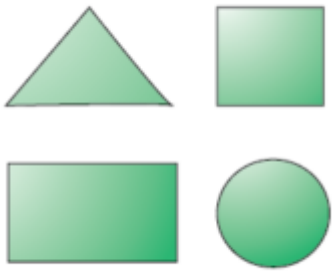
MATHEMATICS

Chapter 10: Visualising Solid Shapes

Visualising Solid Shapes

Two-Dimensional Object

A shape with only two dimensions (such as length and width) and no thickness is called two-dimensional shape. Squares, Circles, Triangles, etc are two dimensional objects. Also known as “2D”.



Area

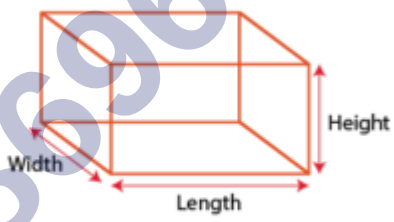
Area is measurement of space enclosed by a closed geometrical figure.

Volume

Volume is measurement of total space occupied by a solid.

Three Dimensional Objects- Solids

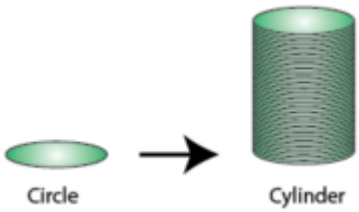
Shapes which can be measured in 3 directions are called three-dimensional shapes. These shapes are also called solid shapes. Length, width, and height (or depth or thickness) are their dimensions.



Formation of Solid Solids

Stacking of 2 dimensional figures, results in solids shapes.

Example: Linear stacking of circles, forms the solid shape called cylinder.

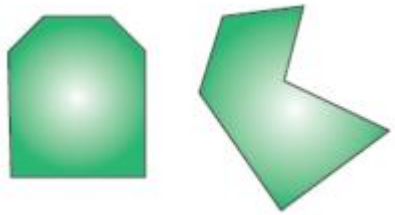


Solids and Their Classification

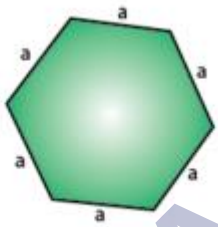
Hexagon

In geometry, hexagon is a polygon with 6 sides. Sum of all interior angles is equal to 720°.

If all the sides of hexagon are equal then it is called regular hexagon, then each interior angle measures 120°.



Irregular hexagons

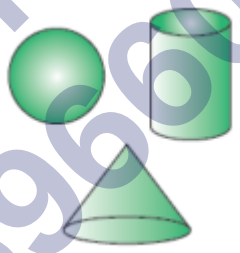


Regular hexagons

Non-Polyhedrons

Solids with curved faces are called Non polyhedrons. They also can be discribed as solids which have sides that are not polygons.

Example: Sphere, Cylinder, Cone, etc.



Polyhedrons

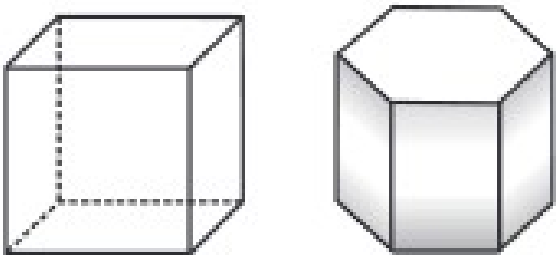
A Polyhedron is a solid in three dimensions with flat polygonal faces, straight edges and sharp corners or vertices. In short, Solids with flat surfaces are called Polyhedrons. (or Polyhedra)

Types of Polyhedrons

Convex Polyhedron:

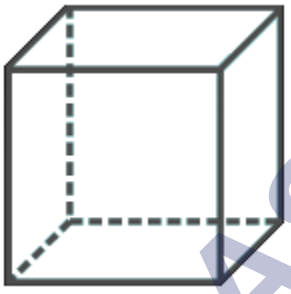
A polyhedron whose surface (comprising its faces, edges and vertices) does not intersect itself.

Line segment joining any two points of the polyhedron lies within its interior part or on surface.



