MATHEMATICS

Chapter 16: Playing with Numbers

Playing with Numbers

General Form of Numbers

If a two-digit number pq needs to represented in general form, then

Pq = 10p + q

Playing with 2 – Digit and 3 – Digit Numbers

Numbers in General form

A two digit number(ab) in its general form, is written as : ab = (10 x a) + (1 x b)

Similarly, a three digit number (abc) is written as:

Abc = (100 x a) + (10 x b) + c

Reversing the 2 digit numbers and adding them

When a two digit number is reversed and added with the number, the resulting number is perfectly divisible by 11 and the quotient is equal to the sum of the digits

For eg: The reverse of 29 is 92.

The sum of 29 and 92 = 29 +92 = 121.

On dividing the sum by 11, we get 121/11 = 11 = 9+2.

So, the sum is divisible by 11 and the quotient is equal to the sum of the digits of the number.

Reversing the 2 digit numbers and Subtracting them

When a two digit number is reversed and the larger number is subtracted from the smaller number, the resulting number is perfectly divisible by 9 and the quotient is equal to the difference of the digits of the number.

For example, the reverse of the number 39 is 93.

Now, 93 > 39.

So, 93 – 39 = 54

On dividing the difference of the two number by 9, we get, 54/9 = 6 = 9−3

So, the difference is divisible by 9 and the quotient is equal to the difference of the digits.

Reversing the 3-digit numbers and Subtracting them

When a three-digit number is reversed and the smaller number is subtracted from the larger number, the resulting number is perfectly divisible by 99 and the quotient is equal to the difference between the first and third digit of the selected number.

For example, the reverse of 123 is 321.

Now, 321 > 123.

321 – 123 = 198

Now,198/99 = 2 = 3 – 1

So, the difference between 123 and 321 is divisible by 99 and the quotient is equal to the difference between 3 and 1.

Taking all the combinations of 3 digit numbers and adding them

If all the combinations of a three-digit number are taken and added together, then the resulting number is perfectly divisible by 111.

Let us take 123.

The various numbers that can be formed using the digits of 123 are 123, 132, 213, 231, 312 and 321.

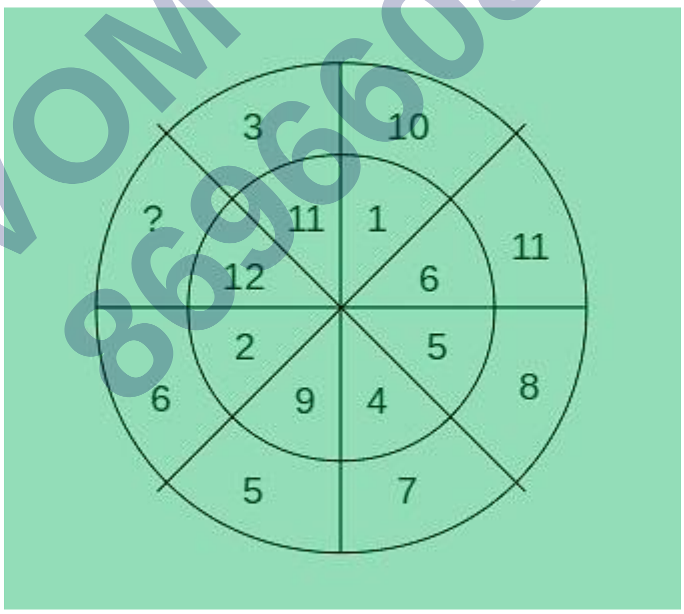
The sum of these numbers is equal to (123 + 132 + 213 + 231 + 312 + 321 = 1332)

Now,1332/111 = 12

So, the sum of all the combinations is divisible by 111.

Puzzles with Digits

Letters for digits



The number system is one of the most innovative and interesting inventions by human beings. Various tricks and puzzles involving numbers have always made us curious and enhanced our process of thinking. Puzzles were created in order to test the knowledge and thinking ability. In this article, we will enhance our reasoning skills by solving questions on puzzles involving numbers and letters.

