MATHEMATICS

Chapter 10: Circles

Circles

1. Introduction to Circle

A **circle** is the locus of a point which lies in the plane in such a manner that its distance from a fixed point in the plane is constant. The fixed point is called the **centre** and the constant distance is called the **radius** of the circle.

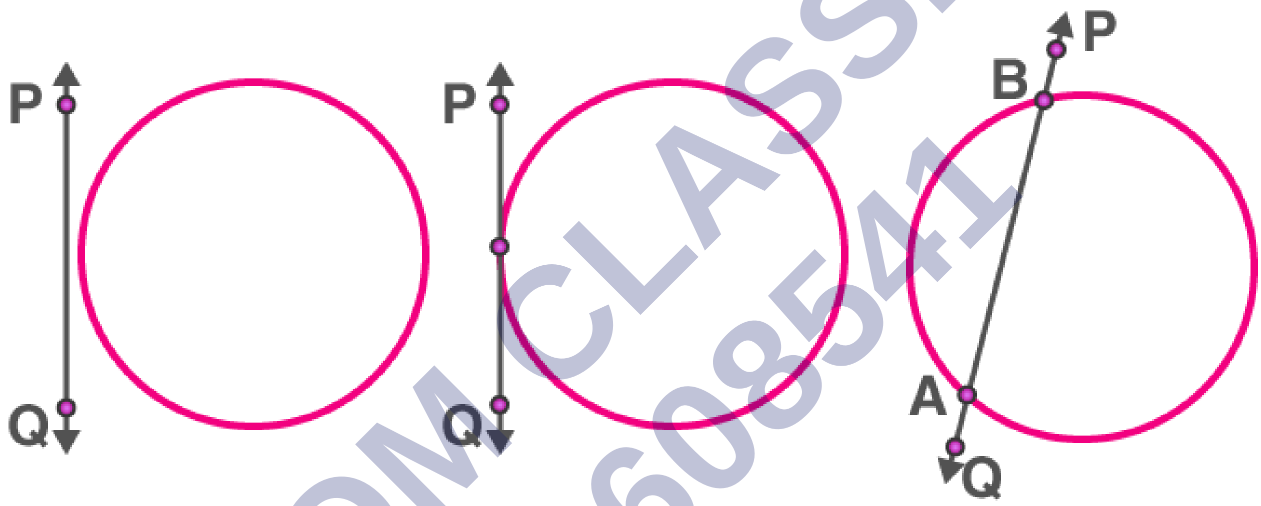
Circle and line in a plane

For a circle and a line on a plane, there can be three possibilities.

they can be non-intersecting

they can have a single common point: in this case, the line touches the circle.

they can have two common points: in this case, the line cuts the circle.

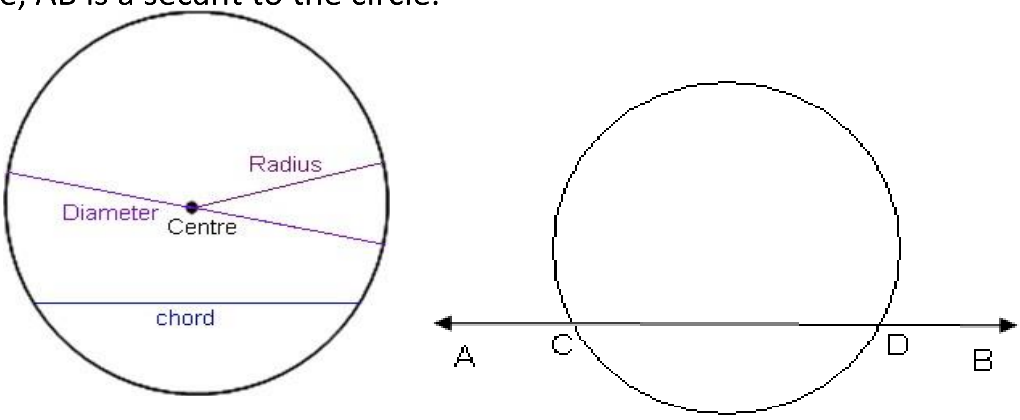


(i) Non intersecting (ii) Touching (iii) Intersecting

2. Parts of the circle

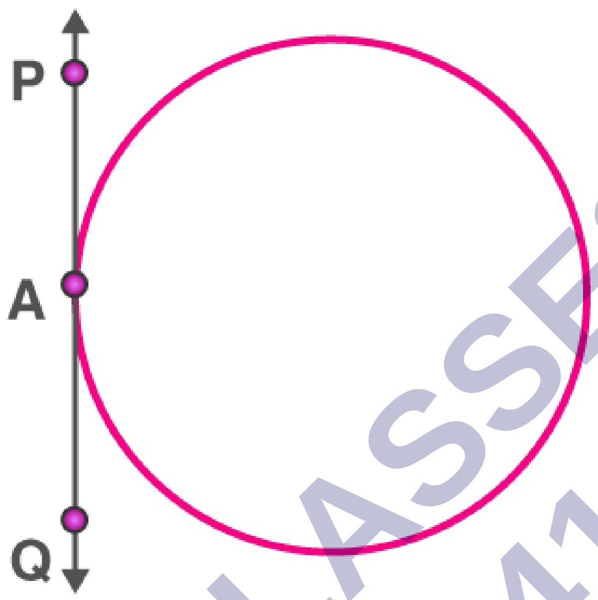
- A line segment that joins any two points lying on a circle is called the **chord** of the circle.
- A chord passing through the centre of the circle is called **diameter** of the circle.
- A line segment joining the centre and a point on the circle is called **radius** of the circle.
- A line which intersects a circle at two distinct points is called a **secant** of the circle.

In the figure, AB is a secant to the circle.



3. Tangent to the circle

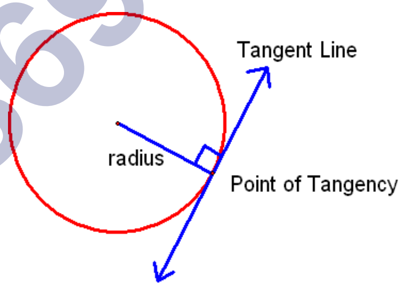
A **tangent** to the circle is a line that intersects the circle (touches the circle) at only one point. The word 'tangent' comes from the Latin word 'tangere', which means to touch. The common point of the circle and the tangent is called **point of contact**.



In the figure, AB is a tangent to the circle and P is the point of contact.

4. Important facts about tangent

- The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact. This point of contact is also called as point of tangency.



- A line drawn through the end of a radius (point on circumference) and perpendicular to it is a tangent to the circle.

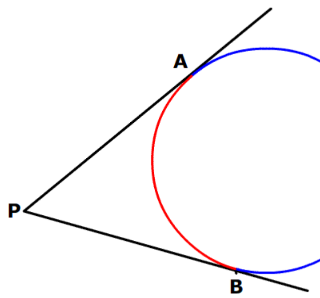
5. Number of tangents on a circle

- There is no tangent possible to a circle from the point (or passing through a point) lying inside the circle.
- There are **exactly two tangents** possible to a circle **through a point outside the circle**.
- At any point on the circle, there can be one and only one tangent possible.