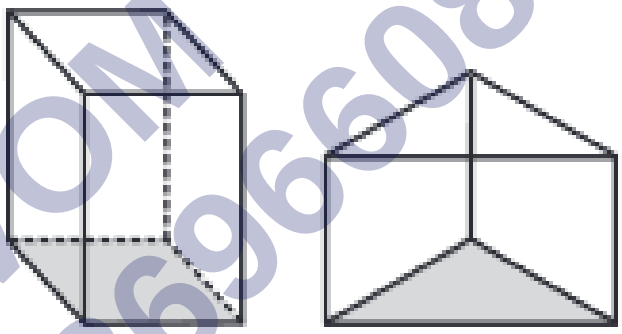
Regular polyhedrons:

A polyhedron is said to be regular if its faces are made up of regular polygons and the same number of faces meet at each vertex.



Important polyhedrons:

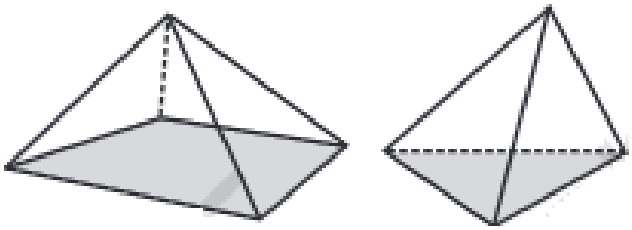
Prism:



Prism is a polyhedron with two parallel opposite faces, called bases, that are congruent polygons, and the lateral faces are parallelograms.

A prism is called a triangular prism if its ends are triangles.

Pyramid



- Pyramid is a solid whose base is a plane rectilinear figure and whose side faces are triangles having a common vertex, called the vertex of the pyramid.
- A pyramid is said to be a regular pyramid if all the sides of its base are equal.
- A pyramid is called a triangular pyramid if its base is a triangle.

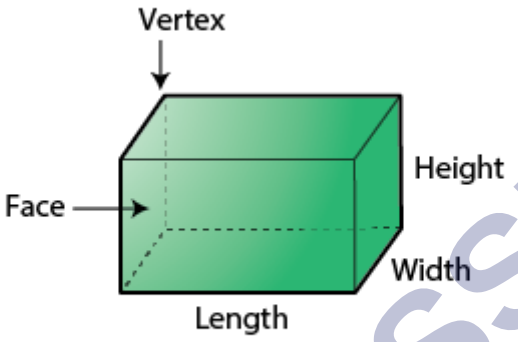
- A triangular pyramid is also called a tetrahedron.

If the base of a pyramid is a quadrilateral, then it is called quadrilateral pyramid.

Solid Cuboid

A cuboid is a solid bounded by six rectangular plane regions.

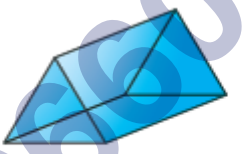
It is formed by stacking rectangles linearly.



Cuboid

Solid Triangular Prism

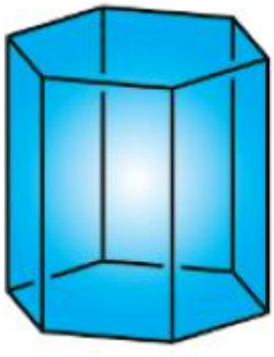
Solid Triangular Prism: A polyhedron with two triangular bases parallel to each other. It is formed by stacking triangles linearly.



Soliod triangular prism

Solid Hexagonal Prism

Solid Hexagonal Prism: A polyhedron with two hexagonal bases parallel to each other. It is formed by stacking hexagons linearly. Each face of hexagonal prism is rectangular in shape.



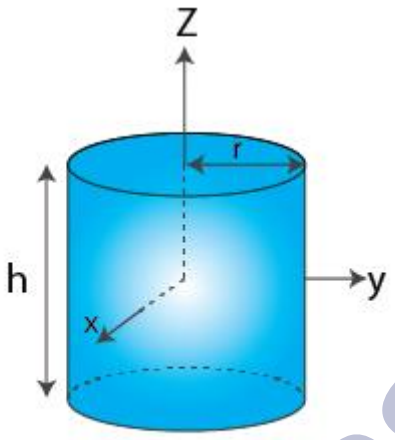
Solid Hexagonal Prism

Solid Cylinder

Solid cylinder can be formed in two different ways:

By stacking rectangles in a circular fashion.

