

# MATHEMATICS

## Chapter 12: Heron's Formula



## Heron's Formula

1. The region enclosed within a simple closed figure is called its **area**.
2. **Area of a triangle** =  $\frac{1}{2} \times \text{base} \times \text{height}$
3. **Area of an equilateral triangle** =  $\frac{\sqrt{3}}{4} a^2$  sq units, where 'a' is the side length of an equilateral triangle.
4. **Semi-perimeter** is half of the perimeter.
5. If a, b and c denote the lengths of the sides of a triangle, then the area of the triangle is calculated by using **Heron's formula**, as given below:  

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}, s = \text{semi-perimeter} = \frac{a+b+c}{2}$$
6. For every triangle, the values of (s - a), (s - b), and (s - c) are positive.
7. Area of a quadrilateral can be calculated by dividing the quadrilateral into two triangles and using Heron's formula for calculating area of each triangle.

### Triangle

The plane closed figure, with three sides and three angles is called as a triangle.

#### Types of triangles:

Based on sides a) Equilateral b) Isosceles c) Scalene

Based on angles a) Acute angled triangle b) Right-angled triangle c) Obtuse angled triangle

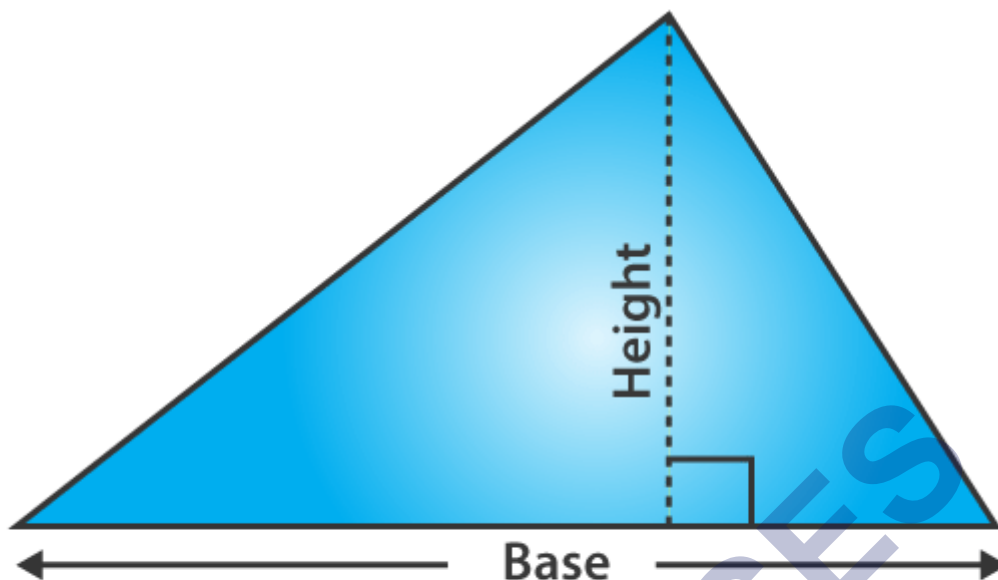
#### Area of a triangle

Area =  $(1/2) \times \text{base} \times \text{height}$

In case of equilateral and isosceles triangles, if the lengths of the sides of triangles are given then, we use Pythagoras theorem in order to find the height of a triangle.

Area of a triangle is the region enclosed by it, in a two-dimensional plane. As we know, a triangle is a closed shape that has three sides and three vertices. Thus, the area of a triangle is the total space occupied within the three sides of a triangle. The general formula to find the area of the triangle is given by half of the product of its base and height.

In general, the term "area" is defined as the region occupied inside the boundary of a flat object or figure. The measurement is done in square units with the standard unit being square meters (m<sup>2</sup>). For the computation of area, there are pre-defined formulas for squares, rectangles, circle, triangles, etc. In this article, we will learn the area of triangle formulas for different types of triangles, along with some example problems.



$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

Example: What is the area of a triangle with base  $b = 3$  cm and height  $h = 4$  cm?

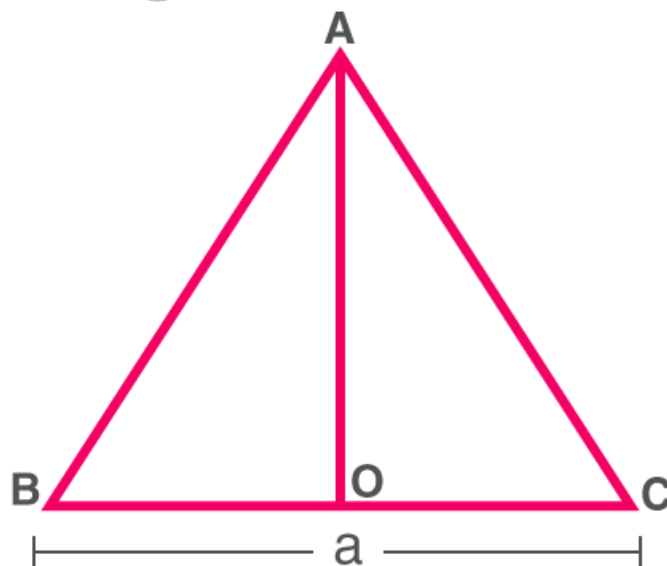
Using the formula,

$$\text{Area of a Triangle, } A = \frac{1}{2} \times b \times h = \frac{1}{2} \times 4 \text{ cm} \times 3 \text{ cm} = 2 \text{ cm} \times 3 \text{ cm} = 6 \text{ cm}^2$$

Apart from the above formula, we have Heron's formula to calculate the triangle's area, when we know the length of its three sides. Also, trigonometric functions are used to find the area when we know two sides and the angle formed between them in a triangle. We will calculate the area for all the conditions given here.

