90°

∠ADM = ∠NBC (AD || BC and BD is transversal.)

So, ΔAMD = ΔCNB (AAS)