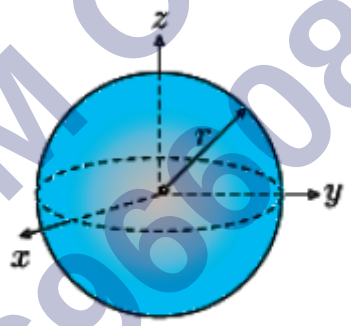
By stacking many circles linearly.



Solid cylinder

Solid Sphere

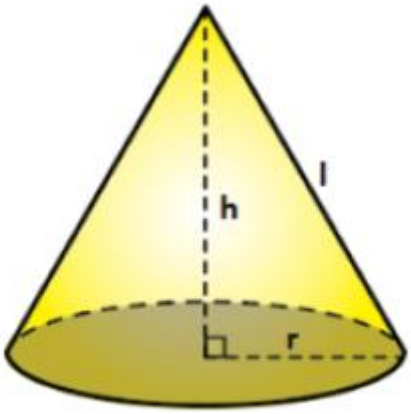
Solid spheres are formed by stacking circles in a circular fashion.



Solid sphere

Solid Cone

Solid Cones are formed by stacking triangles which are right-angled, in a circular fashion with edge which is right angled at the center.



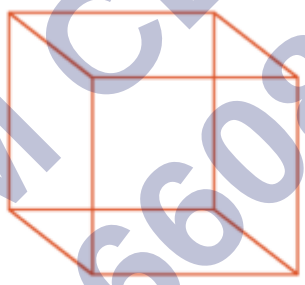
Solid cone

Formation of Hollow Solids

Hollow Solids are obtained by joining two dimensional figures.

Hollow Cuboid

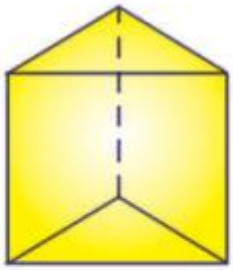
Hollow Cuboid: Formed by joining six rectangles in a specific manner as shown below:



Hollow cuboid

Hollow Triangular Prism

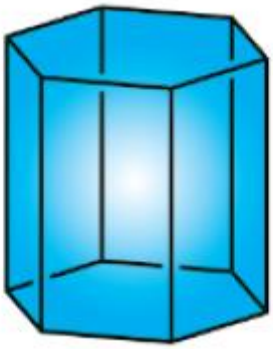
Hollow triangular prism: Formed by joining two triangles and three rectangles in a specific manner as shown below:



Hollow Triangular prism

Hollow Hexagonal Prism

Hollow hexagonal Prism: Formed by joining two hexagons and six rectangles as shown below:

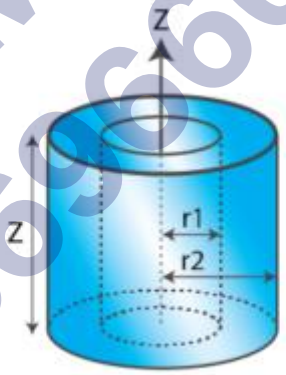


Hollow Hexagonal Prism

Hollow Cylinder

Hollow Cylinder: A cylinder is made by rotating a rectangle around either its length or breadth as shown below.

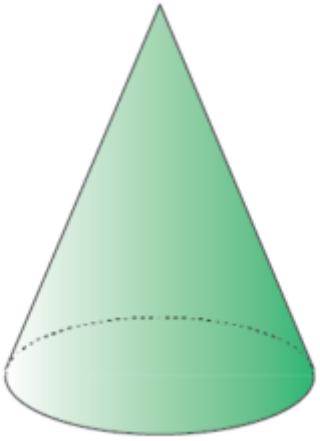
Hollow cylinder



Hollow cylinder

Hollow Cone

Hollow Cone: A circle and a curved sector of a circle are joined together as shown below:



Hollow Cone

Pyramid

Pyramid: All side faces are triangular in shape and base is of the shape of any polygon.