### Area of an equilateral triangle

Consider an equilateral  $\triangle ABC$ , with each side as a unit. Let  $AO$  be the perpendicular bisector of  $BC$ . In order to derive the formula for the area of an equilateral triangle, we need to find height  $AO$ .



Using Pythagoras theorem,

$$AC^2 = OA^2 + OC^2$$

$$OA^2 = AC^2 - OC^2$$

Substitute  $AC = a$ ,  $OC = a/2$  in the above equation.

$$OA^2 = a^2 - a^2/4$$

$$OA = \sqrt{3}a/2$$

We know that the area of the triangle is:

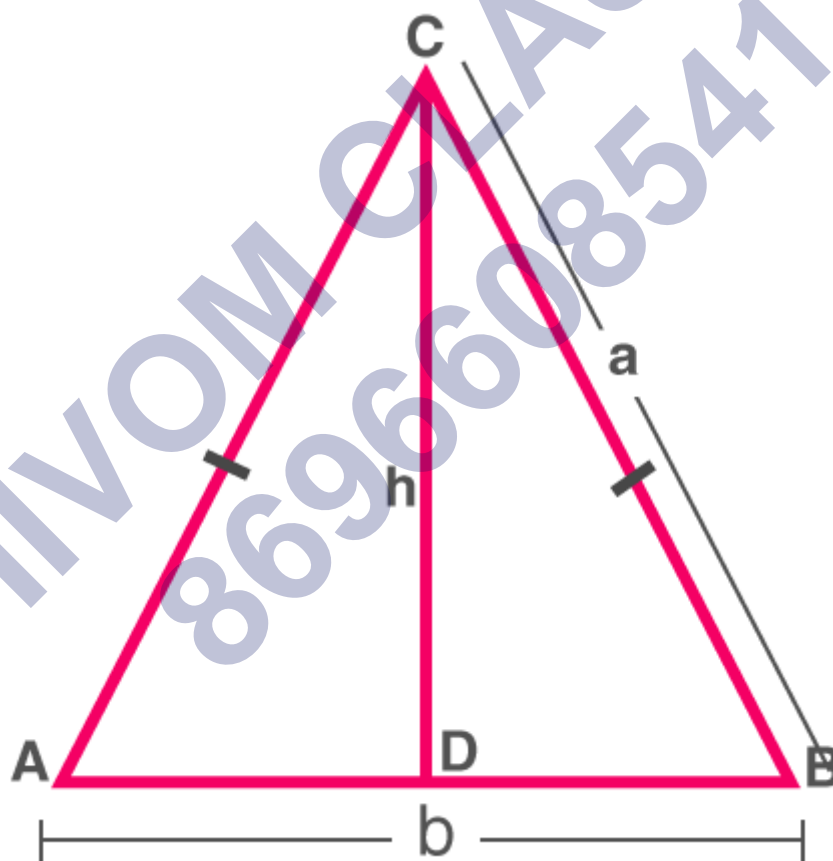
$$A = (1/2) \times \text{base} \times \text{height}$$

$$A = (1/2) \times a \times (\sqrt{3}a/2)$$

$$\therefore \text{Area of Equilateral triangle} = \sqrt{3}a^2/4$$

### Area of an isosceles triangle

Consider an isosceles  $\triangle ABC$  with equal sides as  $a$  units and base as  $b$  units.



Isosceles triangle ABC

The height of the triangle can be found by Pythagoras' Theorem:

$$CD^2 = AC^2 - AD^2$$

$$\Rightarrow h^2 = a^2 - (b^2/4) = (4a^2 - b^2)/4$$

$$\Rightarrow h = (1/2) \sqrt{4a^2 - b^2}$$

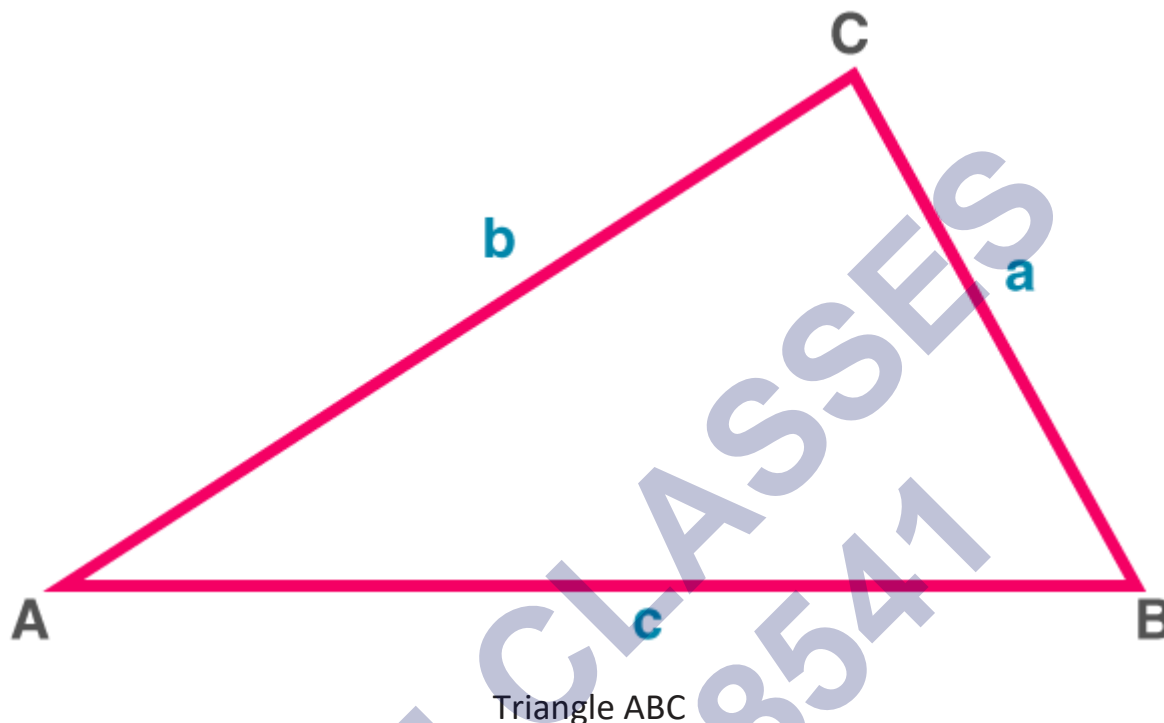
$$\text{Area of triangle is } A = (1/2) bh$$

