MATHEMATICS

Chapter 14: Factorisation

Factorisation

When an algebraic expression is written as the product of two or more expressions, then each of these expressions is called a factor of the given expression.

Factorisation of an expression means writing it as a product of its factors. These factors may be numbers, algebraic variables or algebraic expressions.

An irreducible factor is a factor which cannot be expressed further as a product of factors.

Rearranging the given expression to a convenient form allows us to form groups leading to factorization. This is regrouping.

Division is inverse process of multiplication.

Quotient of two monomials = (quotient of their coefficients) × (quotient of their variables).

While dividing a polynomial by a monomial, divide every term of the polynomial by the monomial. Division of a polynomial by a monomial can also be done by using common factor method and cancelling out the common factors.

While dividing a polynomial by a polynomial, we factorise both the polynomials and cancel out their common factors.

In division of algebraic expressions, we have,

Dividend = Quotient × Divisor + Remainder.

Factors

An expression can be factorised into the product of its factors. These factors can be algebraic expressions, variables and numbers also.

Division of a monomial by another monomial

i) Division of $9x^2$ by 3:

$9x^2 \div 3 = 3(3x^2) / 3 = 3x^2$

ii) Division of $6x^2y$ by 2y:

$6x^2y \div 2y = (6x^2)y / 2y = 2y(3x^2) / 2y = 3x^2$

Division of a polynomial by a monomial

A polynomial $2x^3 + 4x^2 + 6x$is divided by monomial 2x as shown below:

$(2x^3+4x^2+6x)2x = 2x^3 / 2x + 4x^2 2x + 6x / 2x = x^2 + 2x + 3$

Division of a polynomial by a polynomial

Long division method is used to divide a polynomial by a polynomial.

Example: Division of $3x^2 + 3x – 5$ by $(x – 1)$ is shown below:

$$
\begin{array}{r|rrrr}
 & 3x^2 & +3x & -5 & \\
\hline
x-1 & 3x^3 & +0x^2 & -8x & +5 \\
- & 3x^3 & -3x^2 & & \\
\hline
 & & 3x^2 & -8x & +5 \\
 & - & 3x^2 & -3x & \\
\hline
 & & & -5x & +5 \\
 & & - & -5x & +5 \\
\hline
 & & & & 0
\end{array}
$$

Types of Polynomial Division

For dividing polynomials, generally, three cases can arise:

Division of a monomial by another monomial

Division of a polynomial by monomial

Division of a polynomial by binomial

Division of a polynomial by another polynomial

Let us discuss all these cases one by one:

Division of a monomial by another monomial

Consider the algebraic expression 40x2 is to be divided by 10x then

$40x^2/10x = (2 \times 2 \times 5 \times 2 \times x \times x)/ (2 \times 5 \times x)$

Here, 2, 5 and x are common in both the numerator and the denominator.

Hence, $40x^2/10x = 4x$

Division of a polynomial by monomial

The second case is when a polynomial is to be divided by a monomial. For dividing polynomials, each term of the polynomial is separately divided by the monomial (as described above) and the quotient of each division is added to get the result. Consider the following example:

Example: Divide $24x^3 – 12xy + 9x$ by 3x.

Solution: The given expression $24x^3 – 12xy + 9x$ has three terms viz. $24x^3$, – 12xy and 9x. For dividing the polynomial with a monomial, each term is separately divided as shown below:$(24x^3–12xy + 9x)/3x = 8x^2 – 4y + 3$

Division of a Polynomial by Binomial

As we know, binomial is an expression with two terms. Dividing a polynomial by binomial can be done easily. Here, first we need to write the given polynomial in standard form. Now, using the long division method, we can divide the polynomial as given below.

Example: Divide $3x^3 – 8x + 5$ by $x – 1$.

Solution:

The Dividend is $3x^3 – 8x + 5$ and the divisor is $x – 1$.

After this, the leading term of the dividend is divided by the leading term of the divisor i.e., $3x^3 \div x = 3x^2$.

This result is multiplied by the divisor i.e., $3x^2 (x - 1) = 3x^3 - 3x^2$ and it is subtracted from the divisor.