6. Length of the tangent

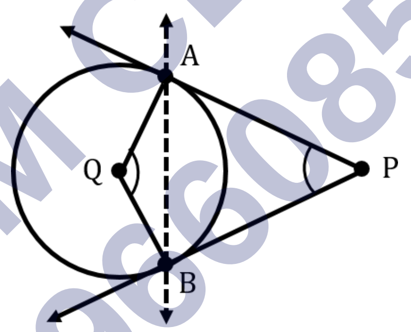
The length of the segment of the tangent from the external point P and the point of contact with the circle is called the **length of the tangent**.

- The lengths of tangents drawn from an external point to the circle are equal.
- The figure shows two equal tangents (*PA* = *PB*) from an external point *P*.



7. Angle between two tangents from an external point

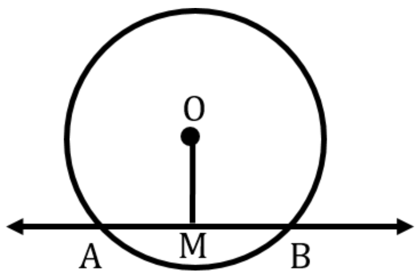
- The centre of a circle lies on the bisector of the angle between the two tangents drawn from an external point.
- Angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.



In the figure, angle P and angle Q are supplementary.

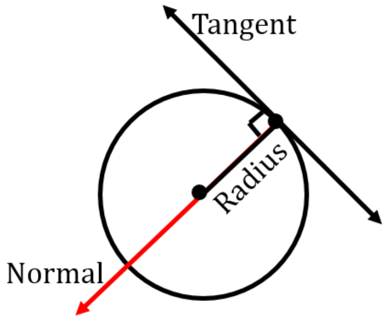
8. Perpendicular from the centre

Perpendicular drawn from the centre to any chord of the circle, divides it into two equal parts. In the figure, OM is perpendicular to AB and AM = MB.



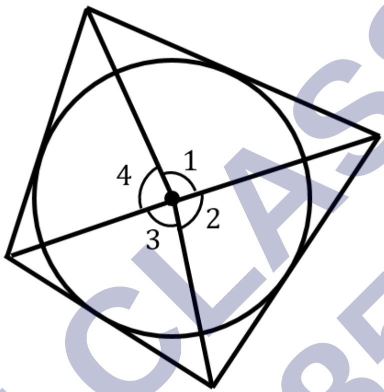
9. Normal to the circle

The line containing the radius through the point of contact is called the normal to the circle at that point.



10. Inscribed circle

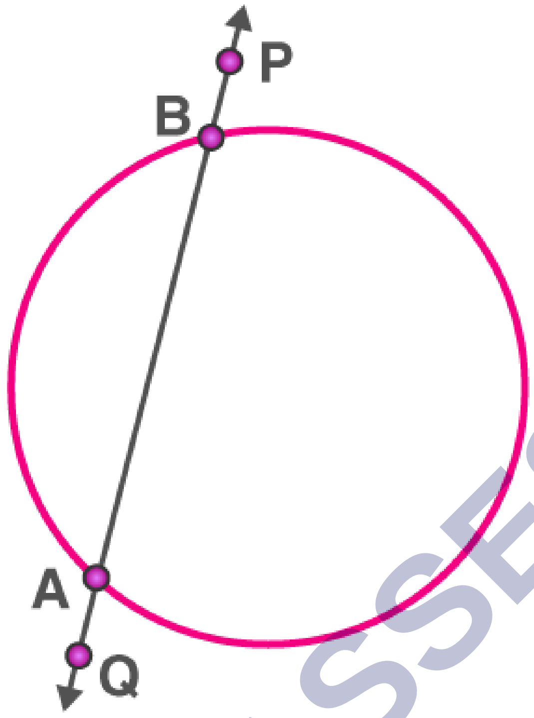
Opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



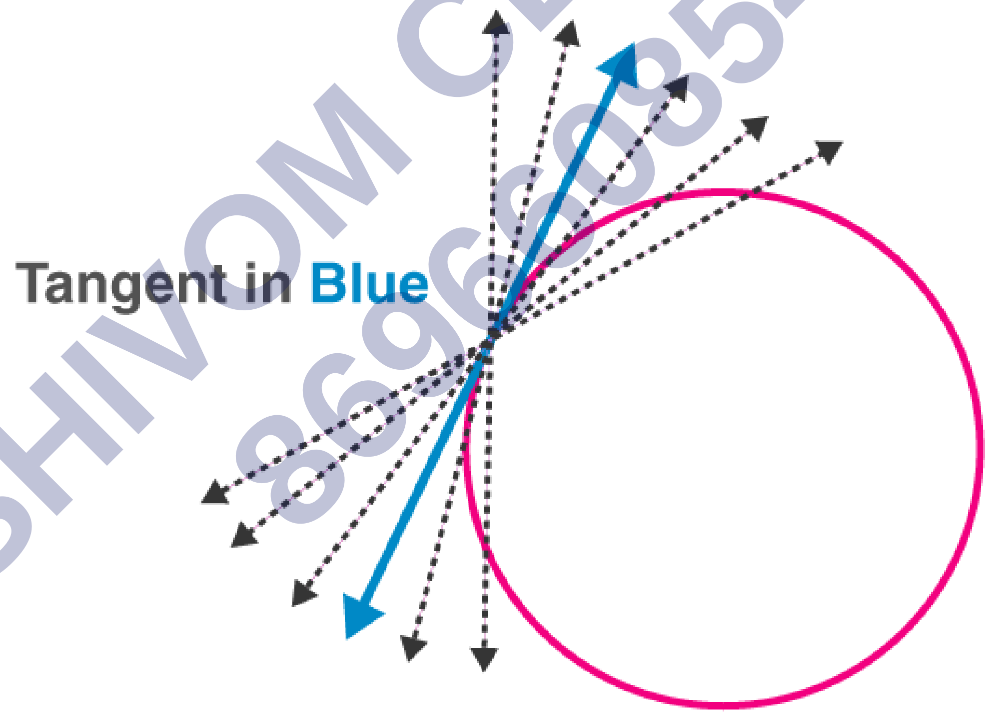
In the figure, angles 1 and 3 are supplementary. Accordingly, angles 2 and 4 are supplementary.

Secant

A secant to a circle is a line that has two points in common with the circle. It cuts the circle at two points, forming a chord of the circle.