$$\therefore A = (1/2) \times b \times (1/2) \sqrt{4a^2 - b^2}$$

$$\therefore A = (1/4) \times b \times \sqrt{4a^2 - b^2}$$

### Area of a triangle – By Heron's formula

Area of a  $\triangle ABC$ , given sides  $a$ ,  $b$ ,  $c$  by Heron's formula (also known as Hero's Formula) is:



Find semi perimeter ( $s$ ) =  $(a + b + c)/2$

$$\text{Area} = \sqrt{[s(s - a)(s - b)(s - c)]}$$

This formula is helpful to find the area of a scalene triangle, given the lengths of all its sides.

### Heron's Formula

Heron's formula is used to find the area of a triangle when we know the length of all its sides. It is also termed as Hero's Formula. We can use Heron's formula to find different types of triangles, such as scalene, isosceles and equilateral triangles. We don't have to need to know the angle measurement of a triangle to calculate its area, by using Heron's formula.

Heron's formula is a formula to calculate the area of triangles, given the three sides of the triangle. This formula is also used to find the area of the quadrilateral, by dividing the quadrilateral into two triangles, along its diagonal.

If  $a$ ,  $b$  and  $c$  are the three sides of a triangle, respectively, then Heron's formula is given by:

$$\text{Area of triangle using three sides} = \sqrt{s(s - a)(s - b)(s - c)}$$

$$\text{Semiperimeter, } s = \text{Perimeter of triangle}/2 = (a + b + c)/2$$

### History of Heron's Formula

Hero of Alexandria was a great mathematician who derived the formula for the calculation of the area of a triangle using the length of all three sides. He also extended this idea to find the area of quadrilateral and also higher-order polygons. This formula has its huge

applications in trigonometry such as proving the law of cosines or the law of cotangents, etc.

### Proof of Heron's Formula

There are two methods by which we can derive Heron's formula.

- First, by using trigonometric identities and cosine rule.
- Secondly, solving algebraic expressions using the Pythagoras theorem.

Let us see one by one both the proofs or derivation.

### Using Cosine Rule

Let us prove the result using the law of cosines:

Let  $a, b, c$  be the sides of the triangle and  $\alpha, \beta, \gamma$  are opposite angles to the sides.

We know that the law of cosines is

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

Again, using trig identity, we have

$$\begin{aligned} \sin \gamma &= \sqrt{1 - \cos^2 \gamma} \\ &= \frac{\sqrt{4a^2b^2 - (a^2 + b^2 - c^2)^2}}{2ab} \end{aligned}$$

Here, Base of triangle =  $a$

Altitude =  $b \sin \gamma$

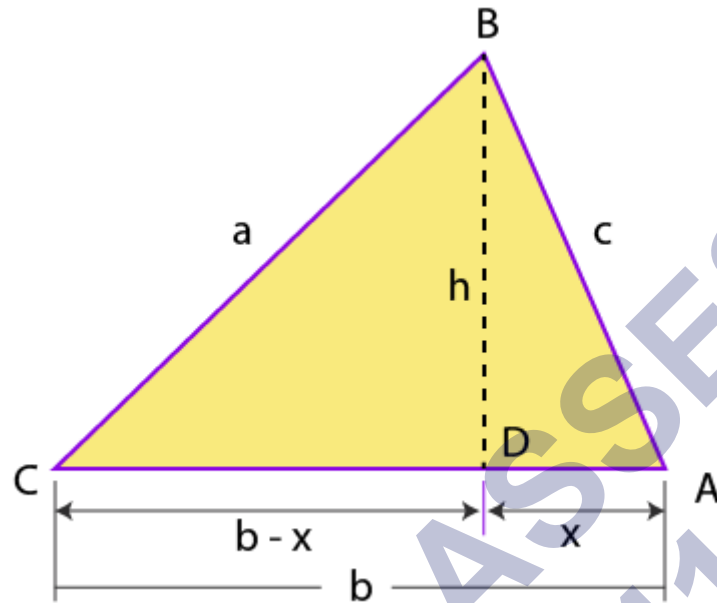
Now,

$$\begin{aligned} A &= \frac{1}{2}(\text{base})(\text{altitude}) \\ &= \frac{1}{2}ab \sin \gamma \\ &= \frac{1}{4} \sqrt{4a^2b^2 - (a^2 + b^2 - c^2)^2} \\ &= \frac{1}{4} \sqrt{(2ab - (a^2 + b^2 - c^2))(2ab + (a^2 + b^2 - c^2))} \\ &= \frac{1}{4} \sqrt{(c^2 - (a - b)^2)((a + b)^2 - c^2)} \\ &= \sqrt{\frac{(c - (a - b))(c + (a - b))((a + b) - c)((a + b) + c)}{16}} \\ &= \sqrt{\frac{(b + c - a)}{2} \frac{(a + c - b)}{2} \frac{(a + b - c)}{2} \frac{(a + b + c)}{2}} \\ &= \sqrt{\frac{(a + b + c)}{2} \frac{(b + c - a)}{2} \frac{(a + c - b)}{2} \frac{(a + b - c)}{2}} \\ &= \sqrt{s(s - a)(s - b)(s - c)}. \end{aligned}$$