Now again, this result is treated as a dividend and the same steps are repeated until the remainder becomes “0” or its degree becomes less than that of the divisor as shown below.

$$
\begin{array}{l|llll}
 & 3x^2 & +3x & -5 & \\
\hline
x-1 & 3x^3 & +0x^2 & -8x & +5 \\
- & 3x^3 & -3x^2 & & \\
\hline
 & & 3x^2 & -8x & +5 \\
 & - & 3x^2 & -3x & \\
\hline
 & & & -5x & +5 \\
 & & - & -5x & +5 \\
\hline
 & & & & 0
\end{array}
$$

Division of Polynomial by Another Polynomial

For dividing a polynomial with another polynomial, the polynomial is written in standard form i.e. the terms of the dividend and the divisor are arranged in decreasing order of their degrees. The method to solve these types of divisions is “Long division”. In algebra, an algorithm for dividing a polynomial by another polynomial of the same or lower degree is called polynomial long division. It is the generalized version of the familiar arithmetic technique called long division. Let us take an example.

Example: Divide $x^2 + 2x + 3x^3 + 5$ by $1 + 2x + x^2$.

Solution:

Let us arrange the polynomial to be divided in the standard form.

$3x^3 + x^2 + 2x + 5$

Divisor = $x^2 + 2x + 1$

Using the method of long division of polynomials, let us divide $3x^3 + x^2 + 2x + 5$ by $x^2 + 2x + 1$.

Step 1: To obtain the first term of the quotient, divide the highest degree term of the dividend, i.e. $3x^3$ by the highest degree term of the divisor, i.e. x^2.

$3x^3/x^2 = 3x$

Now, carry out the division process.

Step 2: Now, to obtain the second term of the quotient, divide the highest degree term of the new dividend, i.e. $-5x^2$ by the highest degree term of the divisor, i.e., x^2.

$-5x^2/x^2 = -5$

Again, carry out the division process with $-5x^2 - x + 5$ (the remainder in the previous step).

Step 3: The remainder obtained from the previous step is $9x + 10$.

The degree of $9x + 10$ is less than the divisor $x^2 + 2x + 1$. So, we cannot continue the division any further.

$$
\begin{array}{r|rrrr}
 & 3x & -5 & & \\
\hline
x^2 + 2x + 1 & 3x^3 & +x^2 & +2x & +5 \\
- & 3x^3 & +6x^2 & +3x & \\
\hline
 & & -5x^2 & -x & +5 \\
 & & - & & \\
 & & -5x^2 & -10x & -5 \\
\hline
 & & & 9x & +10
\end{array}
$$

Polynomial Division Algorithm

If p(x) and g(x) are any two polynomials with $g(x) \neq 0$, then we can find polynomials q(x) and r(x) such that

$p(x) = g(x) \times q(x) + r(x)$

Here,

$r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

This result is called the Division Algorithm for polynomials.

From the previous example, we can verify the polynomial division algorithm as:

$p(x) = 3x^3 + x^2 + 2x + 5$

$g(x) = x^2 + 2x + 1$

Also, quotient $= q(x) = 3x - 1$

remainder $= r(x) = 9x + 10$

Now,

$g(x) \times q(x) + r(x) = (x2 + 2x + 1) \times (3x - 5) + (9x + 10)$

$= 3x^3 + 6x^2 + 3x - 5x^2 - 10x - 5 + 9x + 10$

$= 3x^3 + x^2 + 2x + 5$

= p(x)

Hence, the division algorithm is verified.

Factors of natural numbers

Every number can be expressed in the form of product of prime factors. This is called prime factor form.

Example: Prime factor form of 42 is2 x 3 x 7, where 2, 3 and 7 are factors of 42.

Algebraic expressions

An algebraic expression is defined as the mathematical expression which consists of variables, numbers, and operations. The values of this expression are the not constant. For example: x + 1, p – q, 3x, 2x + 3y, 5a/6b etc.

Factors of algebraic expressions and factorisation

An irreducible factor is a factor which cannot be expressed further as a product of factors.

Algebraic expressions can be expressed in irreducible form.

A number or quantity that when multiplied with another number produces a given number or expression. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.

