MATHEMATICS

Chapter 11: Constructions

Constructions

1. **To divide a line segment internally in a given ratio m: n**, where both m and n are positive integers, we follow the steps given below:

Step 1: Draw a line segment AB of given length by using a ruler.

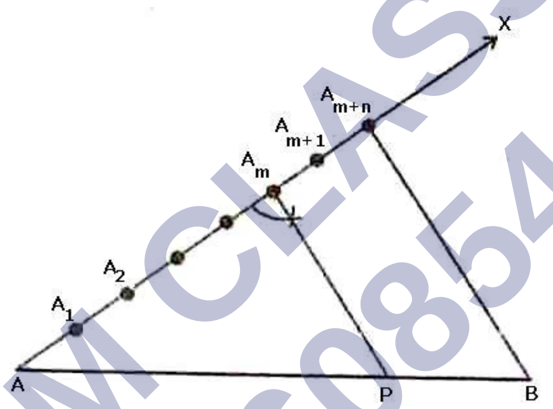
Step 2: Draw any ray AX making an acute angle with AB.

Step 3: Along AX mark off (m + n) points $A_1, A_2, \ldots\ldots A_{m-1}, A_{m+1}, \ldots\ldots, A_{m+n}$, such that $AA_1 = A_1A_2 = A_{m+n-1}\,A_{m+n}$.

Step 4: Join BA_{m+n}

Step 5: Through the point Am, draw a line parallel to Am+nB by making an angle equal to $\angle AA_{m+n}B$ at A_m, intersecting AB at point P.

The point P so obtained is the required point which divides AB internally in the ratio m: n.



Justification

In ΔABA_{m+n}, we observe that A_m P is parallel to A_{m+n} B . Therefore, by Basic Proportionality theorem, we have:

$$\frac{AA_m}{A_mA_{m+n}} = \frac{AP}{PB}$$

$$= \frac{AP}{PB} = \frac{m}{n}\left[\because \frac{AA_m}{A_mA_{m+n}} = \frac{m}{n}, \text{by construction}\right]$$

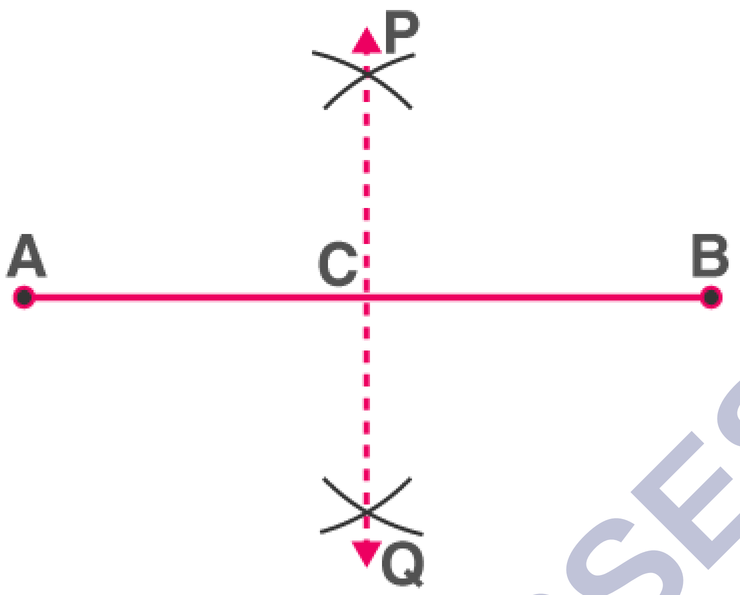
$$= AP : PB = m : n$$

Hence, P divides AB in the ratio m: n.

Bisecting a Line Segment

Step 1: With a radius of more than half the length of the line segment, draw arcs centred at either end of the line segment so that they intersect on either side of the line segment.

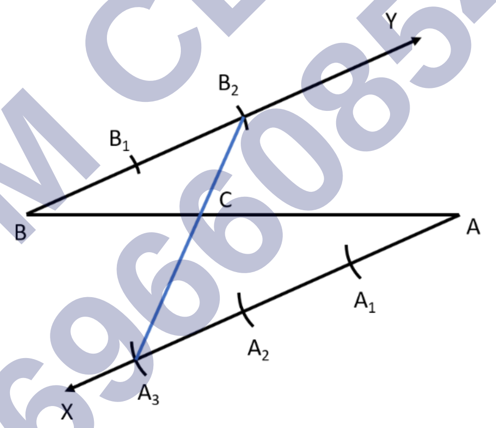
Step 2: Join the points of intersection. The line segment is bisected by the line segment joining the points of intersection.



PQ is the perpendicular bisector of AB

2. Alternative method to divide a line segment internally in a given ratio m: n

Example Find the point C such that it divides BA in ratio 2:3



Steps of Construction:

- Draw any ray XA making an acute angle with BA.
- Draw a ray YB parallel to XA by making ∠YBA equal to ∠XAB.
- Locate the points A1, A2, A3 (m = 3) on AX and B1, B2 (n = 2) on BY such that AA1 = A1A2 = A2A3 = BB1 =B1B2.
- Join A3B2. Let it intersect AB at a point C Then BC : CA = 2:3

Justification

Here $\Delta BB_2C \sim AA_3C$ ...AA test

$$\frac{BB_2}{AA_3} = \frac{BC}{AC} \dots\dots (\text{c.p.s.t.})$$

$$\frac{2}{3} = \frac{BC}{AC}$$

3. The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as a **scale factor**. The scale factor may be less or greater than 1.
4. If the scale factor is less than 1, then the new figure will be smaller in comparison to the given figure.
5. If the scale factor is greater than 1, then the new figure will be bigger in comparison to the given figure.

Construction of Triangle Similar to given Triangle

Consider a triangle ABC. Let us construct a triangle similar to ΔABC such that each of its sides is | | of $\left(\frac{m}{n}\right)^{th}$ the corresponding sides of ? ABC.

Steps of constructions when m < n:

Step 1: Construct the given triangle ABC by using the given data.

Step 2: Take any one of the three side of the given triangle as base. Let AB be the base of the given triangle.

Step 3: At one end, say A, of base AB. Construct an acute angle ?BAX below the base AB.

Step 4: Along AX mark off n points $A_1, A_2, A_3, \ldots\ldots, A_n$ such that

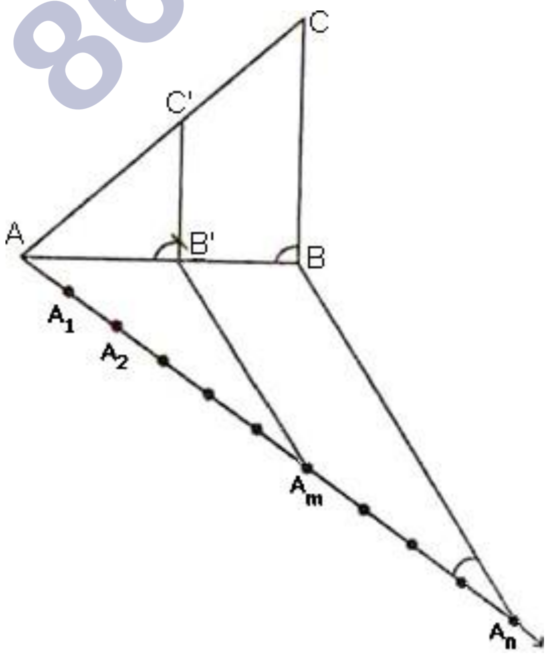
$AA_1 = A_1A_2 = \ldots\ldots = A_{n-1} A_n$

Step 5: Join A_nB

Step 6: Draw A_mB' parallel to A_nB which meets AB at B'.