There are puzzles in which letters take the place of digits in the arithmetic sum, and we find out which letter represents which digit. It is very much similar to cracking a code. The discussion will stick to problems relating to addition and multiplication.

Here we have puzzles in which letters take the place of digits in an arithmetic 'sum', and the problem is to find out which letter represents which digit.

Rules for Solving Puzzles

There are two rules which are to be followed for solving the puzzles:

In a puzzle each letter stands for a single digit. Each digit should be represented by one letter only.

The first digit of a number cannot be zero. For example, we write seventy nine as 79 and not 079.

Illustration 1: Find the value of Q in the following:

$$\begin{array}{cccc} & 4 & Q & 1 \\ + & 3 & 8 & Q \\ \hline & 8 & 0 & 3 \\ \hline \end{array}$$

Solution:

In column one (starting from right), from Q + 1 we get 3 at the units place.

Q + 1 = 3

Q = 2

In the middle column,

Q + 8 gives a number such that it has 0 at its units place. So Q = 2. This is verified by the fact that when Q (2) is added to 8 it results in 10 and hence 1 is carried forward. In the third column 4 + 3 + 1(carry) = 8.

Illustration 2: Find the value of A:

```
    A
  + A
  + A
  ---
  B A
  ---
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Solution:

We have to find the value of A and B. A is any number whose thrice sum with itself is also A. Therefore, the sum of two A's should be 0. This case is possible only for A = 0 or A = 5.

In this case if A = 0, the entire sum will be 0 and hence B = 0.

But this is not possible as it will lead to A = B

We know that different letters represent different digits. Therefore, we take the case in which A = 5.

Hence A + A + A = 5 + 5 + 5 = 15

So, we get B = 1.

For example, if:

4 Q 1

3 8 Q

——— (+)

8 0 3

———

Then the value of Q will be 2 as 2 is the only digit which results in 3 when 1 is added to it. Also, 2+8 =10, 1 as carry and then 1+4+3 =8. So, the value of Q is 2.

Divisibility Rules

Divisibility Rule of 1

Every number is divisible by 1. Divisibility rule for 1 doesn't have any condition. Any number divided by 1 will give the number itself, irrespective of how large the number is. For example, 3 is divisible by 1 and 3000 is also divisible by 1 completely.

Divisibility Rule of 2