## Using Pythagoras Theorem

Area of a Triangle with 3 Sides

Area of  $\Delta ABC$  is given by



$$A = \frac{1}{2} bh \quad \text{--- (i)}$$

Draw a perpendicular BD on AC

Consider a  $\Delta ADB$

$$x^2 + h^2 = c^2$$

$$x^2 = c^2 - h^2 \quad \text{--- (ii)}$$

$$\Rightarrow x = \sqrt{c^2 - h^2} \quad \text{--- (iii)}$$

Consider a  $\Delta CDB$ ,

$$(b - x)^2 + h^2 = a^2$$

$$(b - x)^2 = a^2 - h^2$$

$$b^2 - 2bx + x^2 = a^2 - h^2$$

Substituting the value of  $x$  and  $x^2$  from equation (ii) and (iii), we get

$$b^2 - 2b\sqrt{c^2 - h^2} + c^2 - h^2 = a^2 - h^2$$

$$b^2 + c^2 - a^2 = 2b\sqrt{c^2 - h^2}$$

Squaring on both sides, we get;

$$(b^2 + c^2 - a^2)^2 = 4b^2(c^2 - h^2)$$

$$\frac{(b^2+c^2-a^2)^2}{4b^2} = c^2 - h^2$$

$$h^2 = c^2 - \frac{(b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{4b^2c^2 - (b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{(2bc)^2 - (b^2+c^2-a^2)^2}{4b^2}$$

$$h^2 = \frac{[2bc+(b^2+c^2-a^2)][2bc-(b^2+c^2-a^2)]}{4b^2}$$

$$h^2 = \frac{[(b^2+2bc+c^2)-a^2][a^2-(b^2-2bc+c^2)]}{4b^2}$$

$$h^2 = \frac{[(b+c)^2-a^2] \cdot [a^2-(b-c)^2]}{4b^2}$$

$$h^2 = \frac{[(b+c)+a][(b+c)-a] \cdot [a+(b-c)][a-(b-c)]}{4b^2}$$

$$h^2 = \frac{(a+b+c)(b+c-a)(a+c-b)(a+b-c)}{4b^2}$$

The perimeter of a  $\Delta ABC$  is

$$P = a + b + c$$

$$\Rightarrow h^2 = \frac{P(P-2a)(P-2b)(P-2c)}{4b^2}$$

$$\Rightarrow h = \sqrt{P(P-2a)(P-2b)(P-2c)} \cdot 2b$$

Substituting the value of h in equation (i), we get;

$$A = \frac{1}{2}b \frac{\sqrt{P(P-2a)(P-2b)(P-2c)}}{2b}$$

$$A = \frac{1}{4} \sqrt{(P(P-2a)(P-2b)(P-2c))}$$

$$A = \sqrt{\frac{1}{16} P(P-2a)(P-2b)(P-2c)}$$

$$A = \sqrt{\frac{P}{2} \left(\frac{P-2a}{2}\right) \left(\frac{P-2b}{2}\right) \left(\frac{P-2c}{2}\right)}$$

Semi perimeter(s) =

$$\frac{\text{perimeter}}{2} = \frac{P}{2}$$

$$\Rightarrow A = \sqrt{s(s-a)(s-b)(s-c)}$$

Note: Heron's formula is applicable to all types of triangles and the formula can also be derived using the law of cosines and the law of Cotangents.

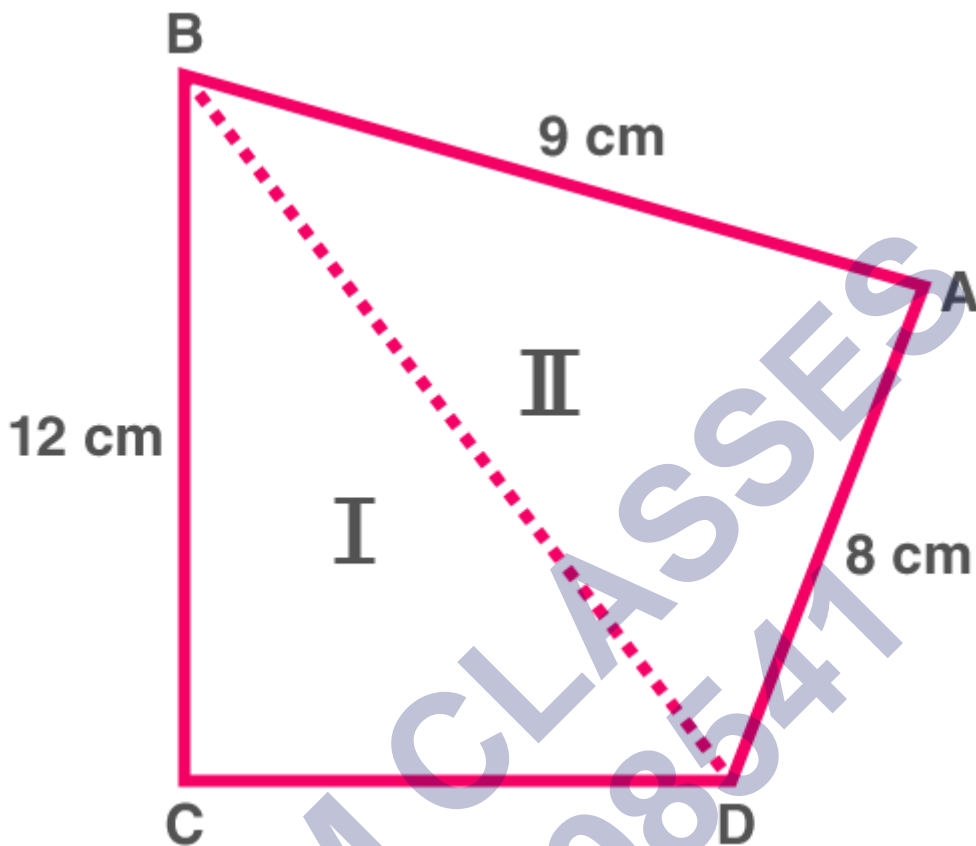
### Area of any polygon – By Heron's formula

For a quadrilateral, when one of its diagonal value and the sides are given, the area can be calculated by splitting the given quadrilateral into two triangles and use the Heron's formula.