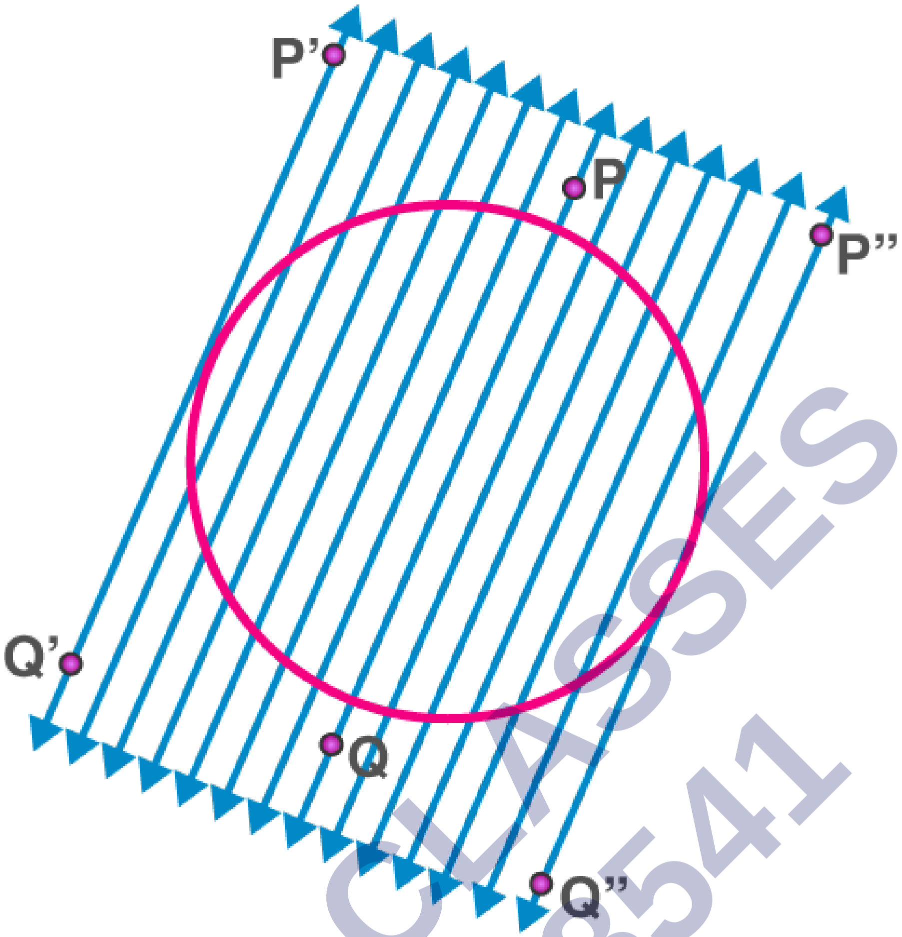
Tangent as a special case of Secant



The tangent to a circle can be seen as a special case of the secant when the two endpoints of its corresponding chord coincide.

Two parallel tangents at most for a given secant

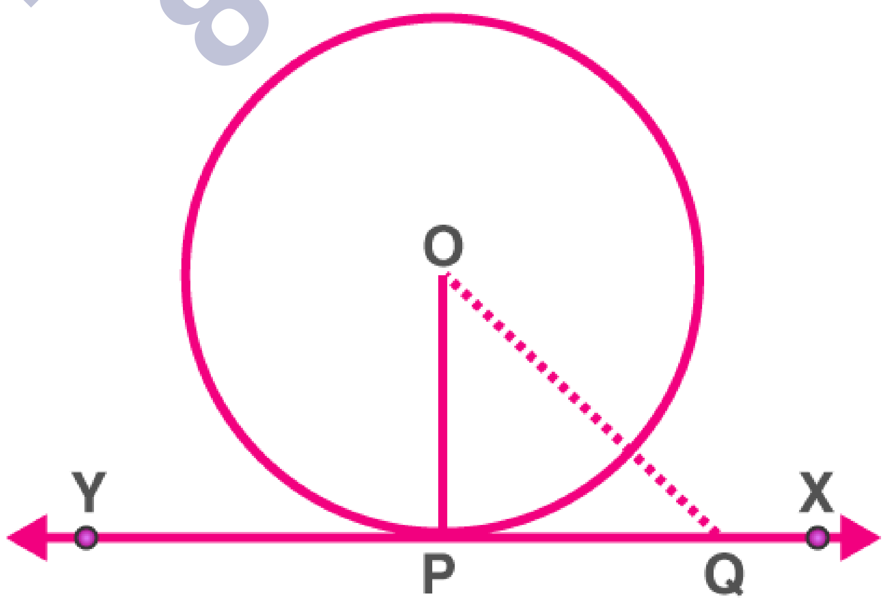
For every given secant of a circle, there are exactly two tangents which are parallel to it and touches the circle at two diametrically opposite points.



Theorems

Tangent perpendicular to the radius at the point of contact

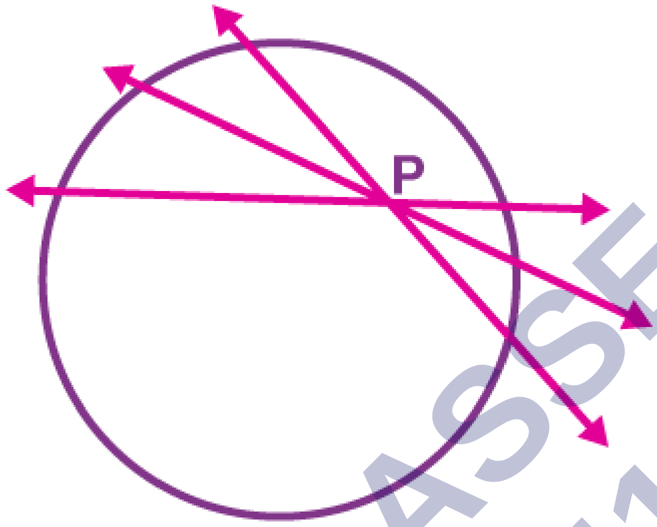
Theorem: The theorem states that "the tangent to the circle at any point is the perpendicular to the radius of the circle that passes through the point of contact".



Here, O is the centre and OP⊥XY.

The number of tangents drawn from a given point

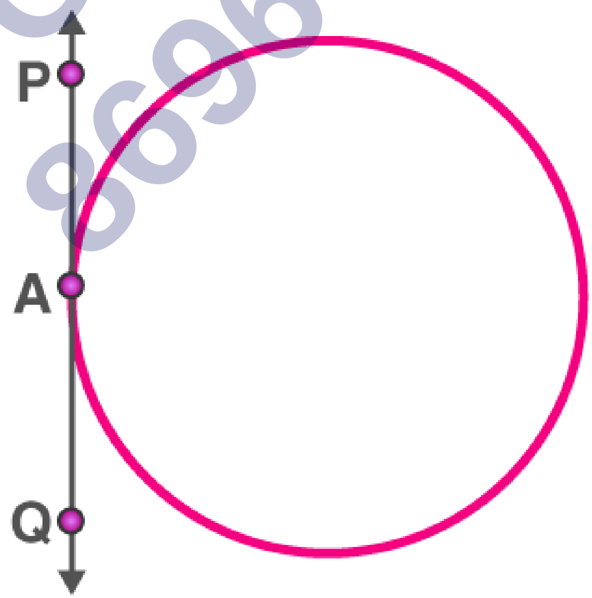
If the point is in an interior region of the circle, any line through that point will be a secant. So, no tangent can be drawn to a circle which passes through a point that lies inside it.