$12 = 1 \times 12$

$12 = 2 \times 6$

$12 = 3 \times 4$

Any number can be expressed in the form of its factors as shown above.

In terms of its prime factors, 12 can be expressed as:

$12 = 2 \times 3 \times 2$

Similarly, an algebraic expression can also be expressed in the form of its factors. An algebraic expression consists of variables, constants and operators. An algebraic expression consists of terms separated by an addition operation. Consider the following algebraic expression:

$3xyz - 16x^2 - yz$

This expression consists of 3 terms

$3xyz$ $-16x^2$ and $-yz$

Each term of this algebraic expression can be expressed in the form of its factors as:

$3xyz = 3.x.y.z$

$-16x^2 = -1.2.2.2.2.x.x$

$-yz = -1.y.z$

Algebraic Expressions can be factorized using many methods. The most common methods used for factorization of algebraic expressions are:

Factorization using common factors

Factorization by regrouping terms

Factorization using identities

Factorization using common factors

To factorize an algebraic expression, the highest common factors of the terms of the given algebraic expression are determined and then we group the terms accordingly. In simple terms, the reverse process of expansion of an algebraic expression is its factorization.

To understand this more clearly let us take an example.

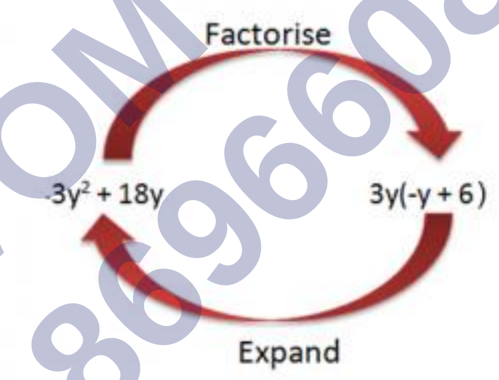
Example-

$-3y^2 + 18y$

Solution- The algebraic expression can be re-written as

$-3y^2 + 18y = -3.y.y + 3.6.y$

$\Rightarrow -3y^2 + 18y = -3.y(y–6)$



Consider the algebraic expression 3y (-y + 6), if we expand this we will get $-3y^2 + 18y$.

Factorization by regrouping terms

In some algebraic expressions, not every term may have a common factor. For instance, consider the algebraic expression 12a + n - na – 12. The terms of this expression do not have a particular factor in common but the first and last term has a common factor of ‘12’ similarly second and third term has n as a common factor. So the terms can be regrouped as:

$\Rightarrow$12a + n – na – 12= 12a – 12 + n – an

$\Rightarrow$12a – 12 – an + n = 12(a -1) –n(a -1)

After regrouping it can be seen that (a - 1) is a common factor in each term,

$\Rightarrow 12a + n - na - 12 = (a-1)(12 - n)$

Thus, by regrouping terms we can factorize algebraic expressions.

Factorizing Expressions using standard identities

An equality relation which holds true for all the values of variables in mathematics is known as an identity. Consider the following identities:

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a-b)^2 = a^2 + b^2 - 2ab$

$a^2 - b^2 = (a+b)(a-b)$

On substituting any value of a and b, both sides of the given equations remain the same. Therefore, these equations are called identities.

Example: Factorize

$9x^2 + 4m^2 + 12mx$

Solution: Observe the given expression carefully. This expression has three terms and all the terms are positive. Moreover, the first and the second term are perfect squares. The expression fits the form

$(a+b)^2 = a^2 + b^2 + 2ab$

where a = 3x, b = 2m.

$9x^2 + 4m^2 + 12mx = (3x)^2 + (2m)^2 + 2.3x.2m$

Therefore,

$9x^2 + 4m^2 + 12mx = (3x + 2m)^2$

Thus, the required factorization of **$9x^2 + 4m^2 + 12mx$ is $(3x + 2m)^2$** using standard identities.

Method of Common Factors

Factorisation by common factors

To factorise an algebraic expression, the highest common factors are determined.

Example: Algebraic expression $-2y^2 + 8y$ can be written as 2y (-y + 4), where 2y is the highest common factor in the expression.