Example: A park, in the shape of a quadrilateral ABCD, has  $\angle C = 90^\circ$ ,  $AB = 9$  cm,  $BC = 12$  cm,  $CD = 5$  cm and  $AD = 8$  cm. How much area does it occupy?

⇒ We draw the figure according to the information given.



The figure can be split into 2 triangles  $\triangle BCD$  and  $\triangle ABD$

From  $\triangle BCD$ , we can find  $BD$  (Using Pythagoras' Theorem)

$$BD^2 = 12^2 + 5^2 = 169$$

$$BD = 13\text{cm}$$

$$\text{Semi-perimeter for } \triangle BCD \ S_1 = (12 + 5 + 13)/2 = 15$$

$$\text{Semi-perimeter } \triangle ABD \ S_2 = (9 + 8 + 13)/2 = 15$$

Using Heron's formula  $A_1$  and  $A_2$  will be:

$$A_1 = \sqrt{[15(15 - 12)(15 - 5)(15 - 13)]}$$

$$A_1 = \sqrt{(15 \times 3 \times 10 \times 2)}$$

$$A_1 = \sqrt{900} = 30 \text{ cm}^2$$

Similarly,

$$A_2 \text{ will be } 35.49 \text{ cm}^2.$$

$$\text{The area of the quadrilateral } ABCD = A_1 + A_2 = 65.49 \text{ cm}^2$$

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By Heron's formula

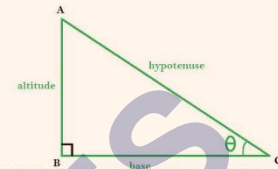
Area of triangle =  $\sqrt{s(s-a)(s-b)(s-c)}$

where,  $s = \frac{a+b+c}{2}$

Here,  $s = \frac{122+22+120}{2} = 132 \text{ cm}$

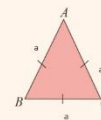
$$\begin{aligned} \therefore \text{Area} &= \sqrt{132(132-122)(132-22)(132-120)} \text{ cm}^2 \\ &= \sqrt{132(10)(110)(12)} \text{ cm}^2 \\ &= 1320 \text{ cm}^2 \end{aligned}$$

$$\frac{1}{2} \times \text{Base} \times \text{Altitude}$$



Right - angled triangle

Equilateral triangle



$$\frac{\sqrt{3}}{4} a^2 ; a = \text{side of triangle}$$

Area

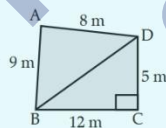
Heron's Formula

Heron's Formula

Find area of triangle  
of sides 122cm,  
22cm, 120cm

Applications

Area of quadrilateral  
ABCD with given  
dimensions :-



Area of ABCD = Area of  $\triangle ABD$  + area of  $\triangle BCD$

$$\begin{aligned} \text{Here, area of } \triangle BCD &= \frac{1}{2} \times BC \times CD \\ &= \frac{1}{2} \times 12 \times 5 = 30 \text{ m}^2 \end{aligned}$$

$$\text{Here } BD = \sqrt{BC^2 + DC^2}$$

$$BD = \sqrt{12^2 + 5^2} = 13 \text{ m}$$

$$\text{Area of } \triangle ABD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2} = \frac{9+8+13}{2} \text{ m} = 15 \text{ m}$$

$$\text{Area} = \sqrt{15(15-9)(15-8)(15-13)} \text{ m}^2$$

$$ABD = \sqrt{15(6)(7)(2)} \text{ m}^2$$

$$= 35.496 \text{ m}^2$$

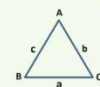
$$\therefore \text{Area of ABCD} = (30+35.496) \text{ m}^2 = 65.496 \text{ m}^2$$

**Heron's Formula**

Area of a triangle with its sides as  $a$ ,  $b$  and  $c$  is calculated by using Heron's Formula stated as

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

Where,  $S$  = semi - perimeter =  $\frac{a+b+c}{2}$ .



**Applications of Heron's Formula**

Area of a quadrilateral whose sides and one diagonal are given, can be calculated by dividing the quadrilateral into two triangles and then using the Heron's Formula to find the area of the two triangles thus formed.



Then we add the area of two triangles to get the area of the quadrilateral.

## Important Questions

### Multiple Choice questions-

Question 1. An isosceles right triangle has area  $8\text{cm}^2$ . The length of its hypotenuse is

- (a)  $\sqrt{32}\text{cm}$
- (b)  $\sqrt{16}\text{cm}$
- (c)  $\sqrt{48}\text{cm}$
- (d)  $\sqrt{24}\text{cm}$

Question 2. The perimeter of an equilateral triangle is  $60\text{m}$ . The area is

- (a)  $10\sqrt{3}\text{m}^2$
- (b)  $15\sqrt{3}\text{m}^2$
- (c)  $20\sqrt{3}\text{m}^2$
- (d)  $100\sqrt{3}\text{m}^2$

Question 3. The sides of a triangle are  $56\text{cm}$ ,  $60\text{cm}$  and  $52\text{cm}$  long. Then the area of the triangle is

- (a)  $1322\text{cm}^2$
- (b)  $1311\text{cm}^2$
- (c)  $1344\text{cm}^2$
- (d)  $1392\text{cm}^2$

Question 4. The area of an equilateral triangle with side  $2\sqrt{3}\text{cm}$  is