Step 7: From B' draw B'C' | | BC meeting AC at C'.

Triangle AB'C' is the required triangle each of whose sides is $\left(\frac{m}{n}\right)^{th}$ of the corresponding side of ΔABC.



Justification

Since A_m B'|| A_n B. Therefore

$\frac{AB'}{B'B} = \frac{AA_m}{A_mA_n}$ [by basic proportionality theorem]

$$\Rightarrow \frac{AB'}{B'B} = \frac{m}{n-m}$$

$$\Rightarrow \frac{B'B}{AB'} = \frac{n-m}{m}$$

Now, $\frac{AB}{AB'} = \frac{AB'+B'B}{AB'}$

$$\Rightarrow \frac{AB}{AB'} = 1 + \frac{B'B}{AB'} = 1 + \frac{n-m}{m} + \frac{n}{m}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{m}{n}$$

In triangles ABC and AB’C’, we have

∠BAC = ∠B' AC'

And ∠ABC = ∠AB' C'

So, by AA similarity criterion, we have

ΔAB'C' ~ ΔABC

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{m}{n}$$

Steps of construction when m > n:

Step 1: Construct the given triangle by using the given data.

Step 2: Take any one of the three sides of the given triangle and consider it as the base. Let AB be the base of the given triangle.

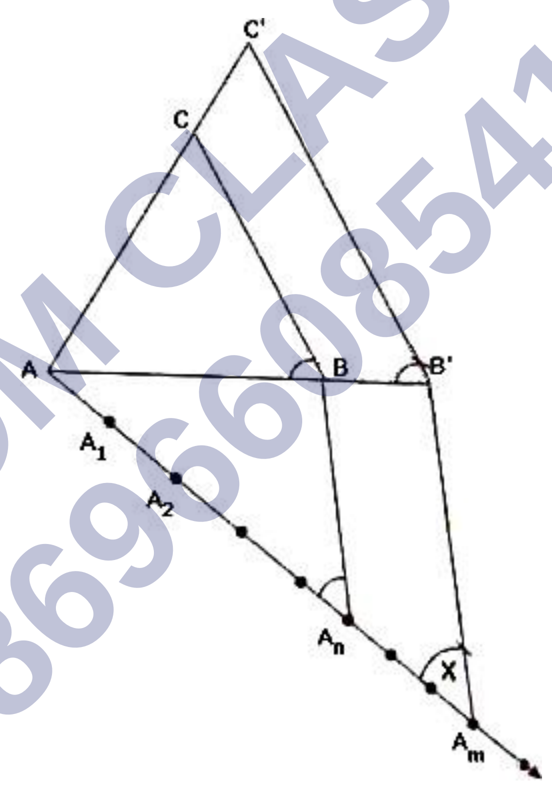
Step 3: At one end, say A, of base AB. Construct an acute angle ∠BAX below base AB i.e., on the opposite side of the vertex C.

Step 4: Along AX mark off m (large of m and n) points A1, A2, A3,.........Am of AX such that

$AA_1 = A_1A_2 = \ldots\ldots\ldots = A_{m-1}A_m$.

Step 5: Join A_nB to B and draw a line through A_m parallel to A_nB, intersecting the extended line segment AB at B'.

Step 6: Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. Step 7: ΔAB'C' so obtained is the required triangle.



Justification

Consider triangle ABC and AB' C'. We have:

$\angle BAC = \angle B'AC'$

$\angle ABC = \angle AB'C'$

So, by AA similarity criterion,

$\Delta ABC \sim \Delta AB'C'$

$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$

$\Delta A A_m B'$, $A_nB \,||\, A_mB'$

$\therefore \frac{AB}{BB'} = \frac{AA_n}{A_nA_m}$

$\Rightarrow \frac{BB'}{AB} = \frac{A_nA_m}{AA_n}$

$\Rightarrow \frac{BB'}{AB} = \frac{m-n}{n}$

$\Rightarrow \frac{AB'-AB}{AB} = \frac{m-n}{n}$

$\Rightarrow \frac{AB'}{AB} - 1 = \frac{m-n}{n}$

$\Rightarrow \frac{AB'}{AB} = \frac{m}{n}$

From (i) and (ii), we have

$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{m}{n}$

The tangent to a circle is a line that intersects the circle at exactly one point.

Tangent to a circle is perpendicular to the radius through the point of contact.

Construction of Triangle to a Circle from a point outside the Circle

Construction of a tangent to a circle from a point outside the circle, when its centre is known

The steps of constructions are as follows:

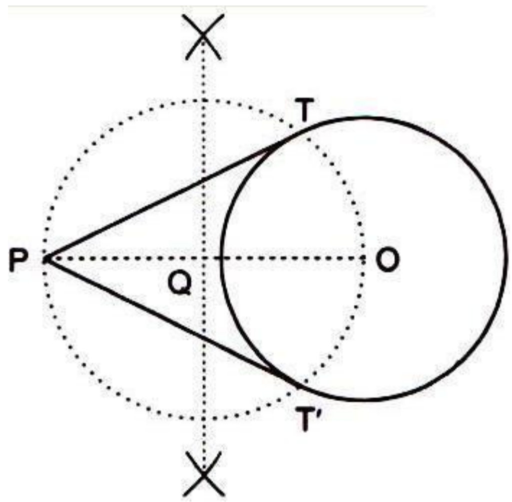
Step 1: Join the centre O of the circle to the point P.

Step 2: Draw perpendicular bisector of OP intersecting OP at Q.

Step 3: With Q as centre and radius OQ, draw a circle. This circle has OP as its diameter.

Step 4: Let this circle intersect the first circle at two points T and T'. Join PT and P T' .

PT and P T' are the two tangents to the given circle from the point P.



Justification

Join OT and OT'

It can be seen that ∠PTO is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

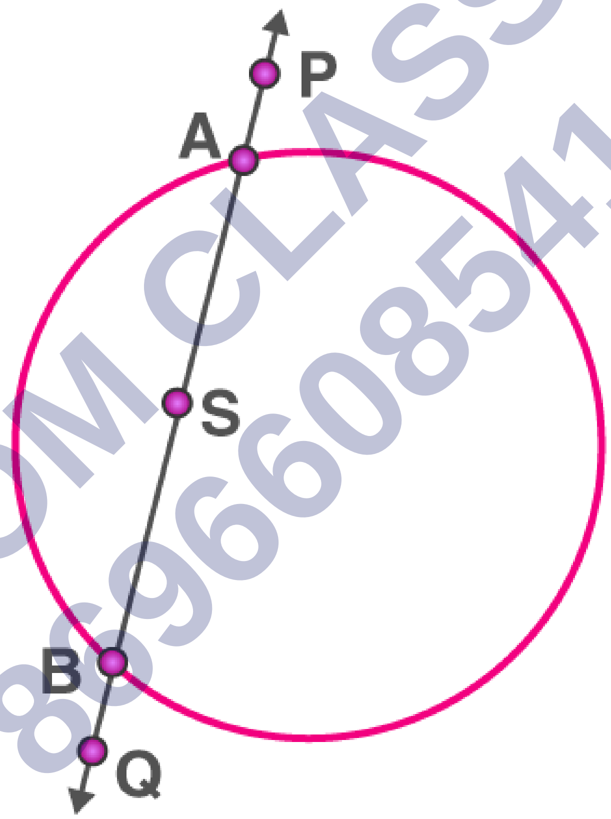
∴∠ PTO = 90°

⇒ OT ⊥ PT

Since OT is the radius of the circle, PT has to be a tangent of the circle. Similarly, PT' is a tangent of the circle.