(i) Triangular pyramid

(i) Triangular pyramid

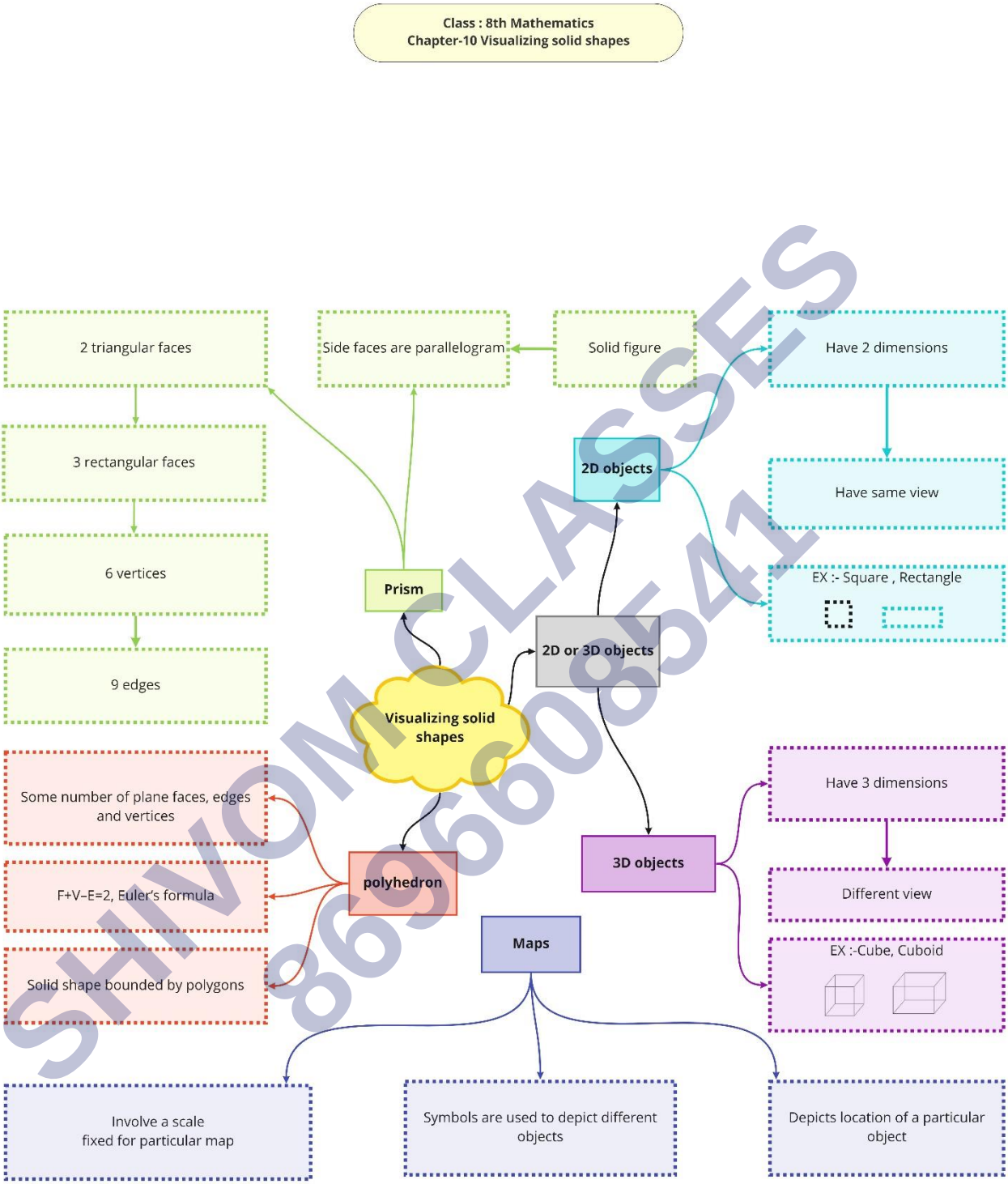
Euler's formula

For any polyhedron: **F + V = E + 2**

where

F = number of faces,

V = number of vertices, E = number of edges.



Important Questions

Multiple Choice Questions:

Question 1. The name of the shape is

(a) cylinder

(b) square

(c) circle

(d) triangle.