- (a)  $5.196\text{cm}^2$
- (b)  $0.866\text{cm}^2$
- (c)  $3.496\text{cm}^2$
- (d)  $1.732\text{cm}^2$

Question 5. The length of each side of an equilateral triangle having an area of  $9\sqrt{3}\text{cm}^2$  is

- (a)  $8\text{cm}$
- (b)  $36\text{cm}$
- (c)  $4\text{cm}$
- (d)  $6\text{cm}$

Question 6. If the area of an equilateral triangle is  $16\sqrt{3}\text{cm}^2$ , then the perimeter of the triangle is

- (a)  $48\text{cm}$

- (b) 24cm
- (c) 12cm
- (d) 36cm

Question 7. The sides of a triangle are 35cm, 54cm and 61cm. The length of its longest altitude is

- (a)  $16\sqrt{5}$ cm
- (b)  $10\sqrt{5}$ cm
- (c)  $24\sqrt{5}$ cm
- (d) 28cm

Question 8. The area of an isosceles triangle having base 2cm and the length of one of the equal sides 4 cm is

- (a)  $15\sqrt{\text{cm}^2}$
- (b)  $\sqrt{\frac{15}{2}}\text{cm}^2$
- (c)  $2\sqrt{15}5\text{cm}^2$
- (d)  $4\sqrt{15}\text{cm}^2$

Question 9. The edges of a triangular board are 6 cm, 8cm and 10cm. The cost of painting it at the rate of 9 paise per  $\text{cm}^2$  is

- (a) Rs 2.00
- (b) Rs 2.16
- (c) Rs 2.48
- (d) Rs 3.00

Question 10. The base of a right triangle is 48cm and its hypotenuse is 50cm. The area of the triangle is

- (a)  $168\text{cm}^2$
- (b)  $252\text{cm}^2$
- (c)  $336\text{cm}^2$
- (d)  $504\text{cm}^2$

### Very Short:

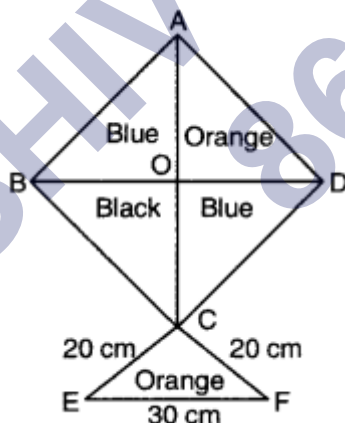
1. Find the area of an equilateral triangle having side 6cm.
2. If the perimeter of an equilateral triangle is 90m, then find its area.
3. If every side of a triangle is doubled, then find the percent increase in area of triangle so formed.
4. If the length of a median of an equilateral triangle is x cm, then find its area.

### Short Questions:

1. Find the area of a triangle whose sides are 11m, 60m and 61m.
2. Suman has a piece of land, which is in the shape of a rhombus. She wants her two sons to work on the land and produce different crops. She divides the land in two equal parts by drawing a diagonal. If its perimeter is 400 m and one of the diagonals is of length 120 m, how much area each of them will get for his crops?
3. The perimeter of a triangular field is 144m and its sides are in the ratio 3 : 4 : 5. Find the length of the perpendicular from the opposite vertex to the side whose length is 60m.
4. Find the area of the triangle whose perimeter is 180 cm and two of its sides are of lengths 80 cm and 18 cm. Also, calculate the altitude of the triangle corresponding to the shortest side.

### Long Questions:

1. Calculate the area of the shaded region.
2. The sides of a triangular park are 8m, 10m and 6m respectively. A small circular area of diameter 2m is to be left out and the remaining area is to be used for growing roses. How much area is used for growing roses? (Use  $\pi = 3.14$ )
3. OPQR is a rhombus, whose three vertices P, Q and R lie on the circle with Centre O. If the radius of the circle is 12cm, find the area of the rhombus.
4. How much paper of each shade is needed to make a kite given in the figure, in which ABCD is a square with diagonal 60cm?



### Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.
  - a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

- b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- c) Assertion is correct statement but reason is wrong statement.
- d) Assertion is wrong statement but reason is correct statement.

**Assertion:** the area of a triangle is  $6 \text{ cm}^2$  whose sides are 3 cm, 4 cm and 5 cm respectively.

**Reason:** area of triangle =  $\sqrt{s(s-a)(s-b)(s-c)}$

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

- a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
- b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- c) Assertion is correct statement but reason is wrong statement.
- d) Assertion is wrong statement but reason is correct statement.

**Assertion:** the area of an equilateral triangle having each side 4 cm is  $4\sqrt{3} \text{ cm}^2$

**Reason:** Area of an equilateral triangle =  $(\sqrt{3}/4) \times a^2$

### Answer Key:

#### MCQ:

1. (a)  $\sqrt{3}2\text{cm}$
2. (d)  $100\sqrt{3}\text{m}^2$
3. (c)  $1344\text{cm}^2$
4. (a)  $5.196\text{cm}^2$
5. (d) 6cm
6. (b) 24cm
7. (c)  $24\sqrt{5}\text{cm}$
8. (a)  $15\sqrt{\text{cm}}^2$
9. (b) Rs 2.16
- 10.(c)  $336\text{cm}^2$

#### Very Short Answer:

1. Area of an equilateral triangle =  $\frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times 6 \times 6 = 9\sqrt{3}\text{cm}^2$
- 2.

$$\text{Side of an equilateral triangle} = \frac{\text{Perimeter}}{3} = \frac{90}{3} = 30 \text{ m}$$

$$\begin{aligned} \therefore \text{Its area} &= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times (30)^2 \\ &= \frac{\sqrt{3}}{4} \times 30 \times 30 = 225\sqrt{3} \text{ m}^2 \end{aligned}$$

3. Let the sides of the given triangle be, a units, b units and c units.

$$\therefore \text{Its area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ sq. units}$$

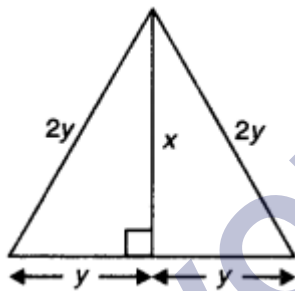
Now, new sides of the triangle are 2a units, 2b units and 2c units.

$$\begin{aligned} \text{Thus, its area} &= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)} \\ &= 4\sqrt{s(s-a)(s-b)(s-c)} \text{ sq. units} \end{aligned}$$

$$\text{Total increase in area} = 3\sqrt{s(s-a)(s-b)(s-c)} \text{ sq. units}$$

Hence, percent increase = 300%

4.