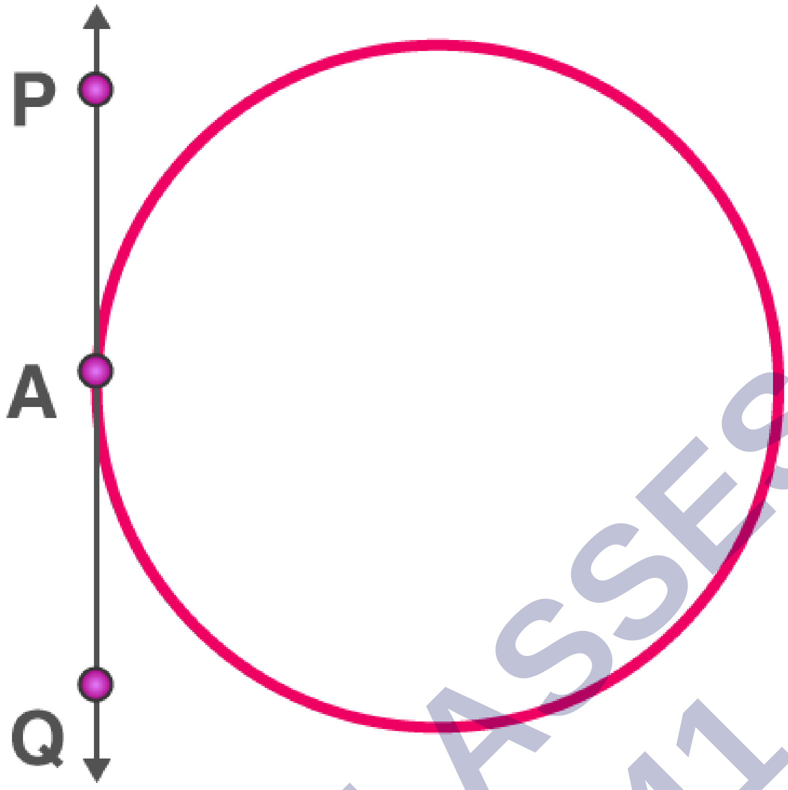
Number of Tangents to a circle from a given point

If the point is in an interior region of the circle, any line through that point will be a secant. So, in this case, there is no tangent to the circle.



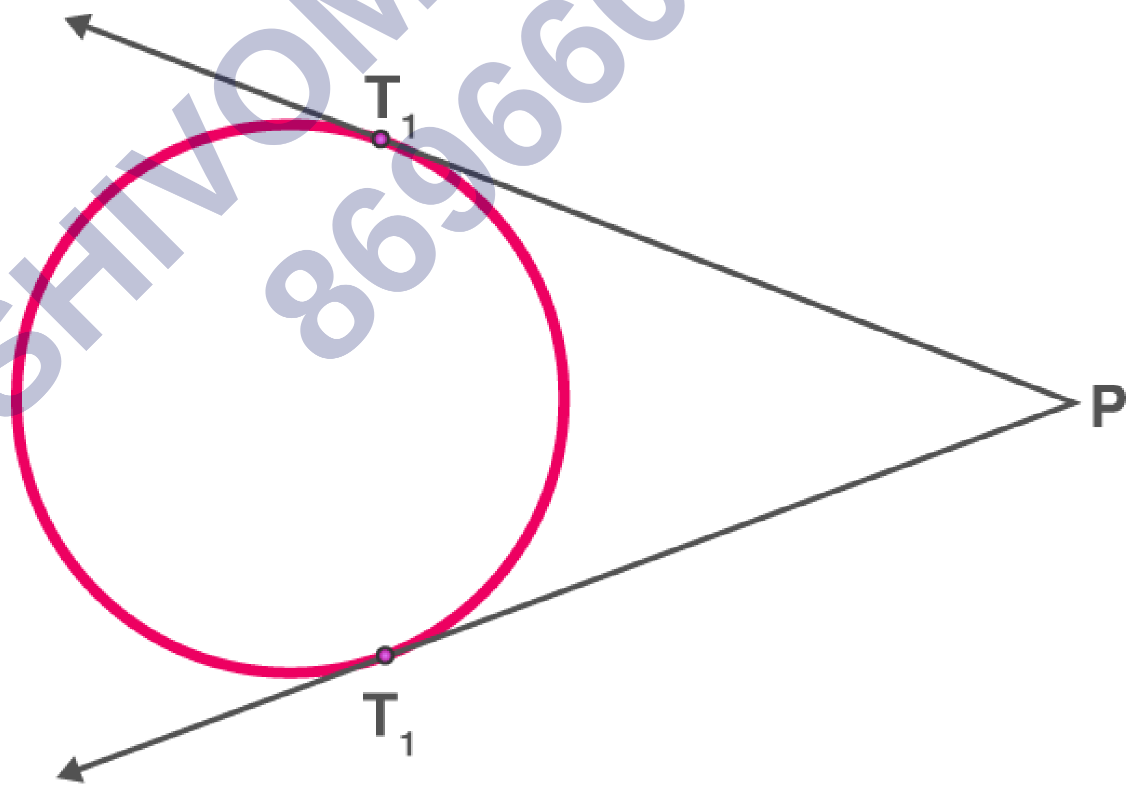
AB is a secant drawn through the point S

When the point lies on the circle, there is accurately only one tangent to a circle.



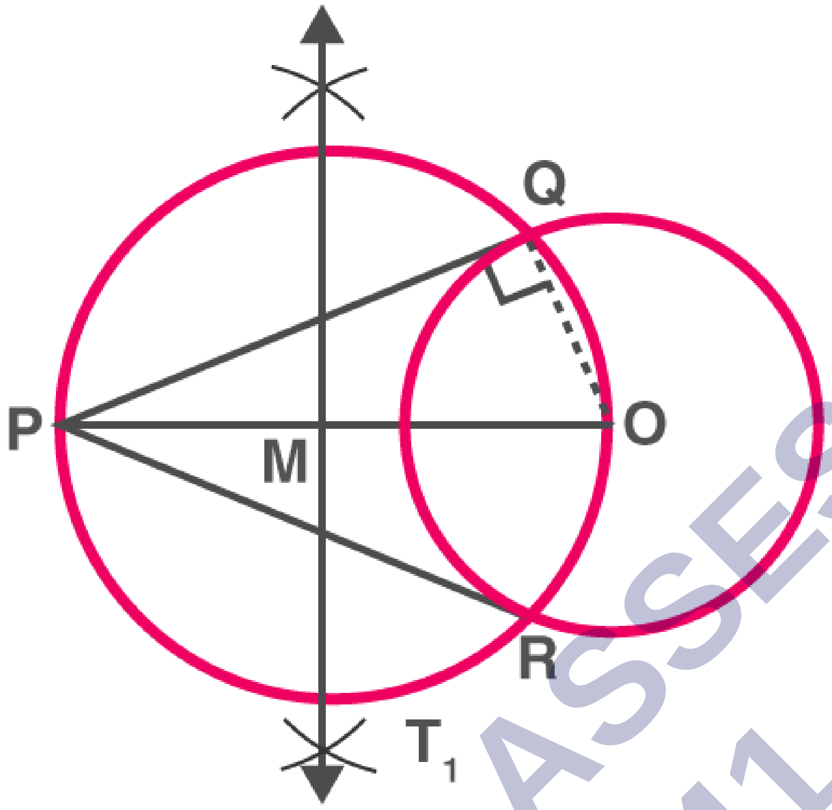
PQ is the tangent touching the circle at A

When the point lies outside of the circle, there are exactly two tangents to a circle.