Question 2. The name of the shape is

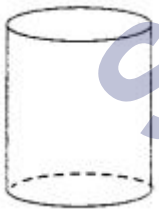
(a) sphere

(b) cylinder

(c) cone

(d) triangle.

Question 3. The name of the shape is



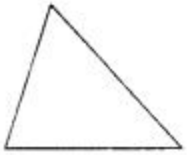
(a) cone

(b) sphere

(c) cuboid

(d) cylinder.

Question 4. The name of the shape



(a) triangle

(b) cone

(c) cylinder

(d) sphere

Question 5. The name of the shape is

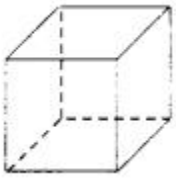
(a) cone

(b) circle

(c) cylinder

(d) cube.

Question 6. The name of the shape is



(a) cuboid

(b) cube

(c) square

(d) cylinder.

Question 7. The name of the shape is

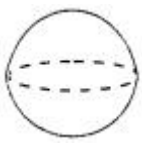
(a) cube

(b) cuboid

(c) sphere

(d) square.

Question 8. The name of the shape is



(a) circle

(b) sphere

(c) cylinder

(d) cone.

Question 9. A cuboid has how many faces ?

(a) 2

(b) 4

(c) 6

(d) 3.

Question 10. How many faces does a cube have ?

(a) 6

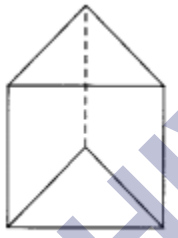
(b) 4

(c) 3

(d) 2.

Very Short Questions:

1. Draw any four 3-dimensional figures.
2. Verify Euler's formula for a right triangular prism.

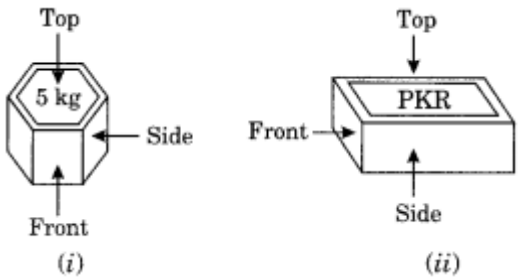


Triangular prism

3. Find the number of vertices of hexagonal prisms.
4. Verify whether a polyhedron can have 10 faces, 20 edges and 15 vertices.
5. If F = 18 and V = 10, then find the value of E in Euler's formula.

Short Questions:

1. Draw the front, side and top views of the following 3-D figures.



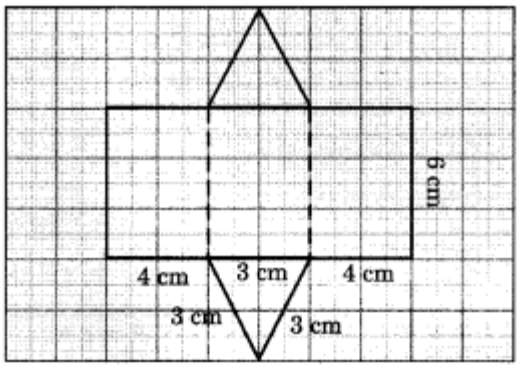
2. Draw the nets of the following polyhedrons.

(i) Cuboid

(ii) Triangular prism with a base equilateral triangle.

(iii) Square pyramid.

3. The given net is made up of two equilateral triangles and three rectangles.



(i) Name the solid it represents.

(ii) Find the number of faces, edges and vertices.

4. Using Euler's formula, fill in the blanks:

	Faces	**Vertices**	**Edges**
(a)	6	8	—
(b)	—	10	15
(c)	4	—	6
(d)	5	6	—
(e)	8	12	—
(f)	7	7	—

5. Name the solids that have:

(i) 4 faces

(ii) 8 triangular faces

(iii) 6 faces

(iv) 1 curved surface

(v) 5 faces and 5 vertices

(vi) 6 rectangular faces and 2 hexagonal faces

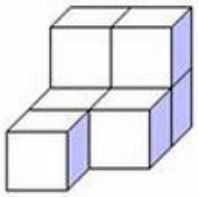
Long Questions:

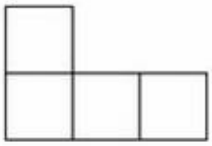
1. Complete the table:

Solid	F	V	E	F + V	E + 2
Cuboid	—	—	—	—	—
Triangular pyramid	—	—	—	—	—
Triangular prism	—	—	—	—	—
Pyramid with square base	—	—	—	—	—
Prism with square base	—	—	—	—	—

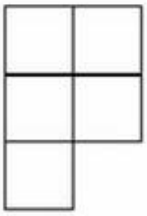
2. Use isometric dot paper to sketch a rectangular prism with length 4 units, height 2 units and width 3 units.