Factorisation by regrouping terms

In some algebraic expressions, it is not possible that every term has a common factor. Therefore, to factorise those algebraic expressions, terms having common factors are grouped together.

Example:

= 12a + n – na – 12

= 12a – 12 + n - na

= 12 (a - 1) – n (a - 1)

= (12 - n) (a - 1)

(12 - n) and (a - 1) are factors of the expression 12a + n – na – 12

Method of Identities

Algebraic identities

The algebraic equations which are true for all values of variables in them are called algebraic identities.

Some of the identities are,

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

Factorisation using algebraic identities

Algebraic identities can be used for factorization

Example: (i) $9x^2 + 12xy + 4y^2$

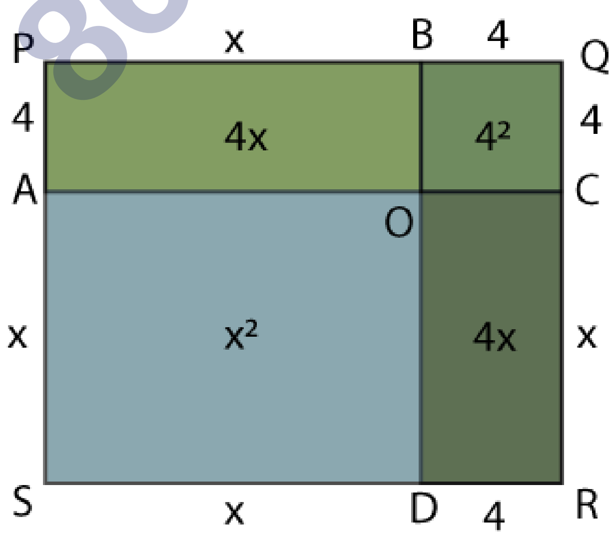
$= (3x)^2 + 2 \times 3x \times 2y + (2y)^2$