No tangent can be drawn to a circle from a point inside it

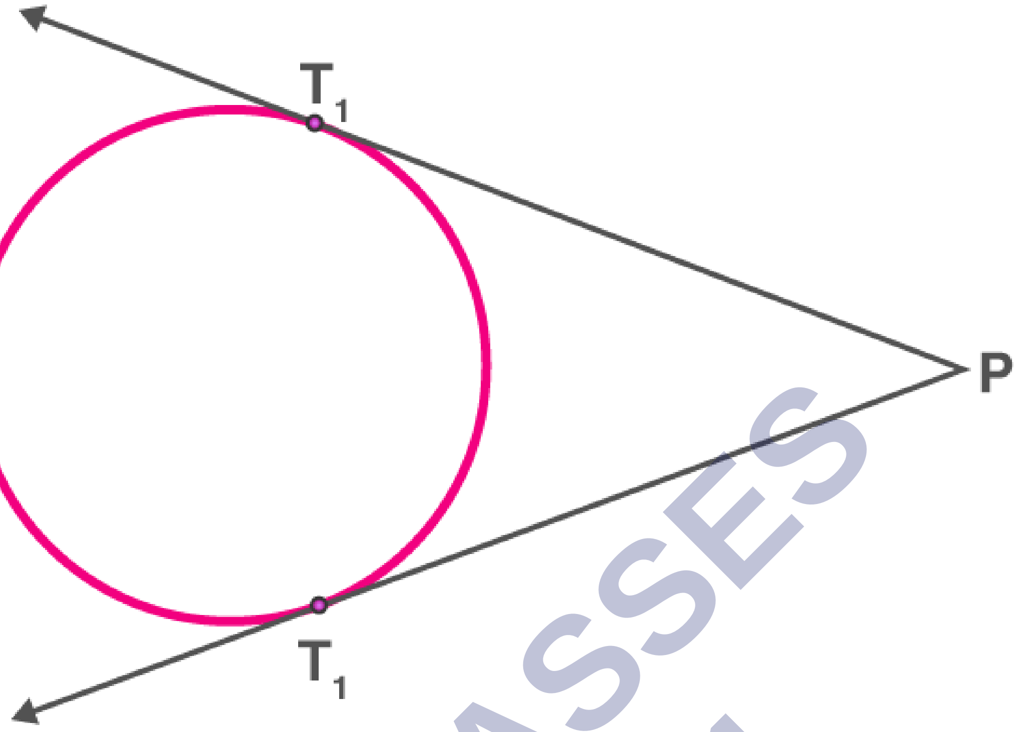
AB is a secant drawn through the point S

When a point of tangency lies on the circle, there is exactly one tangent to a circle that passes through it.



A tangent passing through a point lying on the circle

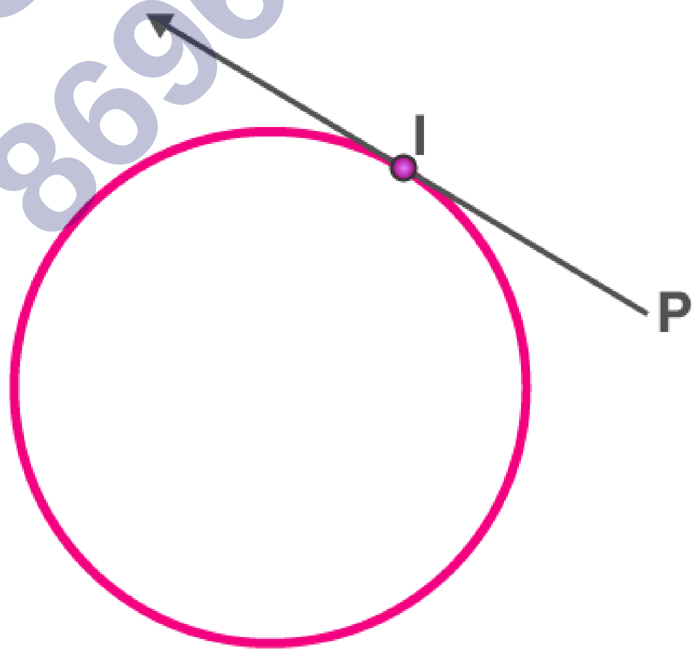
When the point lies outside of the circle, there are accurately two tangents to a circle through it



Tangents to a circle from an external point

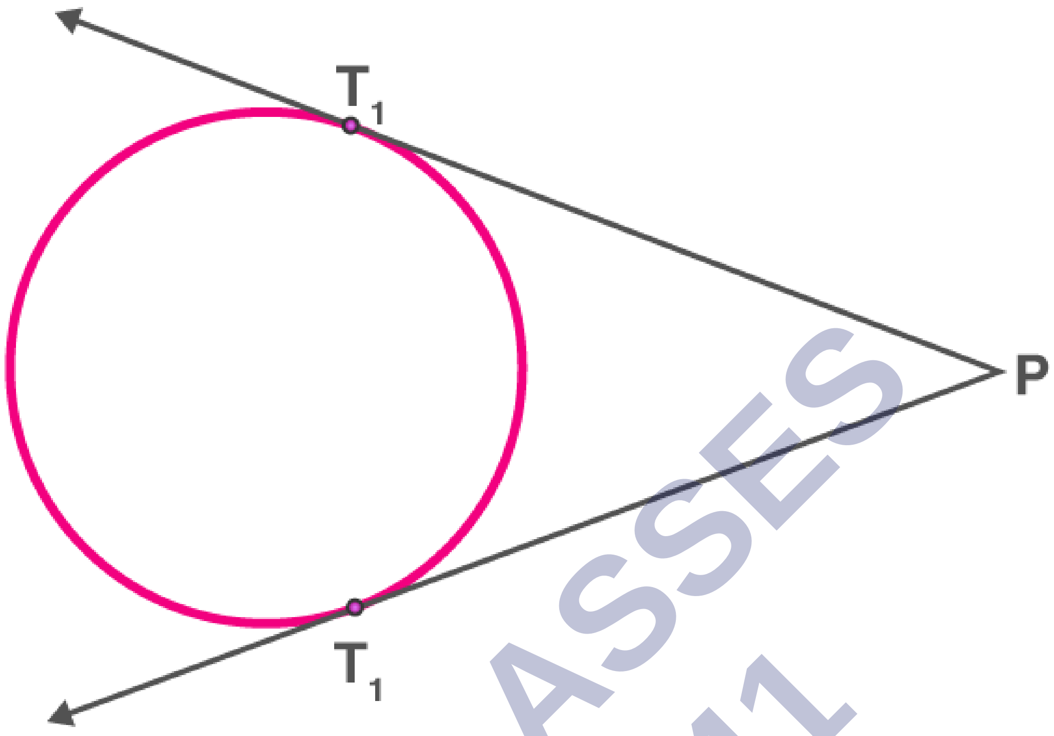
Length of a tangent

The length of the tangent from the point (Say P) to the circle is defined as the segment of the tangent from the external point P to the point of tangency I with the circle. In this case, PI is the tangent length.



Lengths of tangents drawn from an external point

Theorem: Two tangents are of equal length when the tangent is drawn from an external point to a circle.



$PT_1 = PT_2$

Thus, the two important theorems in Class 10 Maths Chapter 10 Circles are:

Theorem 10.1: The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Theorem 10.2: The lengths of tangents drawn from an external point to a circle are equal.