PT1 and PT2 are tangents touching the circle at T1 and T2

Drawing tangents to a circle from a point outside the circle



To construct the tangents to a circle from a point outside it.

Consider a circle with centre O and let P be the exterior point from which the tangents to be drawn.

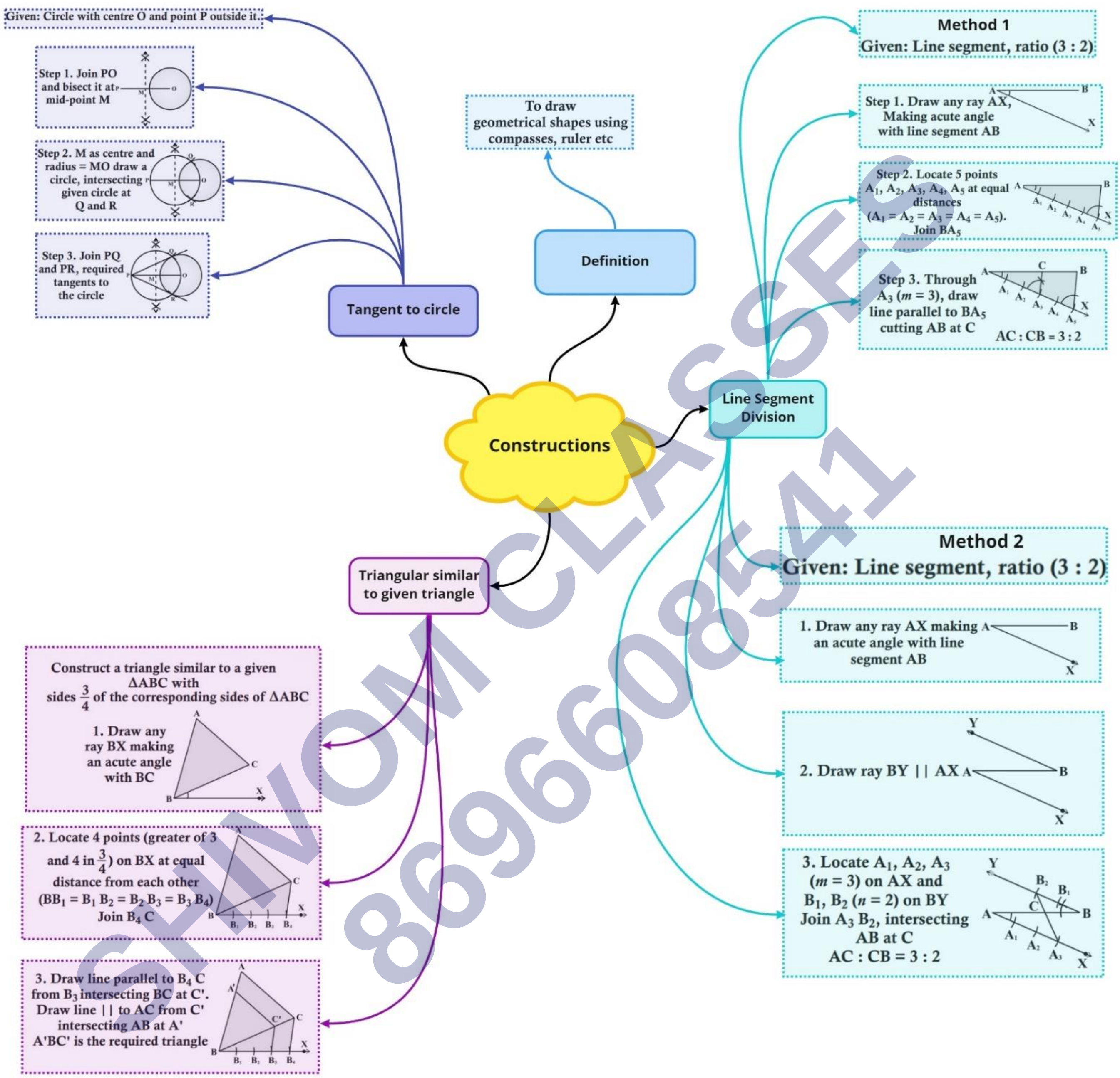
Step 1: Join the PO and bisect it. Let M be the midpoint of PO.

Step 2: Taking M as the centre and MO(or MP) as radius, draw a circle. Let it intersect the given circle at the points Q and R.

Step 3: Join PQ and PR

Step 4:PQ and PR are the required tangents to the circle.

Class : 10th mathematics
Chapter- 11 : Constructions



Important Questions

Multiple Choice questions-

1. To divide a line segment AB in the ratio p : q (p, q are positive integers), draw a ray AX so that ∠BAX is an acute angle and then mark points on ray AX at equal distances such that the minimum number of these points is

(a) greater of p and q

(b) p + q

(c) p + q – 1

(d) pq

2. To draw a pair of tangents to a circle which are inclined to each other at an angle of 35°. It is required to draw tangents at the end points of those two radii of the circle, the angle between which is

(a) 105°

(b) 70°

(c) 140°

(d) 145°

3. To divide a line segment AB in the ratio 5 : 7, first a ray AX is drawn so that ∠BAX is an acute angle and then at equal distances points are marked on the ray AX such that the minimum number of these points is

(a) 8

(b) 10

(c) 11

(d) 12

4. To divide a line segment AB in the ratio 4 : 7, ray AX is drawn first such that ∠BAX is an acute angle and then points A_1, A_2, A_3,……… are located at equal distances on the ray AX and the point B is joined to

(a) A_{12}

(b) A_{11}

(c) A_{10}

(d) A_9

5. To divide a line segment AB in the ratio 5 : 6, draw a ray AX such that ∠BAX is an acute angle, then draw a ray B4 parallel to AX and the points A_1, A_2, A_3, …….. and B_1, B_2, B_3,………. are located at equal distances on ray AX and B4, respectively. Then the points joined are:

(a) A_5 and B_6

(b) A_6 and B_5

(c) A_4 and B_5

(d) A_5 and B_4

6. To construct a triangle similar to a given ΔABC with its sides $\frac{3}{7}$ of the corresponding sides of ΔABC, first draw a ray BX such that ∠CBX is an acute angle and X lies on the opposite side of A with respect to BC. Then locate points B_1, B_2, B_3, on BX at equal distances and next step is to join

(a) B_{10} to C

(b) B_3 to C

(c) B_7 to C

(d) B_4 to C

7. To construct a triangle similar to a given ΔABC with its sides $\frac{8}{5}$ of the corresponding sides of ΔABC draw a ray BX such that ∠CBX is an acute angle and X is on the opposite side of A with respect to BC. Then minimum number of points to be located at equal distances on ray BX is

(a) 5

(b) 8

(c) 13

(d) 3

8. To draw a pair of tangents to a circle which are inclined to each other at an angle of 60°, it is required to draw tangents at end points of those two radii of the circle, the angle between them should be:

(a) 135°

(b) 90°

(c) 60°

(d) 120°

9. To construct a pair of tangents to a circle at an angle of 60° to each other, it is needed to draw tangents at endpoints of those two radii of the circle, the angle between them should be:

(a) 100°

(b) 90°

(c) 180°

(d) 120°

10. A pair of tangents can be constructed from a point P to a circle of radius 3.5 cm situated at a distance of ___________ from the centre.