3. Identify the given views of the block:

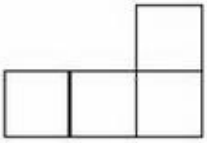




(a)



(b)



(c)

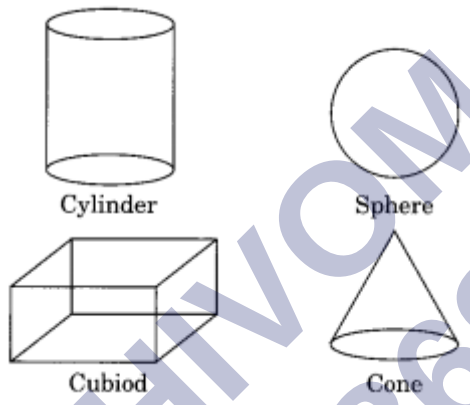
(d)

Answer Key-

Multiple Choice Questions:

1. (b) square
2. (c) cone
3. (d) cylinder
4. (a) triangle
5. (b) circle
6. (b) cube
7. (b) cuboid
8. (b) sphere
9. (c) 6
10. (a) 6

Very Short Answer:

1. 

2. Number of vertices (V) = 6
   Number of faces (F) = 5
   and number of edges (E) = 9
   Euler's formula:
   V + F – E = 2
   ⇒ 6 + 5 – 9 = 2
   ⇒ 2 = 2
   Hence, the formula is verified.
3. Number of vertices = 2 × Number of sides = 2 × 6 = 12
4. We have
   Number of faces F = 10

Number of edges E = 20

and number of vertices V = 15

Euler's formula:

V + F – E = 2

⇒ 15 + 10 – 20 = 2

⇒ 5 ≠ 2

Hence, it is not possible to have a polyhedron satisfying the above data.

5. We know that

V + F – E = 2

⇒ 10 + 18 – E = 2

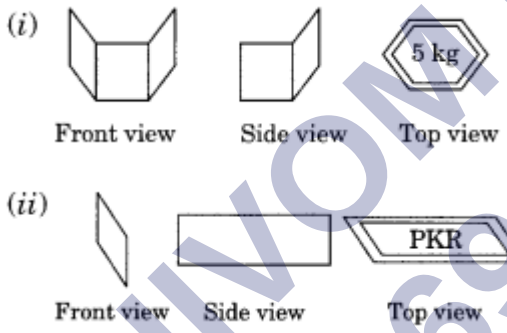
⇒ 28 – E = 2

⇒ E = 28 – 2 = 26

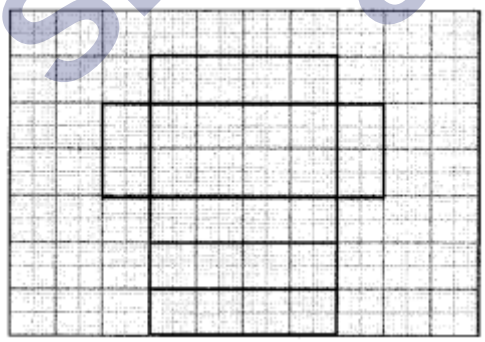
Hence, the required value of E = 26

Short Answer:

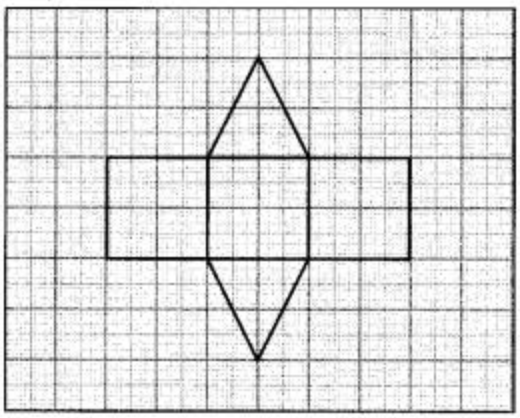
1.