$= (3x+4y)^2$

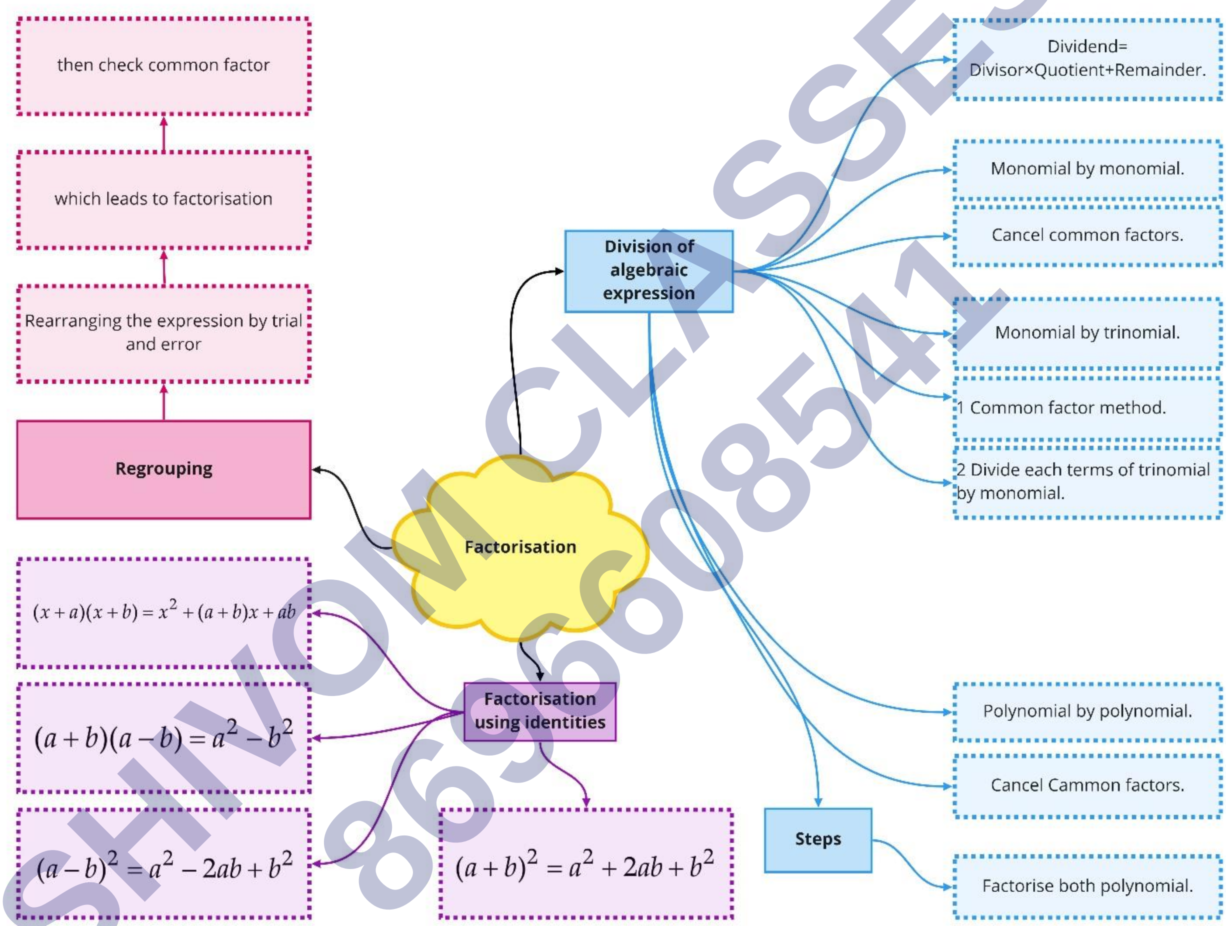
(ii) $4a^2 - b^2 = (2a-b)(2a+b)$

Visualisation of factorisation

The algebraic expression $x^2 + 8x + 16$ can be written as $(x + 4)^2$. This can be visualised as shown below:



Class : 8th Mathematics
Chapter-14 Factorisation



Important Questions

Multiple Choice Questions:

Question 1. The common factor of x^2y^2 and x^3y^3 is

(a) x^2y^2

(b) x^3y^3

(c) x^2y^3

(d) x^3y^2.

Question 2. The common factor of x^3y^2 and x^4y is

(a) $x^{43}y^2$

(b) x^4y

(c) x^3y^2

(d) x^3y.

Question 3. The common factor of a^2 m^4 and a^4m^2 is

(a) a^4m^4

(b) a^2m^2

(c) a^2m^4

(d) a^4m^2

Question 4. The common factor of p^3q^4 and p^4q^3 is

(a) p^4q^4

(b) p^4q^3

(c) p^3q^3

(d) p^3q^4

Question 5. The common factor 12y and 30 is

(a) 6

(b) 12

(c) 30

(d) 6y.

Question 6. The common factor of 2x, $3x^3$, 4 is

(a) 1

(b) 2

(c) 3

(d) 4.

Question 7. The common factor of 10ab, 30bc, 50ca is

(a) 10

(b) 30

(c) 50

(d) abc.

Question 8. The common factor of $14a^2b$ and $35a^4b^2$ is

(a) a^4b^2

(b) $35a^4b^2$

(c) $14a^2b$

(d) $7a^2b$.

Question 9. The common factor of $8a^2b^4c^2$, $12a^4bc^4$ and $20a^3b^4$ is

(a) a^4b^4

(b) a^2b^2

(c) $4a^2b^2$

(d) $4a^2b$.

Question 10. The common factor of $6a^3b^4c^2$, $21a^2b$ and $15a^3$ is

(a) $3a^2$

(b) $3a^3$

(c) $6a^3$

(d) $6a^2$

Very Short Questions:

1. Find the common factors of the following terms.

(a) $25x^2y$, $30xy^2$

(b) $63m^3n$, $54mn^4$

2. Factorise the following expressions.

(a) $54m^3n + 81m^4n^2$

(b) $15x^2y^3z + 25x^3y^2z + 35x^2y^2z^2$

3. Factorise the following polynomials.

(a) $6p(p – 3) + 1 (p – 3)$

(b) $14(3y – 5z)^3 + 7(3y – 5z)^2$

4. Factorise the following:

(a) $p^2q - pr^2 - pq + r^2$

(b) $x^2 + yz + xy + xz$

5. Factorise the following polynomials.

(a) $xy(z^2 + 1) + z(x^2 + y^2)$

(b) $2axy^2 + 10x + 3ay^2 + 15$

Short Questions:

1. Factorise the following expressions.

(a) $x^2 + 4x + 8y + 4xy + 4y^2$

(b) $4p^2 + 2q^2 + p^2q^2 + 8$

2. Factorise:

(a) $a^2 + 14a + 48$

(b) $m^2 - 10m - 56$

3. Factorise:

(a) $x^4 - (x - y)^4$

(b) $4x^2 + 9 - 12x - a^2 - b^2 + 2ab$

4. Factorise the following polynomials.

(a) $16x^4 - 81$

(b) $(a - b)^2 + 4ab$

5. Factorise:

(a) $14m^5n^4p^2 - 42m^7n^3p^7 - 70m^6n^4p^3$

(b) $2a^2(b^2 - c^2) + b^2(2c^2 - 2a^2) + 2c^2(a^2 - b^2)$

Long Questions:

1. Factorise:

(a) $(x + y)^2 - 4xy - 9z^2$

(b) $25x^2 - 4y^2 + 28yz - 49z^2$

2. Evaluate the following divisions:

(a) $(3b - 6a) \div (30a - 15b)$

(b) $(4x2 - 100) \div 6(x + 5)$

3. Simplify the following expressions:

(a) $\frac{(x-1)(x-2)(x^2-9x+14)}{(x-7)(x^2-3x+2)}$

(b) $\frac{(x^2-8x+12)(x^2-16)}{(x^2-36)(x^2-4)}$

4. Factorise the given expressions and divide that as indicated.

(a) $39n^3(50n^2 - 98) \div 26n^2(5n - 7)$

(b) $44(p^4 - 5p^3 - 24p^2) \div 11p(p - 8)$

5. If one of the factors of $(5x^2 + 70x - 160)$ is $(x - 2)$. Find the other factor.

Answer Key-

Multiple Choice Qquestions:

1. (a) x^2y^2
2. (d) x^3y.
3. (b) a^2m^2
4. (c) p^3q^3
5. (a) 6
6. (a) 1
7. (a) 10
8. (d) $7a^2b$.
9. (d) $4a^2b$.
10. (a) $3a^2$

Very Short Answer:

1. (a) $25x^2y$, $30xy^2$

$25x^2y = 5 \times 5 \times x \times x \times y$

$30xy^2 = 2 \times 3 \times 5 \times x \times y \times y$

Common factors are $5 \times x \times y = 5\,xy$

(b) $63m^3n$, $54mn^4$

$63m^3n = 3 \times 3 \times 7 \times m \times m \times m \times n$

$54mn^4 = 2 \times 3 \times 3 \times 3 \times m \times n \times n \times n \times n$

Common factors are $3 \times 3 \times m \times n = 9mn$

2. (a) $54m^3n + 81m^4n^2$

$= 2 \times 3 \times 3 \times 3 \times m \times m \times m \times n + 3 \times 3 \times 3 \times 3 \times m \times m \times m \times m \times n \times n$

$= 3 \times 3 \times 3 \times m \times m \times m \times n \times (2 + 3\,mn)$

$= 27m^3n\,(2 + 3mn)$

(b) $15x^2y^3z + 25\,x^3y^2z + 35x^2y^2z^2 = 5x^2y^2z\,(\,3y + 5x + 7)$

3. (a) $6p(p - 3) + 1\,(p - 3) = (p - 3)\,(6p + 1)$

(b) $14(3y - 5z)^3 + 7(3y - 5z)^2$

$= 7(3y - 5z)^2\,[2(3y - 5z) + 1]$

$= 7(3y - 5z)^2\,(6y - 10z + 1)$

4. (a) $p^2q - pr^2 - pq + r^2$

$= (p^2q - pq) + (-pr^2 + r^2)$

$= pq(p - 1) - r^2(p - 1)$

$= (p - 1)\,(pq - r^2)$

(b) $x^2 + yz + xy + xz$

$= x^2 + xy + xz + yz$

$= x(x + y) + z(x + y)$

$= (x + y)\,(x + z)$

5. (a) $xy(z^2 + 1) + z(x^2 + y^2)$

$= xyz^2 + xy + 2x^2 + zy^2$

$= (xyz^2 + zx^2) + (xy + zy^2)$

$= zx(yz + x) + y(x + yz)$

$= zx(x + yz) + y(x + yz)$

$= (x + yz)\,(zx + y)$

(b) $2axy^2 + 10x + 3ay^2 + 15$

$= (2axy^2 + 3ay^2) + (10x + 15)$

$= ay^2(2x + 3) + 5(2x + 3)$

$= (2x + 3)\,(ay^2 + 5)$

Short Answer:

1. (a) $x^2 + 4x + 8y + 4xy + 4y^2$

$= (x^2 + 4xy + 4y^2) + (4x + 8y)$

$= (x + 2y)^2 + 4(x + 2y)$

$= (x + 2y)(x + 2y + 4)$

(b) $4p^2 + 2q^2 + p^2q^2 + 8$

$= (4p^2 + 8) + (p^2q^2 + 2q^2)$

$= 4(p^2 + 2) + q^2(p^2 + 2)$

$= (p^2 + 2)(4 + q^2)$

2. (a) $a^2 + 14a + 48$

$= a^2 + 6a + 8a + 48$

$[6 + 8 = 14 ; 6 \times 8 = 48]$

$= a(a + 6) + 8(a + 6)$

$= (a + 6)(a + 8)$

(b) $m^2 - 10m - 56$

$= m^2 - 14m + 4m - 56$

$[14 - 4 = 10; 4 \times 4 = 56]$

$= m(m - 14) + 6(m - 14)$

$= (m - 14)(m + 6)$

3. (a) $x^4 - (x - y)^4$

$= (x^2)^2 - [(x - y)^2]^2$

$= [x^2 - (x - y)^2] [x^2 + (x - y)^2]$

$= [x + (x - y] [x - (x - y)] [x^2 + x^2 - 2xy + y^2]$

$= (x + x - y) (x - x + y)[2x^2 - 2xy + y^2]$

$= (2x - y) y(2x^2 - 2xy + y^2)$

$= y(2x - y) (2x^2 - 2xy + y^2)$

(b) $4x^2 + 9 - 12x - a^2 - b^2 + 2ab$

$= (4x^2 - 12x + 9) - (a^2 + b^2 - 2ab)$

$= (2x - 3)^2 - (a - b)^2$

$= [(2x - 3) + (a - b)] [(2x - 3) - (a - b)]$

$= (2x - 3 + a - b)(2x - 3 - a + b)$

4. (a) $16x^4 - 81$

$= (4x^2)^2 - (9)^2$

$= (4x^2 + 9)(4x^2 - 9)$

$= (4x^2 + 9)[(2x)^2 - (3)^2]$

$= (4x^2 + 9)(2x + 3)(2x - 3)$

(b) $(a - b)^2 + 4ab$

$= a^2 - 2ab + b^2 + 4ab$

$= a^2 + 2ab + b^2$

$= (a + b)^2$

5. (a) $14m^5n^4p^2 - 42m^7n^3p^7 - 70m^6n^4p^3$

$= 14m^5n^3p^2(n - 3m^2p^5 - 5mnp)$

(b) $2a^2(b^2 - c^2) + b^2(2c^2 - 2a^2) + 2c^2(a^2 - b^2)$

$= 2a^2(b^2 - c^2) + 2b^2(c^2 - a^2) + 2c^2(a^2 - b^2)$

$= 2[a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)]$

$$= 2\left[\cancel{a^2b^2} - \cancel{a^2c^2} + \cancel{b^2c^2} - \cancel{a^2b^2} + \cancel{a^2c^2} - \cancel{b^2c^2}\right]$$

$= 2 \times 0$

$= 0$

Long Answer:

1. (a) $(x + y)^2 - 4xy - 9z^2$

$= x^2 + 2xy + y^2 - 4xy - 9z^2$

$= (x^2 - 2xy + y^2) - 9z^2$

$= (x - y)^2 - (3z)^2$

$= (x - y + 3z)(x - y - 3z)$

(b) $25x^2 - 4y^2 + 28yz - 49z^2$

$= 25x^2 - (4y^2 - 28yz + 49z^2)$

$= (5x)^2 - (2y - 7)^2$

$= (5x + 2y - 7)[5x - (2y - 7)]$

$= (5x + 2y - 7)(5x - 2y + 7)$

2.