If a number is even or a number whose last digit is an even number i.e. 2,4,6,8 including 0, it is always completely divisible by 2.

Example: 508 is an even number and is divisible by 2 but 509 is not an even number, hence it is not divisible by 2. Procedure to check whether 508 is divisible by 2 or not is as follows:

- Consider the number 508
- Just take the last digit 8 and divide it by 2
- If the last digit 8 is divisible by 2 then the number 508 is also divisible by 2.

Divisibility Rules for 3

Divisibility rule for 3 states that a number is completely divisible by 3 if the sum of its digits is divisible by 3.

Consider a number, 308. To check whether 308 is divisible by 3 or not, take sum of the digits (i.e. 3 + 0 + 8 = 11). Now check whether the sum is divisible by 3 or not. If the sum is a multiple of 3, then the original number is also divisible by 3. Here, since 11 is not divisible by 3, 308 is also not divisible by 3.

Similarly, 516 is divisible by 3 completely as the sum of its digits i.e. 5 + 1 + 6 = 12, is a multiple of 3.

Divisibility Rule of 4

If the last two digits of a number are divisible by 4, then that number is a multiple of 4 and is divisible by 4 completely.

Example: Take the number 2308. Consider the last two digits i.e. 08. As 08 is divisible by 4, the original number 2308 is also divisible by 4.

Divisibility Rule of 5

Numbers, which last with digits, 0 or 5 are always divisible by 5.

Example: 10, 10000, 10000005, 595, 396524850, etc.

Divisibility Rule of 6

Numbers which are divisible by both 2 and 3 are divisible by 6. That is, if the last digit of the given number is even and the sum of its digits is a multiple of 3, then the given number is also a multiple of 6.

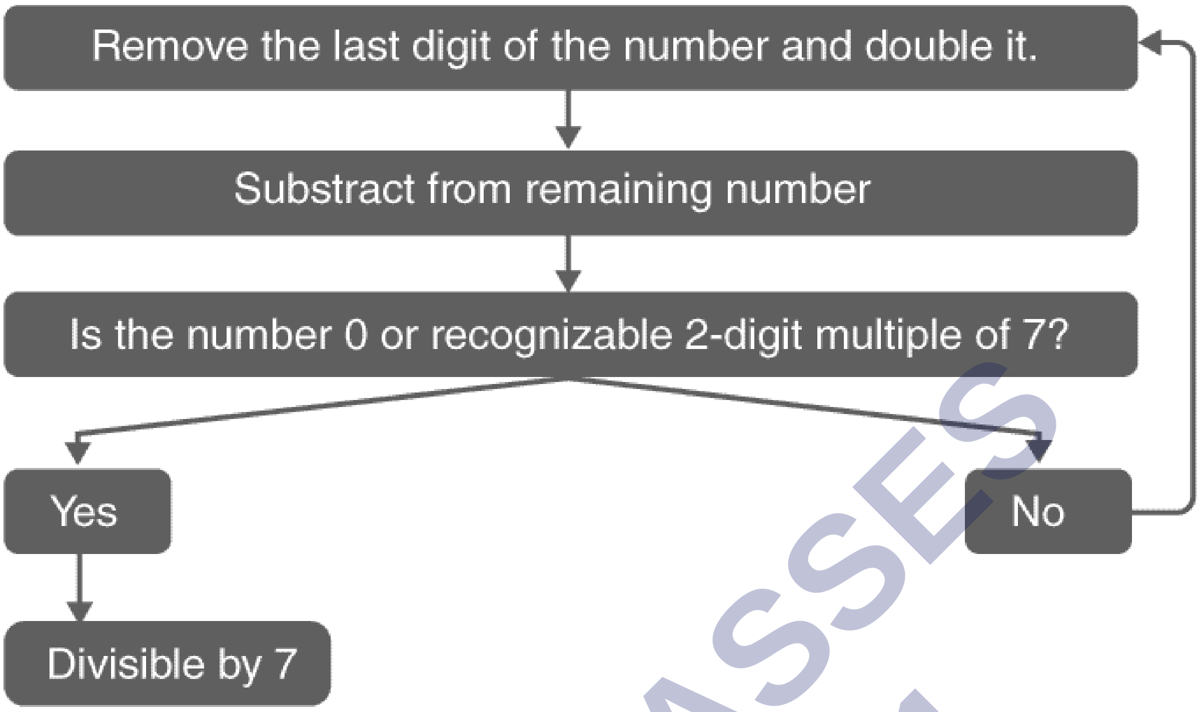
Example: 630, the number is divisible by 2 as the last digit is 0.

The sum of digits is 6 + 3 + 0 = 9, which is also divisible by 3.

Hence, 630 is divisible by 6.

Divisibility Rules for 7

The rule for divisibility by 7 is a bit complicated which can be understood by the steps given below:



Example: Is 1073 divisible by 7?

From the rule stated remove 3 from the number and double it, which becomes 6.

Remaining number becomes 107, so 107-6 = 101.

Repeating the process one more time, we have 1 x 2 = 2.

Remaining number 10 – 2 = 8.

As 8 is not divisible by 7, hence the number 1073 is not divisible by 7.

Divisibility Rule of 8