Interesting facts about Circles and its properties are listed below:

In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

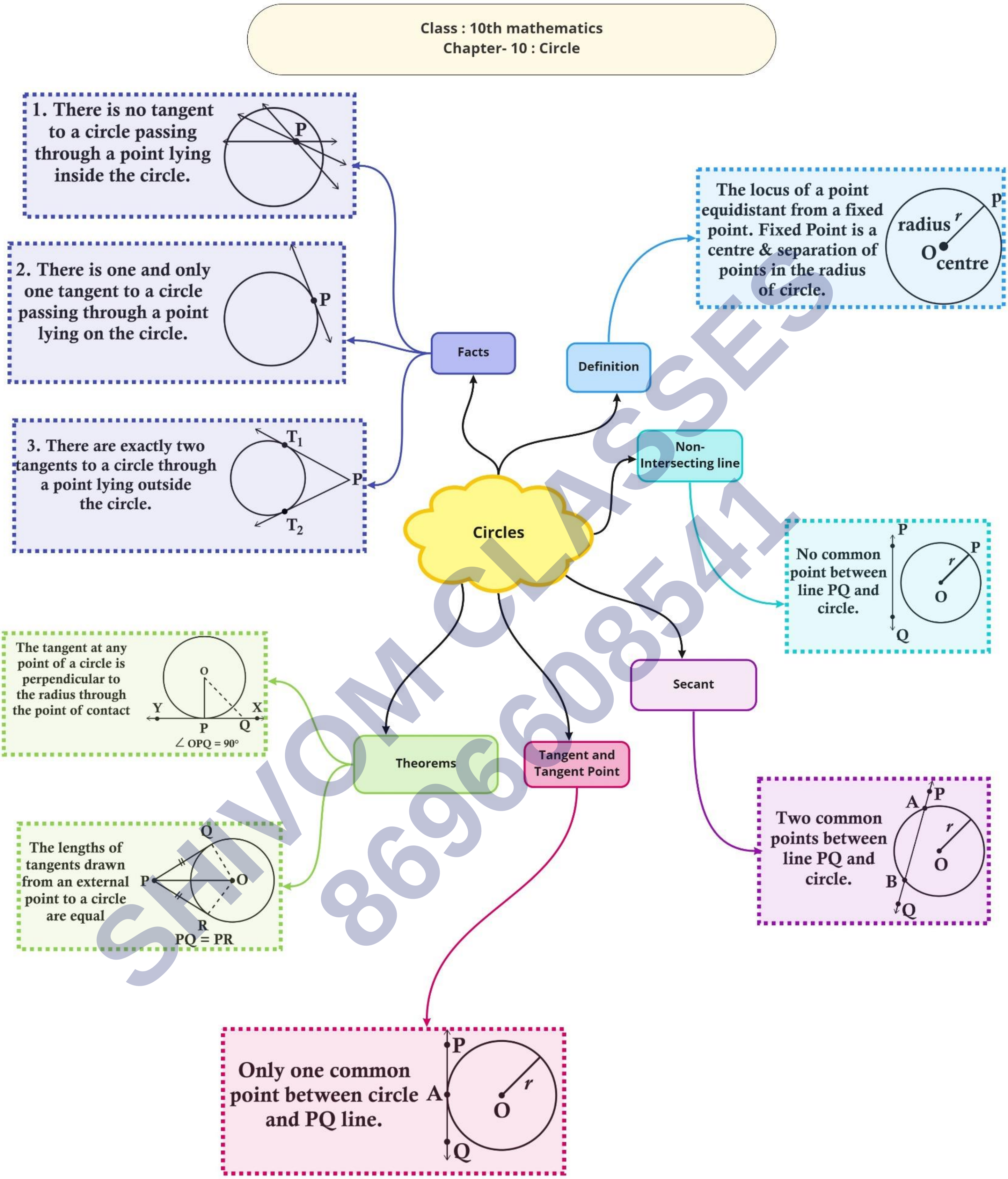
The tangents drawn at the ends of a diameter of a circle are parallel.

The perpendicular at the point of contact to the tangent to a circle passes through the centre.

The angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

The parallelogram circumscribing a circle is a rhombus.

The opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.


Class : 10th mathematics
Chapter- 10 : Circle
1. There is no tangent to a circle passing through a point lying inside the circle.
P
2. There is one and only one tangent to a circle passing through a point lying on the circle.
P
3. There are exactly two tangents to a circle through a point lying outside the circle.
T_1
P
T_2
Facts
Definition
The locus of a point equidistant from a fixed point. Fixed Point is a centre & separation of points in the radius of circle.
radius
r
P
O
centre
Non-Intersecting line
No common point between line PQ and circle.
P
Q
r
O
Circles
Secant
Two common points between line PQ and circle.
A
P
B
Q
O
Theorems
Tangent and Tangent Point
The tangent at any point of a circle is perpendicular to the radius through the point of contact
O
Y
X
P
Q
∠ OPQ = 90°
The lengths of tangents drawn from an external point to a circle are equal
Q
P
O
R
PQ = PR
Only one common point between circle and PQ line.
P
A
O
Q

Important Questions

Multiple Choice questions-

1. Two circle touch each other externally at C and AB is a common tangent to the circles. Then, ∠ACB =

(a) 60°

(b) 45°

(c) 30°

(d) 90°

2. If TP and TQ are two tangents to a circle with centre O so that ∠POQ = 110°, then, ∠PTQ is equal to

(a) 60°

(b) 70°

(c) 80°

(d) 90°

3. Tangents from an external point to a circle are

(a) equal

(b) not equal

(c) parallel

(d) perpendicular

4. Two parallel lines touch the circle at points A and B respectively. If area of the circle is 25 n cm^2, then AB is equal to

(a) 5 cm

(b) 8 cm

(c) 10 cm

(d) 25 cm

5. A line through point of contact and passing through centre of circle is known as

(a) tangent

(b) chord

(c) normal

(d) segment

6. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q

(a) √119 cm

(b) 13 cm

(c) 12 cm

(d) 8.5 cm

7. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is

(a) 60 cm^2

(b) 65 cm^2

(c) 30 cm^2

(d) 32.5 cm^2

8. At point A on a diameter AB of a circle of radius 10 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY at a distance 16 cm from A is

(a) 8 cm

(b) 10 cm

(c) 16 cm

(d) 18 cm

9. The tangents drawn at the extremities of the diameter of a circle are

(a) perpendicular

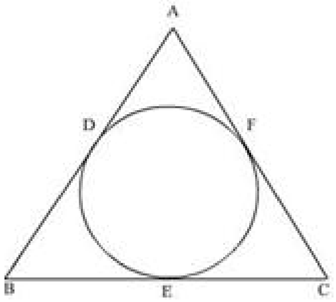
(b) parallel

(c) equal

(d) none of these

10. A circle is inscribed in a ΔABC having AB = 10cm, BC = 12cm and CA = 8cm and

touching these sides at D, E, F respectively. The lengths of AD, BE and CF will be



(a) AD = 4cm, BE = 6cm, CF = 8cm

(b) AD = 5cm, BE = 9cm, CF = 4cm

(c) AD = 3cm, BE = 7cm, CF = 5cm

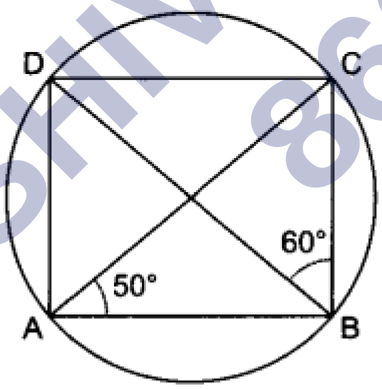
(d) AD = 2cm, BE = 6cm, CF = 7cm

Very Short Questions:

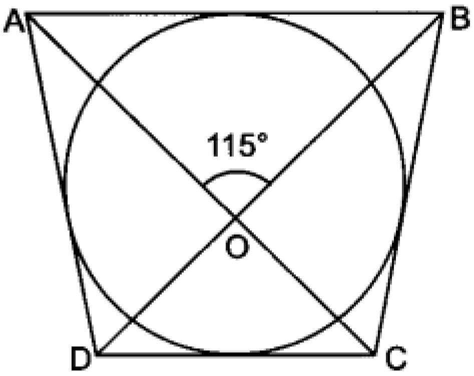
1. If a point P is 17 cm from the centre of a circle of radius 8 cm, then find the length of the tangent drawn to the circle from point P.