(a) 3.5 cm

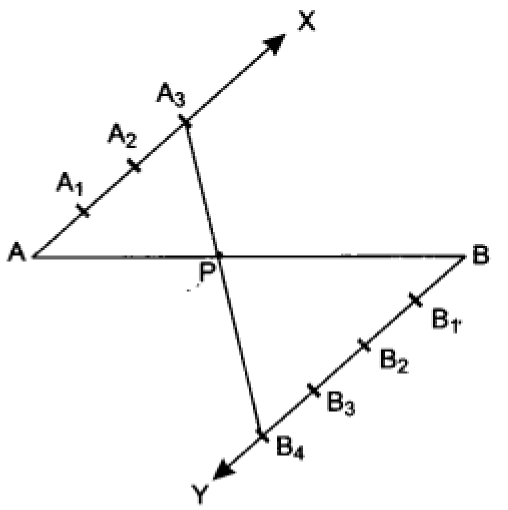
(b) 2.5 cm

(c) 5 cm

(d) 2 cm

Very Short Questions:

1. Is construction of a triangle with sides 8 cm, 4 cm, 4 cm possible?
2. To divide the line segment AB in the ratio 5 : 6, draw a ray AX such that ∠BAX is an acute angle, then draw a ray BY parallel to AX and the point A_1, A_2, A_3... and B_1, B_2, B_3... are located at equal distances on ray AX and BY respectively. Then which points should be joined?
3. To draw a pair of tangents to a circle which are inclined to each other at an angle of 60°, it is required to draw tangents at end points of those two radii of the circle. What should be the angle between them?
4. In Fig. by what ratio does P divide AB internally.



5. Given a triangle with side AB = 8 cm. To get a line segment AB' = 2 of AB, in what ratio will line segment AB be divided?

6. Draw a line segment of length 6 cm. Using compasses and ruler, find a point P on it which divides it in the ratio 3 : 4.

7. Draw a line segment AB of length 7 cm. Using ruler and compasses, find a point P on AB such that $\frac{AP}{AB} = \frac{3}{5}$.

Short Questions :

1. Draw a triangle ABC in which AB = 5 cm, BC = 6 cm and ∠ABC = 60°. Then construct a triangle whose sides are 57 times the corresponding sides of ΔABC.

2. Construct a triangle with sides 5 cm, 5.5 cm and 6.5 cm. Now construct another triangle, whose sides are $\frac{3}{5}$ times the corresponding sides of the given triangle.

3. Construct a right triangle in which the sides, (other than the hypotenuse) are of length 6 cm and 8 cm. Then construct another triangle, whose sides are $\frac{3}{5}$ times the corresponding sides of the given triangle.

4. Draw a triangle PQR such that PQ = 5 cm, ∠P = 120° and PR = 6 cm. Construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔPQR.

5. Draw a triangle ABC with BC = 7 cm, ∠B = 45° and ∠C = 60°. Then construct another triangle, whose sides are $\frac{3}{5}$ times the corresponding sides of ΔABC.

6. Construct a triangle with sides 5 cm, 4 cm and 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of first triangle.

7. Draw a pair of tangents to a circle of radius 3 cm, which are inclined to each other at an angle of 60°.

8. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.

9. Draw a pair of tangents to a circle of radius 4.5 cm, which are inclined to each other at an angle of 45°.

10. Draw two tangents to a circle of radius 3.5 cm, from a point P at a distance of 6.2 cm from its centre.

Long Questions :

1. Draw a triangle ABC with side BC = 7 cm, ∠B = 45° and ΔA = 105°. Then construct a triangle whose sides are $\frac{3}{5}$ times the corresponding sides of ΔABC.

2. Draw a triangle ABC with side BC = 6 cm, ∠C = 30° and ∠A = 105°. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of ΔABC.

3. Draw a triangle with sides 5 cm, 6 cm and 7 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of the first triangle.

4. Construct an isosceles triangle whose base is 6 cm and altitude 4 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the isosceles triangle.

5. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

6. Construct a ΔABC in which AB = 6 cm, ∠A = 30° and ΔB = 60°. Construct another ΔAB'C' similar to ΔABC with base AB' = 8 cm.

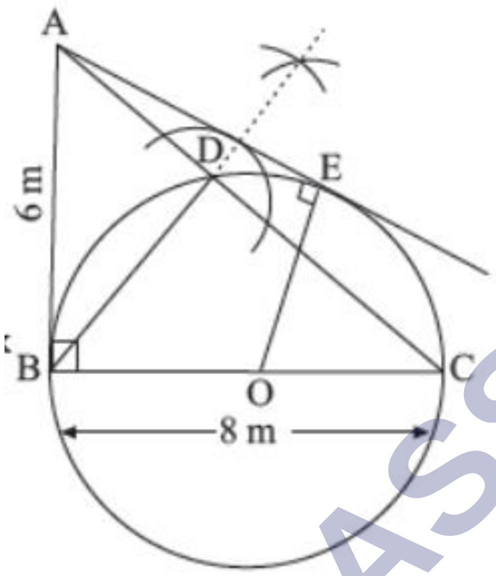
7. Construct a triangle ABC in which AB = 5 cm, BC = 6 cm and ∠ABC = 60°. Now construct another triangle whose sides are $\frac{5}{7}$ times the corresponding sides of ΔABC.

8. Construct a triangle ABC in which BC = 6 cm, AB = 5 cm and ∠ABC = 60°. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔABC.

Case Study Questions:

1. The management of a school decided to arouse interest of their students in Mathematics. So they want to construct some geometrical shapes in one corner of the school premises. They showed a rough sketch of a right triangular structure on a plane

sheet of paper with sides AB = 6 m, BC = 8 m and ∠B = 90°. The diagram shows a perpendicular from the vertex B to the front side AC. They want to build a circular wall through B, C and D but they had certain problems in doing so. So they called on some students of class X to solve this problem. They made some suggestions.



i. Referring to the above, what is the length of perpendicular drawn on side AC from vertex B?

a. 2.6 m

b. 3.0 m

c. 4.8 m

d. 4.0 m

ii. Referring to the above, what is the length of perpendicular drawn on side AC from vertex B?

a. 2.6 m

b. 3.0 m

c. 4.8 m

d. 4.0 m

iii. Referring to the above, the length of tangent AE is

a. 10 m

b. 8 m

c. 12 m

d. 6 m

iv. Referring to the above, what will be the length of AD?

a. 3.6 m

b. 3.8 m

c. 4.8 m

d. 5.6 m

v. Referring to the above, sum of angles ∠BAE and ∠BOE is

a. 120°

b. 180°

c. 90°

d. 60°

2. The construction of a road is in progress. A road already exists through a forest that goes over a circular lake. The engineer wants to build another road through the forest that connects this road but does not go through the lake.