If the last three digits of a number are divisible by 8, then the number is completely divisible by 8.

Example: Take number 24344. Consider the last two digits i.e. 344. As 344 is divisible by 8, the original number 24344 is also divisible by 8.

Divisibility Rule of 9

The rule for divisibility by 9 is similar to divisibility rule for 3. That is, if the sum of digits of the number is divisible by 9, then the number itself is divisible by 9.

Example: Consider 78532, as the sum of its digits (7 + 8 + 5 + 3 + 2) is 25, which is not divisible by 9, hence 78532 is not divisible by 9.

Divisibility Rule of 10

Divisibility rule for 10 states that any number whose last digit is 0, is divisible by 10.

Example: 10, 20, 30, 1000, 5000, 60000, etc.

Divisibility Rules for 11

If the difference of the sum of alternative digits of a number is divisible by 11, then that number is divisible by 11 completely.

In order to check whether a number like 2143 is divisible by 11, below is the following procedure.

Group the alternative digits i.e. digits which are in odd places together and digits in even places together. Here 24 and 13 are two groups.

Take the sum of the digits of each group i.e. 2 + 4 = 6 and 1 + 3 = 4

Now find the difference of the sums; 6 – 4 = 2

If the difference is divisible by 11, then the original number is also divisible by 11. Here 2 is the difference which is not divisible by 11.

Therefore, 2143 is not divisible by 11.

A few more conditions are there to test the divisibility of a number by 11. They are explained here with the help of examples:

If the number of digits of a number is even, then add the first digit and subtract the last digit from the rest of the number.

Example: 3784

Number of digits = 4

Now, 78 + 3 – 4 = 77 = 7 × 11

Thus, 3784 is divisible by 11.

If the number of digits of a number is odd, then subtract the first and the last digits from the rest of the number.

Example: 82907

Number of digits = 5

Now, 290 – 8 – 7 = 275 × 11

Thus, 82907 is divisible by 11.

Form the groups of two digits from the right end digit to the left end of the number and add the resultant groups. If the sum is a multiple of 11, then the number is divisible by 11.

Example: 3774: = 37 + 74 = 111: = 1 + 11 = 12

3774 is not divisible by 11.

253: = 2 + 53 = 55 = 5 × 11

253 is divisible by 11.