2. The length of the tangent to a circle from a point P, which is 25 cm away from the centre, is 24 cm. What is the radius of the circle?

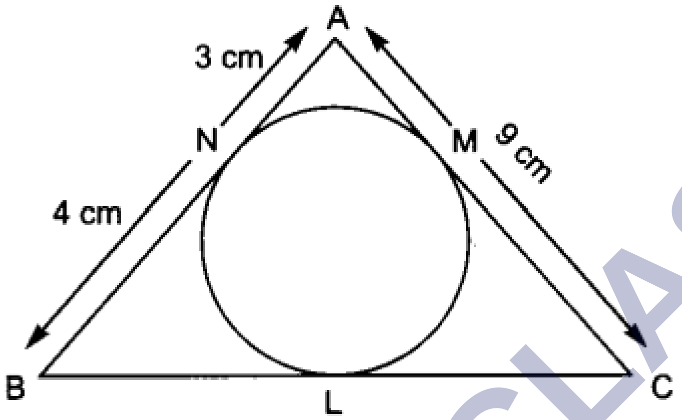
3. In Fig, ABCD is a cyclic quadrilateral. If ∠BAC = 50° and ∠DBC = 60° then find ∠BCD.



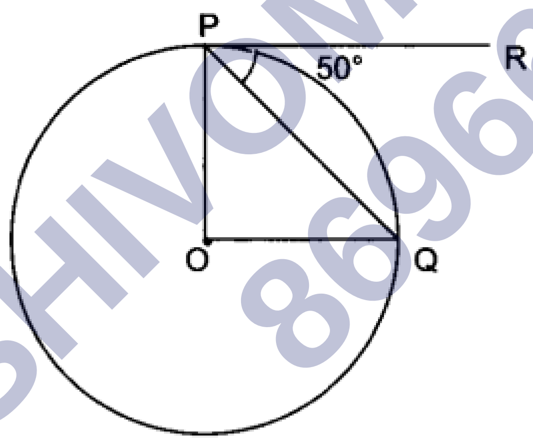
4. In Fig. the quadrilateral ABCD circumscribes a circle with centre O. If ∠AOB = 115°, then find ∠COD.



5. In Fig. AABC is circumscribing a circle. Find the length of BC.



6. In Fig. O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find ∠POQ.

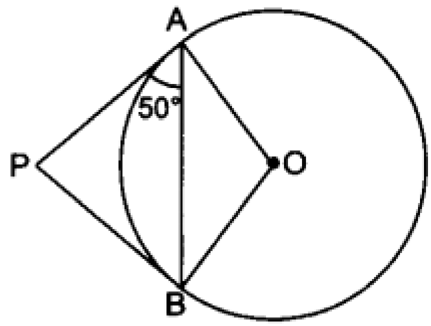


7. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

8. If radii of two concentric circles are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle.

9. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠POR = 120° then find ∠OPQ.

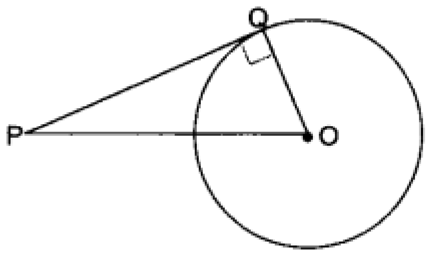
10. From an external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB.



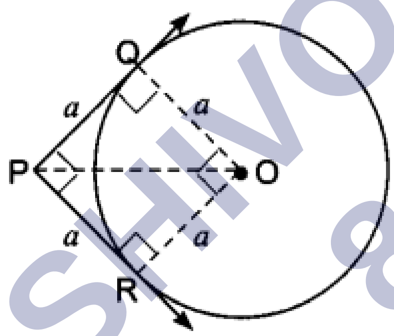
Short Questions :

1. AB is a diameter of a circle and AC is its chord such that ∠BAC = 30°. If the tangent at C intersects AB extended at D, then BC = BD.

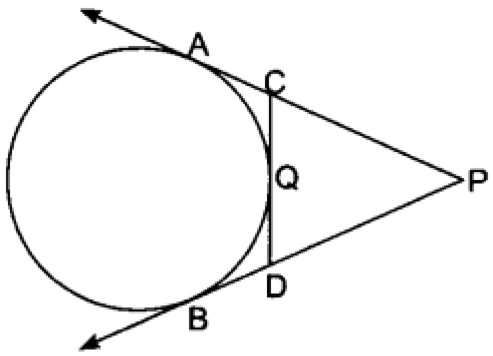
2. The length of tangent from an external point P on a circle with centre O is always less than OP.



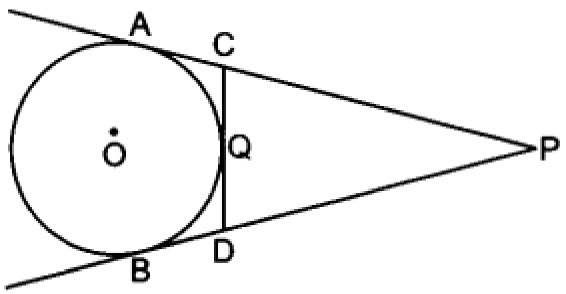
3. If angle between two tangents drawn from a point P to a circle of radius 'a' and centre 0 is 90°, then OP = a√2.



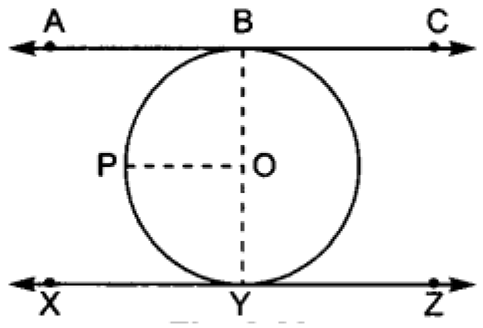
4. In Fig. PA and PB are tangents to the circle drawn from an external point P. CD is the third tangent touching the circle at Q. If PA = 15 cm, find the perimeter of ΔPCD.



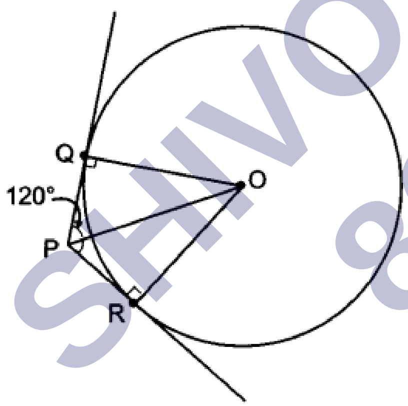
5. In Fig. PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm, then find PC + PD.



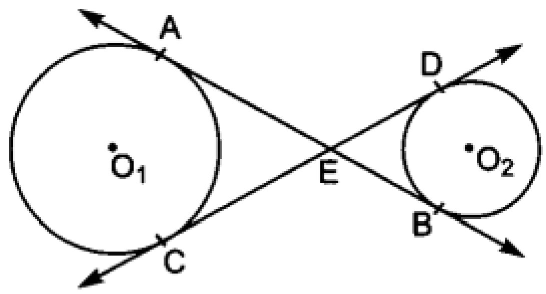
6. Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.