As it turns out, the road the engineer will be building and the road it will connect to both represent characteristics of a circle that have their own name. The road/bridge that already exists is called a secant of the circular lake, and the road the engineer is going to build is called the tangent of the circular lake.

i. Refer to the question (2) if the road under construction, PT is 6 km and it is inclined at an angle of 30° to the line joining the centre, the radius of the lake is

a. $3\sqrt{3}$ km

b. $4\sqrt{3}$ km

c. $2\sqrt{3}$ km

d. $5\sqrt{3}$ km

ii. Refer to the above, if PT = 12 km and PA = 9 km, then the length of existing bridge is

a. 7 km

b. 9 km

c. 12 km

d. 16 km

iii. Refer to the question (3) above, the area of the lake is

a. 12π km^2

b. 16π km^2

c. 18π km^2

d. 9π km^2

iv. Refer to the above if the length of existing bridge is 5 km and the length of the existing road outside the lake is 4 km, then the length of the road under construction is

a. 4 km

b. 6 km

c. 10 km

d. 14 km

v. Refer to the question (3) above, the circumference of the lake is

a. $2\sqrt{3}\,\pi$ km

b. $3\sqrt{3}\,\pi$ km

c. $4\sqrt{3}\,\pi$ km

d. $5\sqrt{3}\,\pi$ km

Assertion Reason Questions-

1. ***Directions:*** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) Both A and R is false.

Assertion: a, b and c are the lengths of three sides of a triangle, then $a+b > c$

Reason: The sum of two sides of a triangle is always greater than the third side.

2. ***Directions:*** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) Both A and R is false.

Assertion: The side lengths 4cm, 4cm and 4cm can be sides of equilateral triangle.

Reason: Equilateral triangle has all its three sides equal.

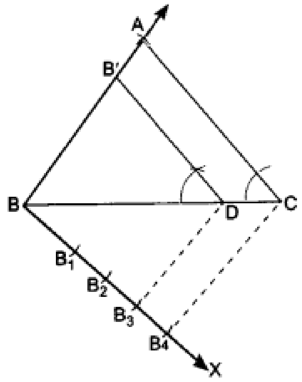
Answer Key

Multiple Choice questions-

1. (b) p + q
2. (d) 145°
3. (d) 12
4. (b) A_{11}
5. (a) A_5 and B_6
6. (c) B_7 to C
7. (b) 8
8. (d) 120°
9. (d) 120°
10. (c) 5 cm

Very Short Answer :

1. No, we know that in a triangle sum of two sides of a triangle is greater than the third side. So the condition is not satisfied.
2. A_5 and B_6.
3. 120˚
4. From Fig. it is clear that there are 3 points at equal distances on AX and 4 points at equal distances on BY. Here P divides AB on joining A3 B4. So P divides internally by 3 : 4.
5.



Given AB = 8 cm

AB' = $\frac{3}{4}$ of AB

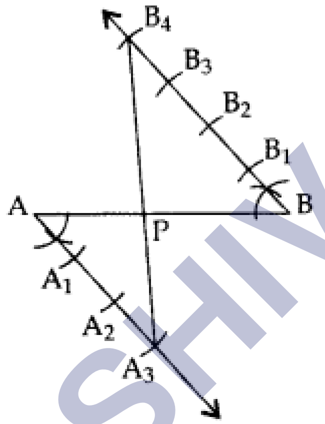
= $\frac{3}{4}$ × 8 = 6 cm

BB' = AB – AB' = 8 – 6 = 2 cm.

⇒ AB': BB' = 6 : 2 = 3 : 1

Hence the required ratio is 3 : 1.

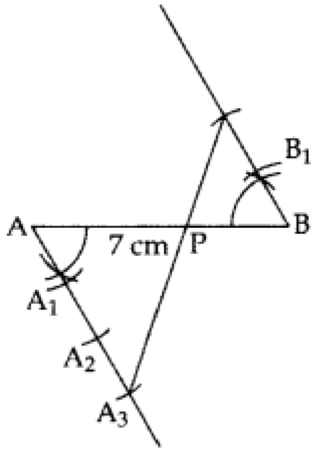
6.



Hence, PA : PB = 3 : 4

7. AB = 7 cm, AB = $\frac{AP}{AB} = \frac{3}{5}$ ... [Given

∴ AP : PB = 3 : 2



Hence, AP : AB = 3 : 5 or $\frac{AP}{AB} = \frac{3}{5}$

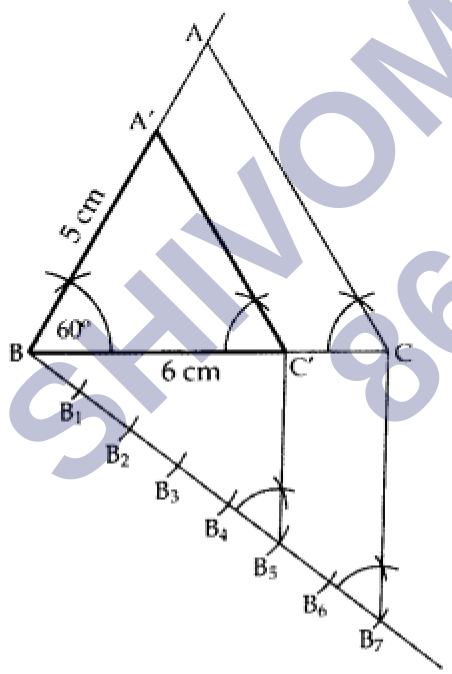
Short Answer :

1. In ΔABC

AB = 5 cm

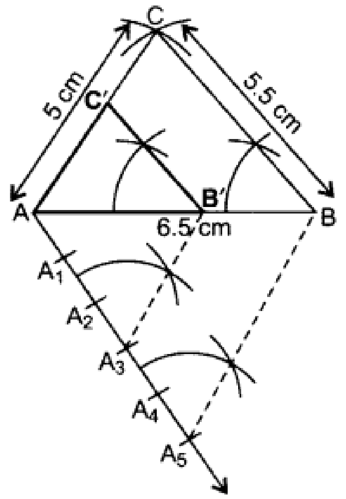
BC = 6 cm

∠ABC = 60°



Hence, ΔA’BC’ is the required Δ.

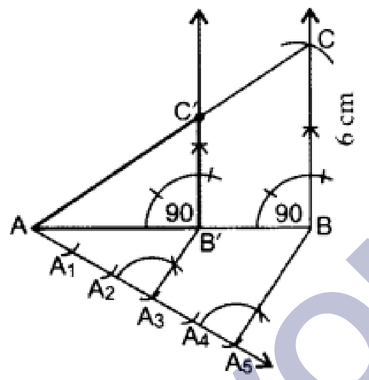
2.



∴ ΔAB'C' is the required Δ.

3. Here AB = 8 cm, BC = 6 cm and

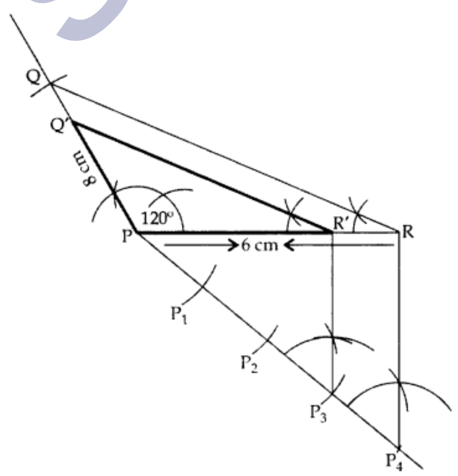
Ratio = $\frac{3}{5}$ of corresponding sides



∴ ΔAB'C' is the required triangle.