(a) $(3b - 6a) \div (30a - 15b)$

$$= \frac{3b-6a}{30a-15b} = \frac{-3(2a-b)}{15(2a-b)}$$

$$= \frac{-1}{5}$$

(b) $(4x^2 - 100) \div 6(x + 5)$

$$= \frac{4x^2-100}{6(x+5)} = \frac{4(x^2-25)}{6(x+5)}$$

$$= \frac{4(x-5)(x+5)}{6(x+5)}$$

$$= \frac{2}{3}(x-5)$$

3.

(a) $$\frac{(x-1)(x-2)(x^2-9x+14)}{(x-7)(x^2-3x+2)}$$

$$= \frac{(x-1)(x-2)(x^2-7x-2x+14)}{(x-7)(x^2-2x-x+2)}$$

$$= \frac{(x-1)(x-2)[x(x-7)-2(x-7)]}{(x-7)[x(x-2)-1(x-2)]}$$

$$= \frac{(x-1)(x-2)(x-7)(x-2)}{(x-7)(x-2)(x-1)} = (x-2)$$

(b) $$\frac{(x^2-8x+12)(x^2-16)}{(x^2-36)(x^2-4)}$$

$$= \frac{(x^2-6x-2x+12)(x-4)(x+4)}{(x-6)(x+6)(x-2)(x+2)}$$

$$= \frac{[x(x-6)-2(x-6)](x-4)(x+4)}{(x-6)(x+6)(x-2)(x+2)}$$

$$= \frac{(x-2)(x-6)(x-4)(x+4)}{(x-6)(x+6)(x-2)(x+2)}$$

$$= \frac{(x-4)(x+4)}{(x+6)(x+2)}$$

4.

(a) $39n^3(50n^2 - 98) \div 26n^2(5n + 7)$

$$= \frac{39n^3(50n^2 - 98)}{26n^2(5n - 7)}$$

$$= \frac{39n^3 \times 2(25n^2 - 49)}{26n^2(5n - 7)}$$

$$= \frac{3 \times 13n^3 \times 2[(5n)^2 - (7)^2]}{2 \times 13n^2(5n - 7)}$$

$$= \frac{3 \times \cancel{13}\,\cancel{n^3}^{\,n} \times \cancel{2}\,(5n + 7)\,\cancel{(5n - 7)}}{\cancel{2} \times \cancel{13}\,\cancel{n^2}\,\cancel{(5n - 7)}}$$

$= 3n(5n + 7)$

(b) $44(p^4 - 5p^3 - 24p^2) \div 11p(p - 8)$

$$= \frac{44(p^4 - 5p^3 - 24p^2)}{11p(p - 8)}$$

$$= \frac{44 \times p^2(p^2 - 5p - 24)}{11p(p - 8)}$$

$$= \frac{44p^2(p^2 - 8p + 3p - 24)}{11p(p - 8)}$$

$$= \frac{44p^2[p(p - 8) + 3(p - 8)]}{11p(p - 8)}$$

$$= \frac{\cancel{44}^{\,4}\,\cancel{p^2}^{\,p}\,\cancel{(p - 8)}(p + 3)}{\cancel{11}\,\cancel{p}\,\cancel{(p - 8)}}$$

$= 4p(p + 3)$

5. Let the other factor be m.

$(x - 2) \times m = 5x^2 + 70x - 160$

$$\Rightarrow \quad m = \frac{5x^2 + 70x - 160}{x - 2}$$

$$= \frac{5(x^2 + 14x - 32)}{x - 2}$$

$$= \frac{5(x^2 + 16x - 2x - 32)}{x - 2}$$

$$= \frac{5[x(x + 16) - 2(x + 16)]}{x - 2}$$

$$= \frac{5(x + 16)\cancel{(x - 2)}}{\cancel{(x - 2)}}$$

$$= 5(x + 16)$$