Let each equal sides of given equilateral triangle be  $2y^2$ . We know that median is also perpendicular bisector.

$$\therefore y^2 + x^2 = 4y^2$$

$$\Rightarrow x^2 = 3y^2$$

$$\Rightarrow x = \sqrt{3}y$$

or

$$\Rightarrow y = \frac{x}{\sqrt{3}}$$

Now, area of given triangle =  $\frac{1}{2}$

$$\times 2y \times X = y \times x = \frac{x}{\sqrt{3}} \times x = \frac{x^2}{\sqrt{3}}$$

### Short Answer:

Ans: 1. Let a = 11m, b = 60m and c = 61m:



$$\therefore s = \frac{a+b+c}{2} = \frac{11+60+61}{2} = \frac{132}{2} = 66 \text{ m}$$

Now,

$$s - a = 66 - 11 = 55 \text{ m}$$

$$s - b = 66 - 60 = 6 \text{ m}$$

$$s - c = 66 - 61 = 5 \text{ m}$$

$$\therefore \text{Area of given triangle} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{66(55)(6)(5)}$$

$$= \sqrt{108900} = 330 \text{ sq. m.}$$

**Ans: 2.** Here, perimeter of the rhombus is 400m.

$$\therefore \text{Side of the rhombus} = \frac{400}{4} = 100 \text{ m}$$

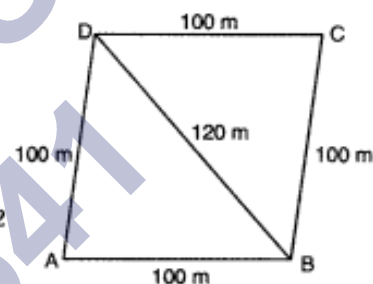
Let diagonal BD = 120m and this diagonal divides the rhombus ABCD into two equal parts.

Now,

$$s = \frac{100 + 120 + 100}{2} = \frac{320}{2} = 160$$

$$\therefore \text{Area of } \triangle ABD = \sqrt{160(160-100)(160-100)(160-120)}$$

$$= \sqrt{160 \times 60 \times 60 \times 40} = 80 \times 60 = 4800 \text{ m}^2$$



Hence, area of land allotted to two sons for their crops is  $4800 \text{ m}^2$  each.

**Ans: 3.**

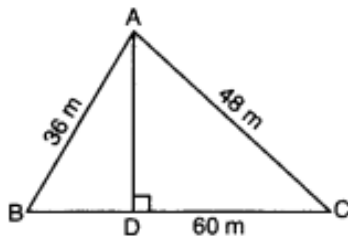
Let the sides of the triangle be  $3x$ ,  $4x$  and  $5x$

$\therefore$  The perimeter of the triangular field = 144m

$$\Rightarrow 3x + 4x + 5x = 144$$

$$\Rightarrow 12x = 144$$





$$\Rightarrow x = \frac{144}{12} = 12 \text{ m}$$

$\therefore$  Sides of the triangle are  $3 \times 12 \text{ m}$ ,  $4 \times 12 \text{ m}$ ,  $5 \times 12 \text{ m}$  i.e., 36 m, 48 m, 60 m

$$\therefore s = \frac{a+b+c}{2} = \frac{36+48+60}{2} = \frac{144}{2} = 72 \text{ m}$$

$$\begin{aligned} \text{Area of the } \Delta ABC &= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{72(72-36)(72-48)(72-60)} \\ &= \sqrt{72(36)(24)(12)} = \sqrt{746496} = 864 \text{ m}^2 \end{aligned}$$

$$\therefore \text{Also, ar } (\Delta ABC) = \frac{1}{2} \times AD \times BC = \frac{1}{2} \times AD \times 60 = 30 \times AD$$

$$\therefore 30 \times AD = 864$$

$$AD = \frac{864}{30} = 28.8 \text{ m}$$

**Ans: 4.** Perimeter of given triangle = 180cm

Two sides are 18cm and 80cm

$\therefore$  Third side =  $180 - 18 - 80 = 82 \text{ cm}$

$$s = \frac{180}{2} = 90 \text{ cm}$$

$$\begin{aligned} \text{Area of triangle} &= \sqrt{90(90-18)(90-80)(90-82)} \\ &= \sqrt{90 \times 72 \times 10 \times 8} = \sqrt{518400} = 720 \text{ cm}^2 \end{aligned}$$

$$\text{Also, } \frac{1}{2} \times 18 \times h = 720$$

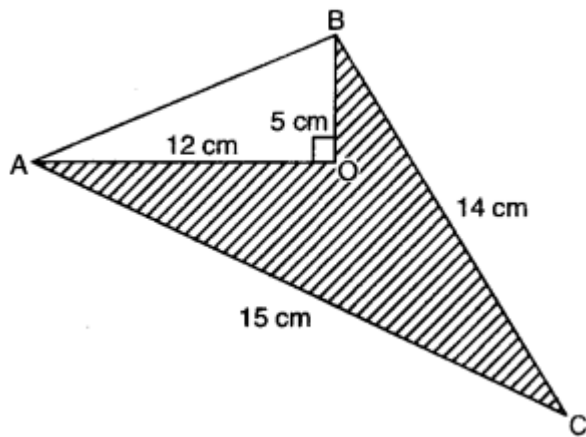
$$h = \frac{720}{9} = 80 \text{ cm}$$

Hence, area of triangle is  $720 \text{ cm}^2$  and altitude of the triangle corresponding to the shortest side is

80cm.