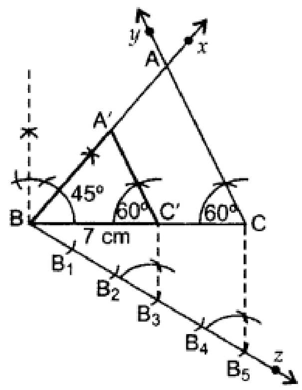
4. In ΔPQR,

PQ = 5 cm, PR = 6 cm, ∠P = 120°



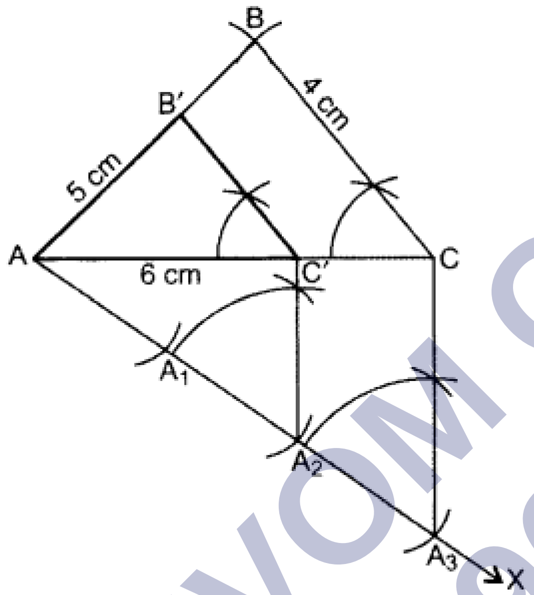
∴ ΔPO'R' is the required Δ.

5. Here, BC = 7 cm, ∠B = 45°, ∠C = 60° and ratio is $\frac{3}{5}$ times of corresponding sides



∴ ΔA'BC' is the required triangle.

6.



Steps of Construction:

Draw ΔABC with AC = 6 cm, AB = 5 cm, BC = 4 cm.

Draw ray AX making an acute angle with AÇ.

Locate 3 equal points A_1, A_2, A_3 on AX.

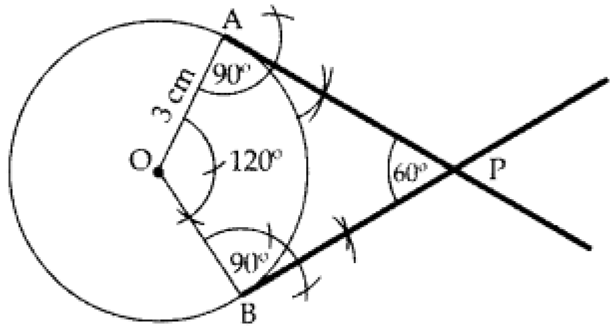
Join CA_3.

Join A_2C' || CA_3.

From point C' draw B'C' || BC.

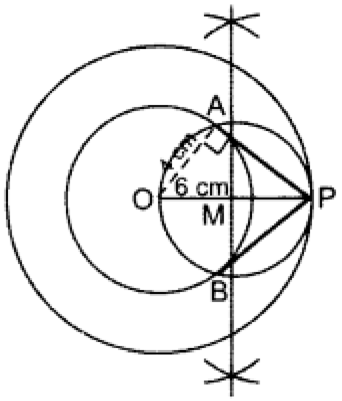
∴ ΔAB'C' is the required triangle.

7.



∴ PA & PB are the required tangents.+

8.



Steps of Construction:

Draw two circles with radius OA = 4 cm and OP = 6 cm with O as centre. Draw ⊥ bisector of OP at M. Taking M as centre and OM as radius draw another circle intersecting the smaller circle at A and B and touching the bigger circle at P. Join PA and PB. PA and PB are the required tangents.

Verification:

In rt. ΔOAP,

$OA^2 + AP^2 = OP^2$ … [Pythagoras' theorem

$(4)^2 + (AP)^2 = (6)^2$

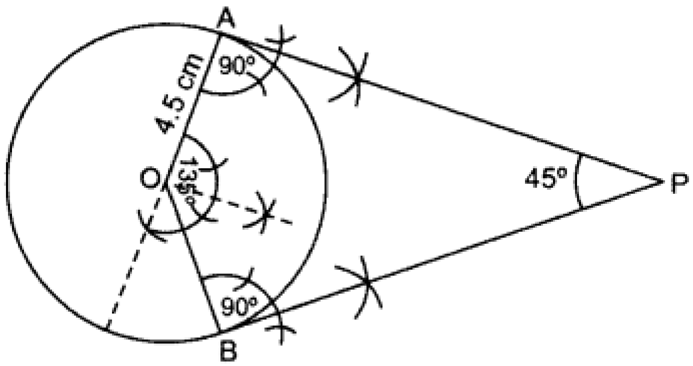
$AP^2 = 36 - 16 = 20$

$AP = +\sqrt{20} = \sqrt{4 \times 5}$

$= 2\sqrt{5}$ 2(2.236) = 4.472 = 4.5 cm

By measurement, ∴ PA = PB = 4.5 cm

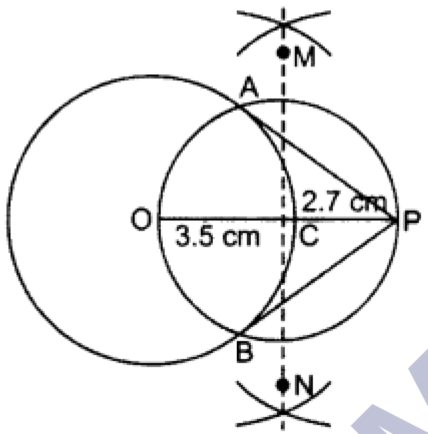
9.



Draw ∠AOB = 135°, ∠OAP = 90°, ∠OBP = 90°

∴ PA and PB are the required tangents.

10. OP = OC + CP = 3.5 + 2.7 = 6.2 cm



Hence AP & PB are the required tangents.

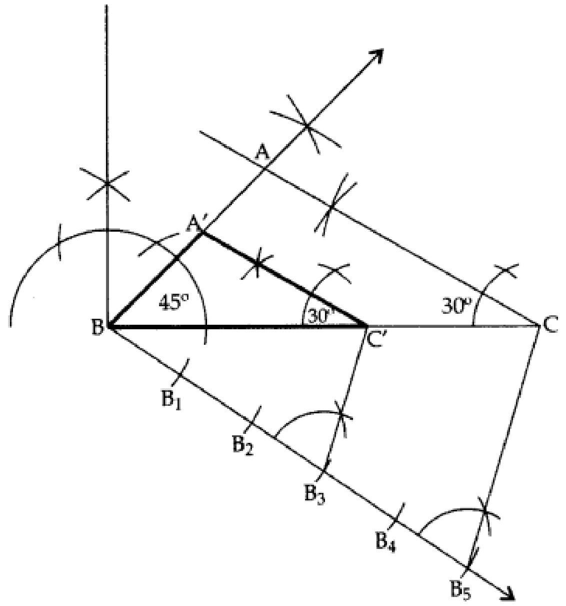
Long Answer :

1. In ΔABC, ∠A + ∠B + ∠C = 180° ... [angle sum property of a Δ

105° + 45° + C = 180°

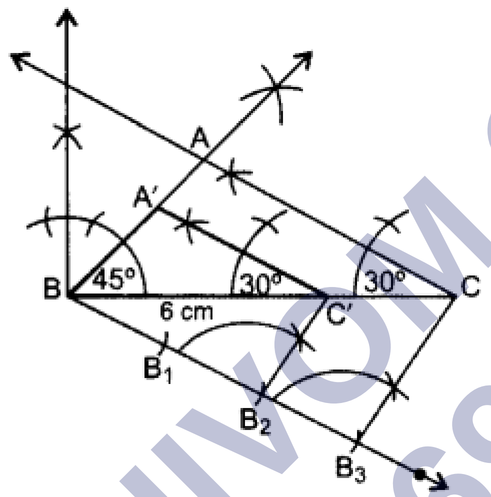
∠C = 180° – 105° – 45o = 30°

BC = 7 cm



∴ ΔA'BC' is the required Δ.