Subtract the last digit of the number from the rest of the number. If the resultant value is a multiple of 11, then the original number will be divisible by 11.

Example: 9647

9647: = 964 – 7 = 957

957: = 95 – 7 = 88 = 8 × 11

Thus, 9647 is divisible by 11.

Divisibility Rule of 12

If the number is divisible by both 3 and 4, then the number is divisible by 12 exactly.

Example: 5864

Sum of the digits = 5 + 8 + 6 + 4 = 23 (not a multiple of 3)

Last two digits = 64 (divisible by 4)

The given number 5846 is divisible by 4 but not by 3; hence, it is not divisible by 12.

Divisibility Rules for 13

For any given number, to check if it is divisible by 13, we have to add four times of the last digit of the number to the remaining number and repeat the process until you get a two-digit number. Now check if that two-digit number is divisible by 13 or not. If it is divisible, then the given number is divisible by 13.

For example: 2795 → 279 + (5 x 4)

→ 279 + (20)

→ 299

→ 29 + (9 x 4)

→ 29 + 36

→65

Number 65 is divisible by 13, 13 x 5 = 65.

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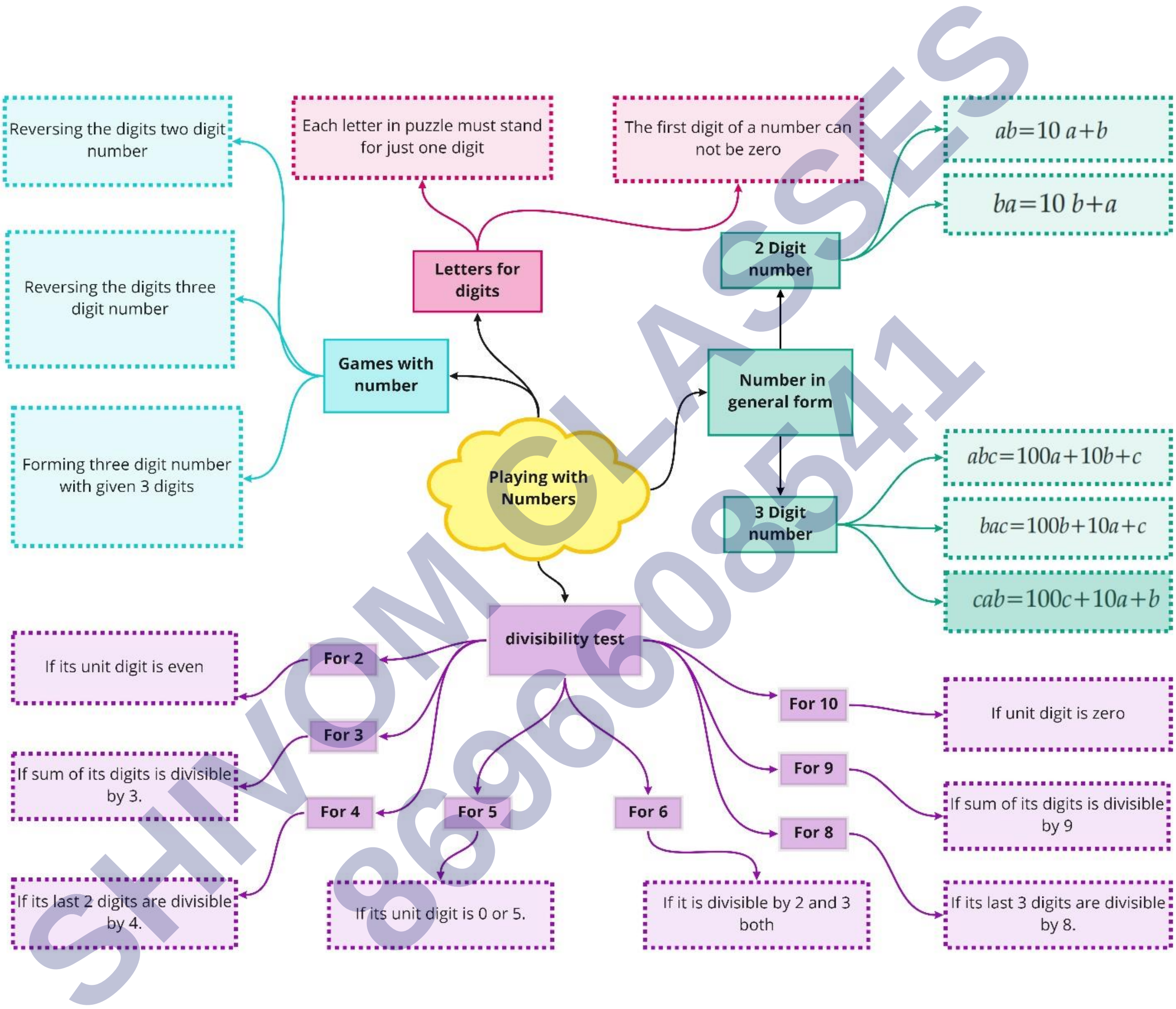


Important Questions

Multiple Choice Questions:

Question 1. The generalised form of the number 52 is

(a) 10 × 5 + 2

(b) 100 × 5 + 2

(c) 10 × 2 + 5

(d) 10 × 5.

Question 2. The generalised form of the number 33 is

(a) 10 × 3 + 3

(b) 10 × 3

(c) 3 + 3

(d) 3 × 3 + 3.

Question 3. The number 10 × 7 + 5 in usual form is

(a) 57

(b) 75

(c) 55

(d) 77.

Question 4. The number 10 × 2 + 7 in usual form is

(a) 72

(b) 22

(c) 77

(d) 21.

Question 5. The generalised form of the number 123 is

(a) 1 × 100 + 2 × 10 + 3

(b) 2 × 100 + 3 × 10 + 1

(c) 3 × 100 + 1 × 10 + 2

(d) none of these.

Question 6. The generalised form of the number 234 is

(a) 2 × 100 + 3 × 10 + 4

(b) 3 × 100 + 4 × 10 + 2

(c) 4 × 100 + 2 × 10 + 3

(d) none of these.

Question 7. The number 3 × 100 + 4 × 10 + 5 in usual form is

(a) 453

(b) 435

(c) 354

(d) 345.

Question 8. The number 5 × 100 + 7 × 10 + 9 in usual form is

(a) 795

(b) 759

(c) 579

(d) 597.

Question 9. The number 100 × a + 10 × 6 + c in usual form is

(a) abc

(b) bca

(c) cab

(d) none of these.

Question 10. The number 100 × b + 10 × c + a in usual form is

(a) bac

(b) bca

(c) cab

(d) cba.

Very Short Questions:

1. Write the following numbers in generalised form.

(a) ab

(b) 85

(c) 132

(d) 1000

2. Write the following in usual form.