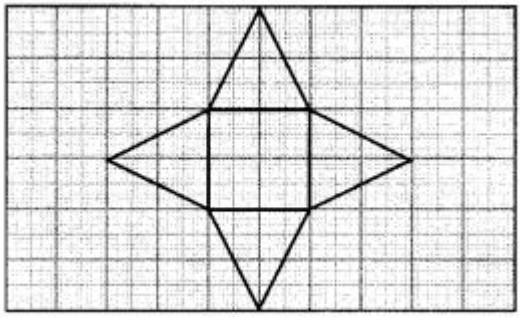
2. (i) The net pattern of cuboid



(ii) The net pattern of a triangular prism



(iii) Net pattern of square pyramid



3. (i) The given figure represents the net prims of the triangular prism

(ii) Number of faces = 5

Number of edges = 9

Number of vertices = 6

4. (a) F + V – E = 2

⇒ 6 + 8 – E = 2

⇒ 14 – E = 2

⇒ E = 14 – 2 = 12

(b) F + V – E = 2

⇒ F + 10 – 15 = 2

⇒ F – 5 = 2

⇒ F = 2 + 5 = 7

(c) F + V – E = 2

⇒ 4 + V – 6 = 2

⇒ V – 2 = 2

⇒ V = 2 + 2 = 4

(d) F +V – E = 2

⇒ 5 + 6 – E = 2

⇒ 11 – E = 2

$\Rightarrow E = 11 - 2 = 9$

(e) $F + V - E = 2$

$\Rightarrow 8 + 12 - E = 2$

$\Rightarrow 20 - E = 2$

$\Rightarrow E = 20 - 2 = 18$

(f) $F + V - E = 2$

$\Rightarrow 7 + 7 - E = 2$

$\Rightarrow 14 - E = 2$

$\Rightarrow E = 14 - 2 = 12$

Hence (a) → 12, (b) → 7, (c) → 4, (d) → 9, (e) → 18, (f) → 12.

5. (i) Tetrahedron

(ii) Regular octahedron

(iii) Cube and cuboid

(iv) Cylinder

(v) Square and a rectangular pyramid

(vi) Hexagonal prism

Long Answer:

1.

Solid	F	V	E	F + V	E + 2
Cuboid	6	8	12	14	14
Triangular pyramid	4	4	6	8	8
Triangular prism	5	6	9	11	11
Pyramid with square base	5	5	8	10	10

Prism with square base	6	8	12	14	14

2. Steps:

1. Draw a parallelogram with sides 4 units and 3 units. This is the top of the prism (Fig. 1).

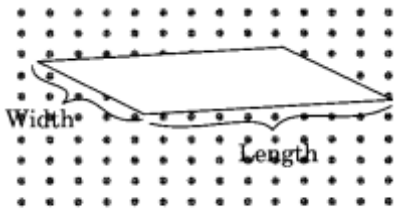


Fig. 1

2. Start at one vertex. Draw a line passing through two dots. Repeat for the other three vertices. Draw the hidden edges as a dashed line (Fig. 2).

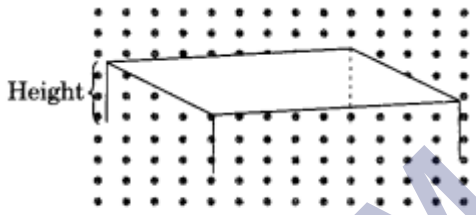


Fig. 2

3. Connect the ends of the lines to complete the prism (Fig. 3).

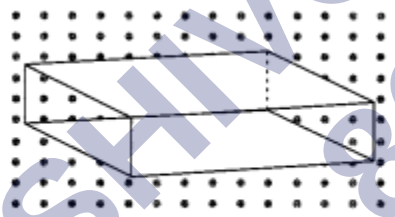


Fig. 3

3.

a. Left View

b. Top View

c. Right View

d. Front View.