2. Here, BC = 6 cm, ∠A = 105° and ∠C = 30°



In ΔABC,

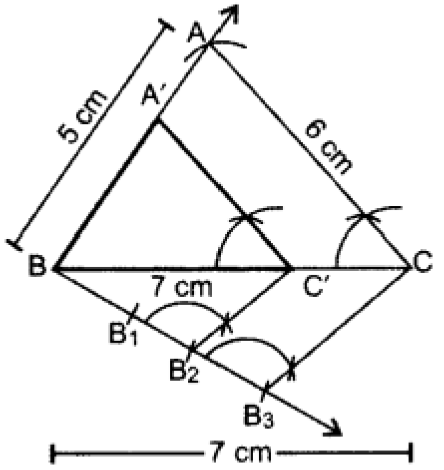
∠A + ∠B + ∠C = 180° ...[Angle-sum-property of a Δ

105° + ∠B + 30° = 180°

∠B = 180° – 105° – 30° = 45°

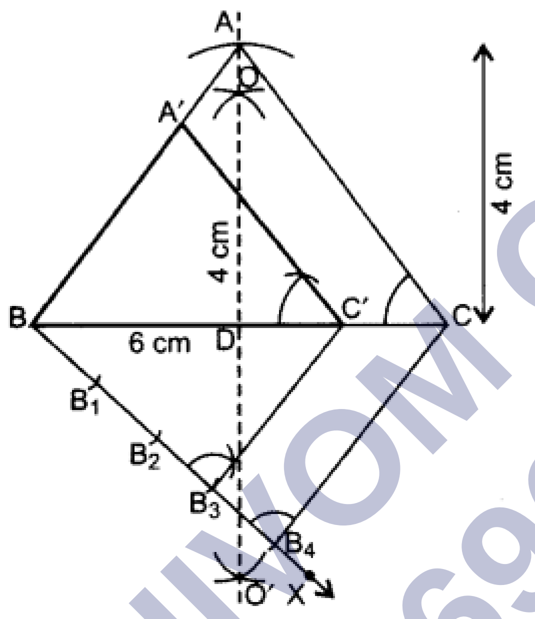
∴ ΔA'BC' is the required Δ.

3. Here, AB = 5 cm, BC = 7 cm, AC = 6 cm and ratio is $\frac{2}{3}$ times of corresponding sides.



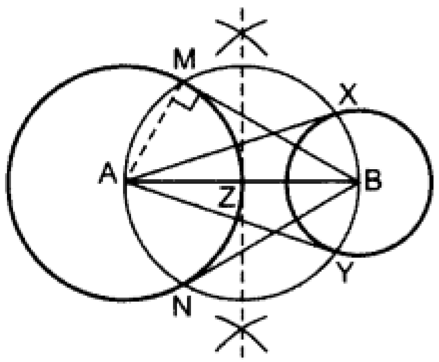
∴ ΔA′BC′ is the required triangle.

4.



∴ ΔA′BC′ is the required triangle.

5.



Step of constructions:

Draw two circles on A and B as asked.

Z is the mid-point of AB.

From Z, draw a circle taking ZA = ZB as radius,

so that the circle intersects the bigger circle at M and N and smaller circle at X and Y.

Join AX and AY, BM and BN.

BM, BN are the required tangents from external point B.

AX, AY are the required tangents from external point A.

Justification:

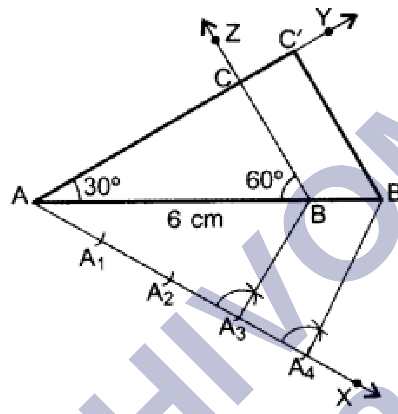
∠AMB = 90° ...[Angle in a semi-circle

Since, AM is a radius of the given circle.

∴BM is a tangent to the circle

Similarly, BN, AX and AY are also tangents.

6.

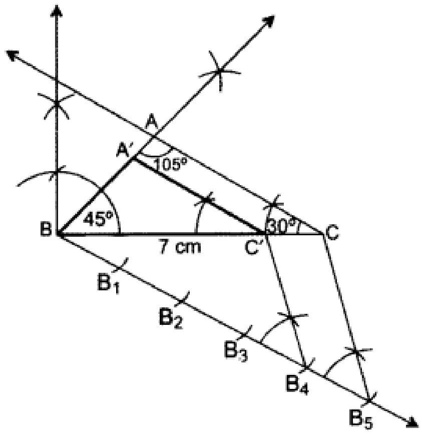


Steps of construction:

- Draw a ΔABC with side AB = 6 cm, ∠A = 30° and ∠B = 60°.
- Draw a ray AX making an acute angle with AB on the opposite side of point C.
- Locate points A_1, A_2, A_3 and A_4 on AX.
- Join A_3B. Draw a line through A_4 parallel to A_3B intersecting extended AB at B'.
- Draw a line parallel to BC intersecting ray AY at C'.

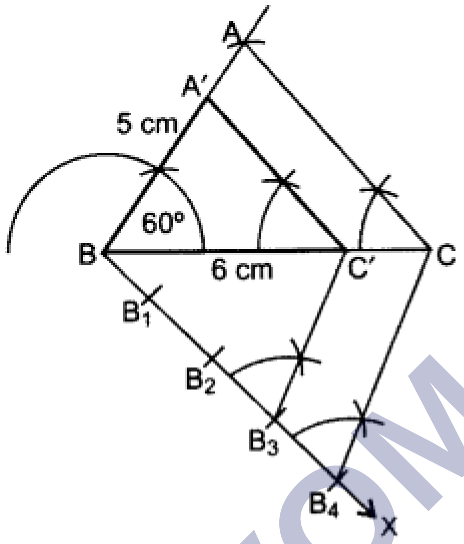
Hence, ΔAB'C' is the required triangle.

7. In ΔABC, AB = 5 cm; BC = 6 cm; ∠ABC = 60°



∴ ΔA'BC' is the required Δ.

8.



Steps of Construction:

- Draw ΔABC with the given data.
- Draw a ray BX downwards making an acute angle with BC.
- Locate 4 points B_1, B_2, B_3, B_4, on BX, such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
- Join CB_4.
- From B_3 draw a line $C'B_3$ || CB_4 intersecting BC at C'.
- From C' draw A'C' || AC intersecting AB at B'.

Then ΔAB'C' in the required triangle.

Justification:

$\Delta A'BC' \sim \Delta ABC$ ...[AA similarty rule

$\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC}$ ...(*i*)

...[Sides are proportional

But $\frac{BC'}{BC} = \frac{BB_3}{BB_4} = \frac{3}{4}$ ...(*ii*)

$\therefore \frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{3}{4}$ ...[From (*i*) & (*ii*)

Case Study Answer:

1. **Answer:**
   i. c.
   ii. d.
   iii. d.
   iv. a.
   v. b.
2. **Answer:**
   i. c.
   ii. a.
   iii. a.
   iv. b.
   v. c.

Assertion Reason Answer-

(a) Both A and R are true and R is the correct explanation of A.

(a) Both A and R are true and R is the correct explanation of A.