(a) 3 × 100 + 0 × 10 + 6

(b) 5 × 1000 + 3 × 100 + 2 × 10 + 1

3. Which of the following numbers are divisible by 3?

(i) 106

(ii) 726

(iii) 915

(iv) 1008

4. Check the divisibility of 34567 by 9.

5. Check the divisibility of 56748 by 3.

6. Check the divisibility of 7986 by 9.

Short Questions:

1. If a, b, c are three digits of a three-digit number, prove that abc + cab + bca is a multiple of 37

2. Complete the magic square given below so that the sum of the numbers in each row or in each column or along each diagonal is 15.

8	1	A
B	5	C
D	E	F

3. Find the values of P and Q from the given addition problem

$$\begin{array}{cccc} & 3 & P & 4 & 3 \\ + & 4 & 2 & 7 & Q \\ \hline & 7 & 9 & 1 & 7 \\ \hline \end{array}$$

4. Find the values of p, q and r in the following multiplication problem.

$$\begin{array}{ccccc} & & 3 & p & 4 \\ & & \times & q & 6 \\ \hline & 2 & 1 & 2 & 4 \\ 1 & 0 & 6 & r & \times \\ \hline 1 & 2 & 7 & 4 & 4 \\ \hline \end{array}$$

5. Observe the following patterns:

1 × 9 – 1 = 8

21 × 9 – 1 = 188

321 × 9 – 1 = 2888

4321 × 9 – 1 = 38888

Find the value of 87654321 × 9 – 1

Long Questions:

1. Prove that the sum of the given numbers and the numbers obtained by reversing their digits is divisible by 11.

(a) 89

(b) ab

(c) 69

(d) 54

2. Prove that the difference of the given numbers and the numbers obtained by reversing their digits is divisible by 9.

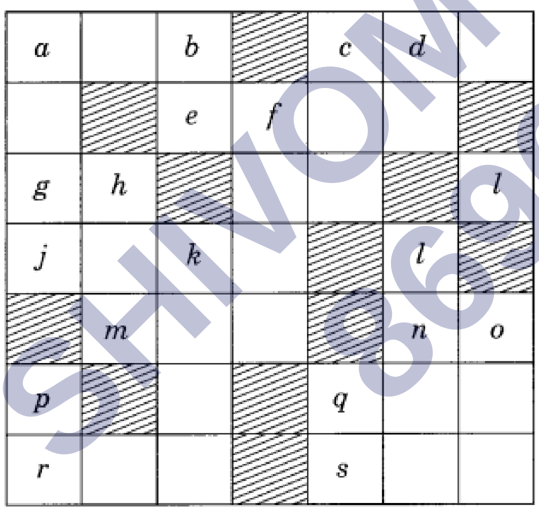
(i) 59

(ii) xy

(iii) xyz

(iv) 203

3. Complete the cross number puzzle with the given column.



Horizontal row	Vertical row
(a) $0.7 \times 8 \times 90$	(a) 64% of 9200
(c) $94.9 - 2.1$	(b) 0.6×75
(e) 80% of 6600	(c) $1079.2 - 90.2$
(g) 0.5×168	(d) 0.4×50
(i) 0% of 125	(f) $512 + 1722$
(j) $10^4 - 1037$	(k) 896×7
(m) $550 - 26$	(l) $(0.36 \times 10^4) + 192$
(n) 100% of 79	(o) $10^2 \times 9$
(q) $360 + 30$	(p) $(3 \times 10^1) + (3 \times 10)$
(r) $8.34 - 5.22$	(q) 70% of 50
(s) $8 \times 5 \times 13$	

4. The product of two 2-digit numbers is 1431. The product of their tens digits is 10 and the product of their units digits is 21. Find the numbers.

Answer Key-

Multiple Choice Questions:

1. (d) Salary of employees.
2. (a) Travelling allowance/rent
3. (b) Travelling allowance and rent
4. (c) 300
5. (b) 200
6. (a) Food
7. (a) Education
8. (a) Rs 1050
9. (a) Rs 1200
10. (c) Rs 5400

Very Short Answer:

1. (a) ab = $10 \times a + 1 \times b = 10a + b$

   (b) $85 = 10 \times 8 + 1 \times 5 = 10 \times 8 + 5$

   (c) $132 = 100 \times 1 + 10 \times 3 + 1 \times 2 = 100 \times 1 + 10 \times 3 + 2$

   (d) $1000 = 1000 \times 1$

2. (a) 3 × 100 + 0 × 10 + 6 = 300 + 0 + 6 = 306

(b) 5 × 1000 + 3 × 100 + 2 × 10 + 1 = 5000 + 300 + 20 + 1 = 5321

3. (i) Sum of the digits of 106 = 1 + 0 + 6 = 7 which is not divisible by 3.

Hence 106 is not divisible by 3.

(ii) Sum of the digits of 726 = 7 + 2 + 6 = 15 which is divisible by 3.

Hence 726 is divisible by 3.

(iii) Sum of the digits of 915 = 9 + 1 + 5 = 15 which is divisible by 3.

Hence 915 is divisible by 3.

(iv) Sum of the digits of 1008 = 1 + 0 + 0 + 8 = 9 which is divisible by 3.

Hence 1008 is divisible by 3.

4. The sum of the digits is 3 + 4 + 5 + 6 + 7 = 25. This number is not divisible by 9. Therefore 34567 is not divisible by 9.

5. The sum of the digits is 5 + 6 + 7 + 4 + 8 = 30 is divisible by 3

By the actual division $\frac{56748}{3} = 18916$

∴ 56748 is divisible by 3.

6. The sum of the digits is 7 + 9 + 8 + 6 = 30

30 is not divisible by 9

Therefore 7986 is not divisible by 9.

Short Answer:

1. We have abc + cab + bca

abc = 100a + 10b + c

cab = 100c + 10a + b

bca = 100b + 10c + a

Adding abc + cab + bca = 111a +111b + 111c

= 111 (a + b+ c)

= 37 × 3 (a + b + c) which is a multiple of 37.

Hence proved.

2. (i) A = 15 – (8 + 1) = 15 – 9 = 6

(ii) F = 15 – (8 + 5) = 15 – 13 = 2

(iii) C = 15 – (A + F) = 15 – (6 + 2) = 15 – 8 = 7

(iv) E = 15 – (1 + 5) = 15 – 6 = 9

(v) D = 15 – (E + F) = 15 – (9 + 2) = 15 – 11 = 4

(vi) B = 15 – (8 + 4) = 15 – 12 = 3

Hence the required square is

8	1	6
3	5	7
4	9	2

3. Here, 3 + Q = 7

⇒ Q = 7 – 3 = 4

Now taking second column, we get

4 + 7 = 11 i.e. 1 is carried over to third column

⇒ 1 + P + 2 = 9

⇒ 3 + P = 9

P = 9 – 3 = 6

Hence the value of P = 6 and Q = 4

4. 6 × 4 = 24, Here 2 is carried over second column

⇒ 6 × p + 2 – 3 × 10 = 2 [∵ 21 – 3 × 6 = 3]

⇒ 6p – 30 = 0

⇒ p = 5

Now the multiplication problem becomes

		3	5	4
		×	q	6
	2	1	2	4
1	0	6	r	×
1	2	7	4	4

Here 2 + r = 4

⇒ r = 2

q × 354 = 1062

⇒ q = 3

Hence, p = 5, q = 3, r = 2

5. From the pattern, we observe that there are as many eights in the result as the first digit from the right which is to be multiplied by 9 and reduced by 1.

87654321 × 9 – 1 = 788888888

Long Answer:

1. (a) Given number = 89