### Long Answer:

**Ans: 1**



$$\begin{aligned}\text{Area of } \triangle AOB &= \frac{1}{2} \times OA \times OB \\ &= \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2\end{aligned}$$

Also,

$$\begin{aligned}AB^2 &= OA^2 + OB^2 \\ &= 12^2 + 5^2 \\ &= 144 + 25 = 169\end{aligned}$$

$$\Rightarrow AB = \sqrt{169} = 13 \text{ cm}$$

Now, in  $\triangle ABC$ , we have

$$a = BC = 14 \text{ cm}, b = CA = 15 \text{ cm}, c = AB = 13 \text{ cm}$$

$$s = \frac{a+b+c}{2} = \frac{14+15+13}{2} = \frac{42}{2} = 21 \text{ cm}$$

$$\begin{aligned}\text{Area of } \triangle ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{21(21-14)(21-15)(21-13)} \\ &= \sqrt{21 \times 7 \times 6 \times 8} \\ &= \sqrt{3 \times 7 \times 7 \times 2 \times 3 \times 2 \times 2 \times 2}\end{aligned}$$

**Ans: 2.** The sides of the triangular park are 8m, 10m and 6m.

$$\therefore s = \frac{a+b+c}{2} = \frac{8+10+6}{2} = \frac{24}{2} = 12 \text{ m}$$

$$\begin{aligned}\text{Area of the park} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{12(12-8)(12-10)(12-6)} \\ &= \sqrt{12 \times 4 \times 2 \times 6} \\ &= \sqrt{2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 3} \\ &= 2 \times 2 \times 2 \times 3 = 24 \text{ m}^2\end{aligned}$$

$$\text{Radius of the circle} = \frac{2}{2} = 1 \text{ m}$$

$$\text{Area of the circle} = \pi r^2 = 3.14 \times 1 \times 1 = 3.14 \text{ m}^2$$

$$\begin{aligned}\therefore \text{Area to be used for growing roses} &= \text{Area of the park} - \text{area of the circle} \\ &= 24 - 3.14 = 20.86 \text{ m}^2\end{aligned}$$

**Ans: 3.** Since diagonals bisect each other at  $90^\circ$ .

∴ In right  $\triangle QLR$ ,  $(LR)^2 + (LQ)^2 = (QR)^2$

$$\Rightarrow \left(\frac{PR}{2}\right)^2 + \left(\frac{OQ}{2}\right)^2 = (QR)^2$$

$$\Rightarrow \left(\frac{PR}{2}\right)^2 = (12)^2 - \left(\frac{12}{2}\right)^2 \quad [\because OQ = r = 12 \text{ cm}]$$

$$\Rightarrow \frac{PR^2}{4} = 144 - 36$$

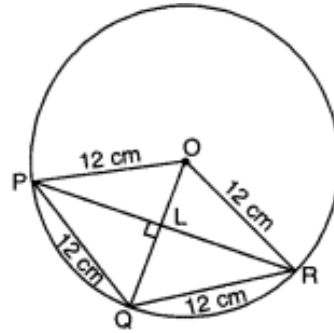
$$\Rightarrow PR^2 = 4 \times 108 = 432$$

$$PR = \sqrt{144 \times 3} = 12\sqrt{3} \text{ cm}$$

Area of rhombus OPQR =  $\frac{1}{2} \times$  product of diagonals

$$= \frac{1}{2} \times OQ \times PR = \frac{1}{2} \times 12 \times 12\sqrt{3}$$

$$= 72\sqrt{3} \text{ cm}^2$$



**Ans: 4.** Since diagonals of a square are of equal length and bisect each other at right angles, therefore,

$$\text{Area of } \triangle AOD = \frac{1}{2} \times 30 \times 30 = 450 \text{ cm}^2$$

$$\begin{aligned} \text{Area of } \triangle AOD &= \text{Area of } \triangle DOC = \text{Area of } \triangle BOC \\ &= \text{Area of } \triangle AOB = 450 \text{ cm}^2 \end{aligned}$$

[ $\because \triangle AOD = \triangle AOB \cong \triangle BOC \cong \triangle COD$ ,

$\therefore$  they<sup>2</sup> have equal area]

Now, area of ACEF (by Heron's formula)

Here  $a = 20 \text{ cm}$ ,  $b = 20 \text{ cm}$  and  $c = 30 \text{ cm}$

$$\Rightarrow s = \frac{20 + 20 + 30}{2} = \frac{70}{2} = 35 \text{ cm}$$

$$\begin{aligned} \text{Area of } \triangle CEF &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{35(35-20)(35-20)(35-30)} \\ &= \sqrt{35(15)(15)(5)} = 75\sqrt{7} \text{ cm}^2 \approx 198.4 \text{ cm}^2 \end{aligned}$$

Now, area of orange shaded paper in kite

$$= \text{Area of } \triangle AOD + \text{Area of } \triangle CEF$$

$$= 450 \text{ cm}^2 + 198.4 \text{ cm}^2$$

$$= 648.4 \text{ cm}^2$$

Area of blue shaded paper in kite

= Area of  $\triangle AOB$  + Area of  $\triangle COD$

=  $450\text{cm}^2 + 450\text{cm}^2 = 900\text{cm}^2$

Area of black shaded paper in kite = Area of  $\triangle BOC = 450\text{cm}^2$ .

### Assertion and Reason Answers-

1. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
2. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

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