7. If from an external point P of a circle with centre 0, two tangents PQ and PR are drawn such that QPR = 120°, prove that 2PQ = PO.



8. In Fig. common tangents AB and CD to two circles with centres , and 0, intersect at E. Prove that AB = CD.

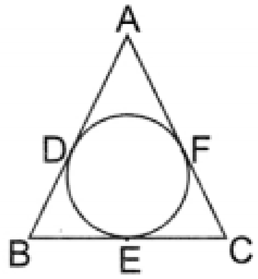


9. The incircle of an isosceles triangle ABC, in which AB = AC, touches the sides BC, CA and AB at D, E and F respectively. Prove that BD = DC.

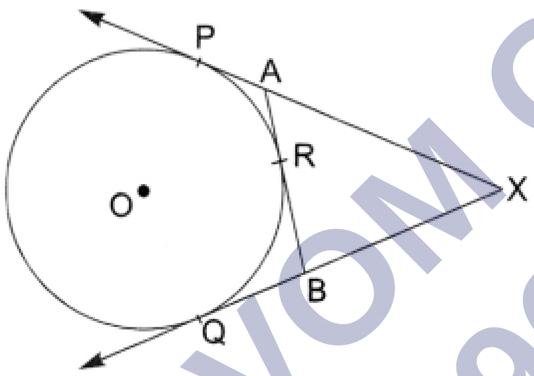
OR

In Fig. if AB = AC, prove that BE = EC.

[Note: D, E, F replace by F, D, E]



10. In Fig. XP and XQ are two tangents to the circle with centre O, drawn from an external point X. ARB is another tangent, touching the circle at R. Prove that XA + AR = XB + BR.



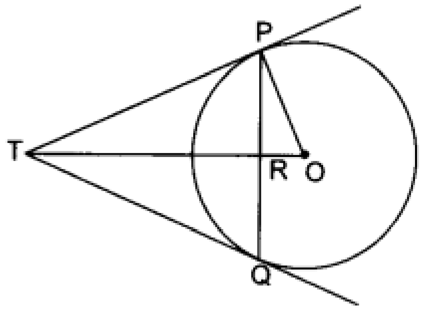
Long Questions :

1. Prove that the tangent to a circle is perpendicular to the radius through the point of contact.

2. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

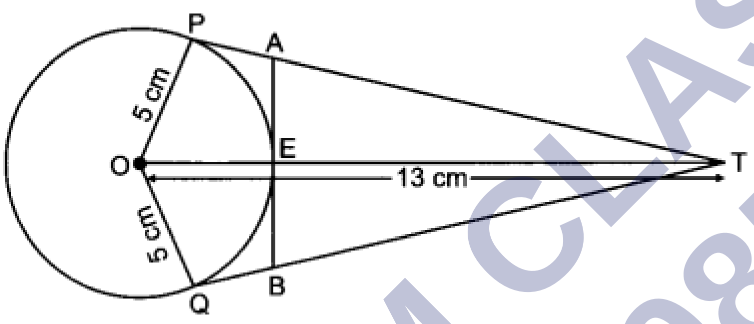
3. Prove that the parallelogram circumscribing a circle is a rhombus.

4. In Fig. PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP.



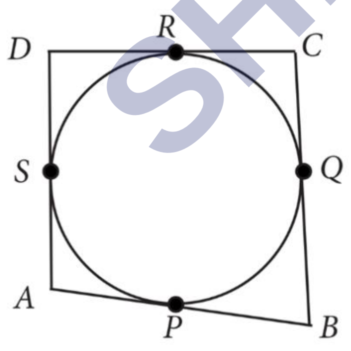
5. If PQ is a tangent drawn from an external point P to a circle with centre O and QOR is a diameter where length of QOR is 8 cm such that ∠POR = 120°, then find OP and PQ.

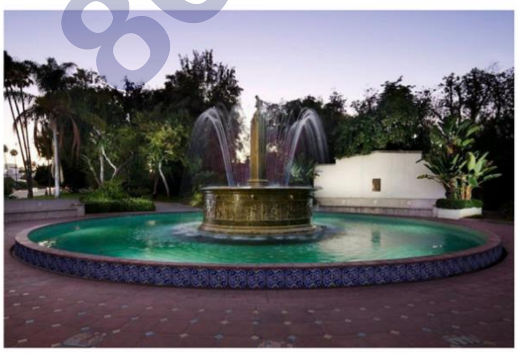
6. In Fig. O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Case Study Questions:

1. In a park, four poles are standing at positions A, B, C and D around the fountain such that the cloth joining the poles AB, BC, CD and DA touches the fountain at P, Q, R and S respectively as shown in the figure.





Based on the above information, answer the following questions.

i. If O is the centre of the circular fountain, then ∠OSA

a. 60º
b. 90º
c. 45º

d. None of these

ii. Which of the following is correct?

a. AS = AP
b. P = BQ
c. CQ = CR
d. All of these

iii. If DR = 7cm and AD= 11cm, then AP =

a. 4cm
b. 18cm
c. 7cm
d. 11cm

iv. If O is the centre of the fountain, with ∠QCS=60°, then ∠QOS

a. 60º
b. 120º
c. 90º
d. 30º

v. Which of the following is correct?

a. AB + BC = CD + DA
b. AB + AD = BC + CD
c. AB + CD = AD + BC
d. All of these

2. Smita always finds it confusing with the concepts of tangent and secant of a circle. But this time she has determined herself to get concepts easier. So, she started listing down the differences between tangent and secant of a circle, along with their relation. Here, some points in question form are listed by Smita in her notes. Try answering them to clear your concepts also.

i. A line that intersects a circle exactly at two points is called:

a. Secant
b. Tangent
c. Chord
d. Both (a) and (b)

ii. Number of tangents that can be drawn on a circle is:

a. 1
b. 0
c. 2
d. Infinite

iii. Number of tangents that can be drawn to a circle from a point not on it, is:

a. 1
b. 2
c. 0
d. Infinite

iv. Number of secants that can be drawn to a circle from a point on it is:

a. Infinite
b. 1
c. 2
d. 0

v. A line that touches a circle at only one point is called:

a. Secant
b. Chord
c. Tangent
d. Diameter

Assertion Reason Questions-

1. ***Directions:*** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Assertion (A): In a circle of radius 6 cm, the angle of a sector is 60°. Then the area of the sector is 132/7 cm^2.

Reason (R): Area of the circle with radius r is πr^2

2. ***Directions:*** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Assertion (A): If the circumference of a circle is 176 cm, then its radius is 28 cm.

Reason (R): Circumference $2\pi \times$ radius.

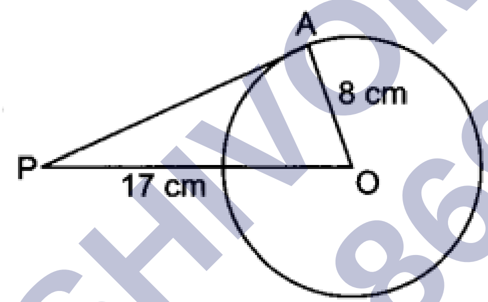
Answer Key-

Multiple Choice questions-

1. (d) 90°
2. (b) 70°
3. (a) equal
4. (c) 10 cm
5. (c) normal
6. (a) √119 cm
7. (a) 60 cm^2
8. (c) 16 cm
9. (b) parallel
10. (a) AD = 4cm, BE = 6cm, CF = 8cm

Very Short Answer :

1.