Number obtained by reversing the order of digits = 98

Sum = 89 + 98 = 187 ÷ 11 = 17

Hence, the required number is 11.

(b) Given number = ab = 10a + b

Number obtained by reversing the digits = 10b + a

Sum = (10a + b) + (10b + a)

= 10a + b + 10b + a

= 11a + 11b

= 11(a + b) ÷ 11

= a + b

(c) Given number = 69

Number obtained by reversing the digits = 96

Sum = 69 + 96 = 165 ÷ 11 = 15

Hence, the required number is 11.

(d) Given number = 54

Number obtained by reversing the digits = 45

Sum = 54 + 45 = 99 ÷ 11 = 9

Hence, the required number is 11.

2. (i) Given number = 59

Number obtained by reversing the digits = 95

Difference = 95 – 59 = 36 ÷ 9 = 4

Hence, the required number is 9.

(ii) Given number = xy = 10x + y

Number obtained by reversing the digits = 10y + x

Difference = (10x + y) – (10y + x)

= 10x + y – 10y – x

= 9x – 9y

= 9(x – y) ÷ 9

= x – y

Hence, the required number is 9.

(iii) Given number = xyz = 100x + 10y + z

Number obtained by reversing the digits = 100z + 10y + x

Difference = (100x + 10y + z) – (100z + 10y + x)

= 100x + 10y + z – 100z – 10y – x

= 99x – 99z

= 99(x – z)

= 99(x – 2) ÷ 9

= 11 (x – z)

Hence, the required number is 9.

(iv) Given number = 203

Number obtained by reversing the digits = 302

Difference = 302 – 203 = 99 ÷ 9 = 11

Hence, the required number is 9.

Horizontal row	Vertical row
(a) 0.7 × 8 × 90 = 504	(a) 64% of 9200 = 5888
(c) 94.9 – 2.1 = 92.8	(b) 0.6 × 75 = 45
(e) 80% of 6600 = 5280	(c) 1079.2 – 90.2 = 989
(g) 0.5 × 168 = 84	(d) 0.4 × 50 = 20
(i) 0% of 125 = 0	(f) 512 + 1722 = 2234
(j) 10^4 – 1037 = 8963	(k) 896 × 7 = 6272
(m) 550 – 26 = 524	(l) (0.36 × 10^4) + 192 = 3792
(n) 100% of 79 = 79	(o) 10^2 × 9 = 900
(q) 360 + 30 = 390	(p) (3 × 10^1) + (3 × 10^0) = 33
(r) 8.34 – 5.22 = 3.12	(q) 70% of 50 = 35
(s) 8 × 5 × 13 = 520	

Hence the complete square is

5	0	4		9	2	8
8		5	2	8	0	
8	4		2	9		0
8	9	6	3		3	
	5	2	4		7	9
3		7		3	9	0
3	1	2		5	2	0

Let the required two 2-digit numbers be 10a + b and 10p + q as per the condition, we have

a × p = 10 and b × q = 21

a = 2 and p = 5 or a = 5 and p = 2

Similarly b × q = 21

b = 3 and q = 7 or b = 7 and q = 3

10p + q = 57 or 10p + q = 53

and 10a + b = 23 or 10a + b = 27

Since the units digit of product 1431 is 1.

Numbers are 57 and 23 or 53 and 27.

Now 57 × 23 = 1311 and 53 × 27 = 1431 which is given.

Hence, the required numbers are 53 and 27.