OA ⊥ PA (∵ radius is ⊥ to tangent at point of contact)

∴ In ΔOAP, we have

$PO^2 = PA^2 + AO^2$

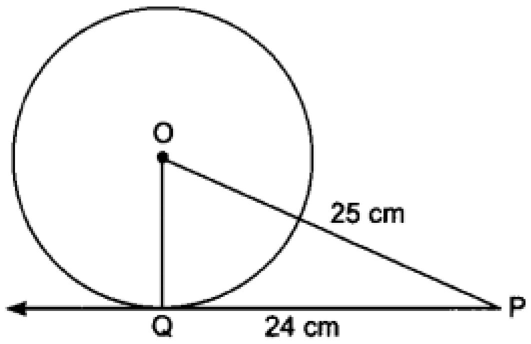
$\Rightarrow (17)^2 = (PA)^2 + (8)^2$

$(PA)^2 = 289 - 64 = 225$

$\Rightarrow PA = \sqrt{225} = 15$

Hence, the length of the tangent from point P is 15 cm.

2.



∵ OQ ⊥ PQ

∴ PQ2 + QO2 = OP2

⇒ 252 = OQ2 + 242

or OQ = √625 – √576

= √49 = 7 cm

3. Here ∠BDC = ∠BAC = 50° (angles in same segment are equal)

In ABCD, we have

∠BCD = 180° – (∠BDC + ∠DBC)

= 180° – (50° + 60°)= 70°

4. ∵ ∠AOB = ∠COD (vertically opposite angles)

∴ ∠COD = 115°

5. AN = AM = 3 cm [Tangents drawn from an external point]

BN = BL = 4 cm [Tangents drawn from an external point]

CL = CM = AC – AM = 9 – 3 = 6 cm

⇒ BC = BL + CL = 4 + 6 = 10 cm.

6. ∠OPQ = 90° – 50° = 40°

OP = OQ [Radii of a circle]

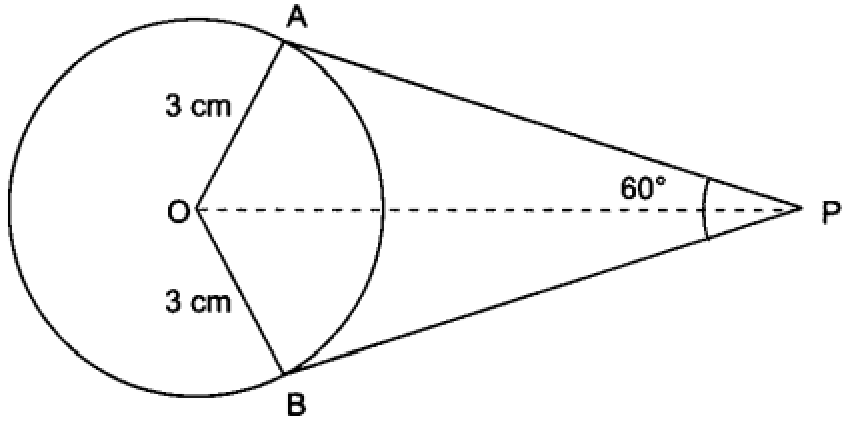
∠OPQ = ∠OQP = 40°

(Equal opposite sides have equal opposite angles)

∠POQ = 180° – ∠OPQ – ∠OQP

= 180° – 40° – 40° = 100°

7.



ΔAOP ≅ ΔBOP (By SSS congruence criterion)

$\angle APO = \angle BPO = \frac{60°}{2} = 30°$

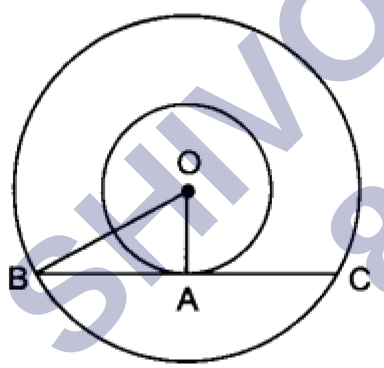
In ΔAOP, OA ⊥ AP

$\therefore \tan 30° = \frac{OA}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AP}$

⇒ AP = 3√3 cm

8.



OA = 4 cm, OB = 5 cm

Also, OA ⊥ BC

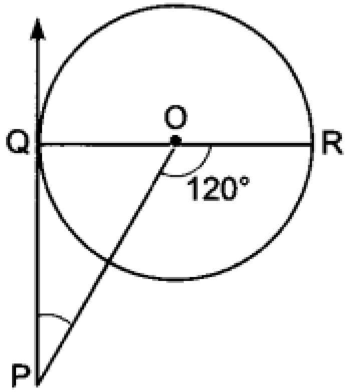
∴ OB2 = OA2 + AB2

⇒ 52 = 42 + AB2

⇒ AB = √25 – √16 = 3 cm

⇒ BC = 2 AB = 2 × 3 = 6 cm

9.



∠OQP = 90°

∠QOP = 180° – 120° = 60°

∠OPQ = 180° – ∠OQP – ∠QOP

= 180° – 90° – 60°

= 30°

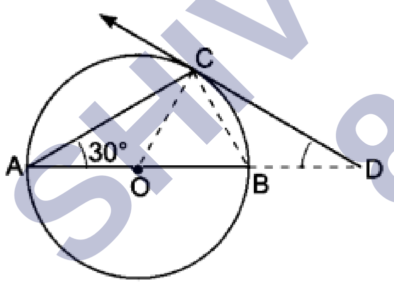
10. ∵ PA = PB ⇒ ∠BAP = ∠ABP = 50°

∴ ∠APB = 180° – 50° – 50° = 80°

∴ ∠AOB = 180° – 80° = 100°

Short Answer :

1.



True, Join OC,

∠ACB = 90° (Angle in semi-circle)

∴ ∠OBC = 180o – (90° + 30°) = 60°

Since, OB = OC = radii of same circle [Fig. 8.16]

∴ ∠OBC = ∠OCB = 60°

Also, ∠OCD = 90°

$\Rightarrow \angle BCD = 90° - 60° = 30°$

Now, $\angle OBC = \angle BCD + \angle BDC$ (Exterior angle property)

$\Rightarrow 60° = 30° + \angle BDC$

$\Rightarrow \angle BDC = 30°$

$\because \angle BCD = \angle BDC = 30°$

$\therefore BC = BD$

2. True, let PQ be the tangent from the external point P.

Then ΔPQO is always a right angled triangle with OP as the hypotenuse. So, PQ is always less than OP.

3. True, let PQ and PR be the tangents

Since $\angle P = 90°$, so $\angle QOR = 90°$

Also, OR = OQ = a

$\therefore$ PQOR is a square

$$\Rightarrow \quad OP = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$$

4. $\because$ PA and PB are tangent from same external point

$\therefore$ PA = PB = 15 cm

Now, Perimeter of ΔPCD = PC + CD + DP = PC + CQ + QD + DP

= PC + CA + DB + DP

= PA + PB = 15 + 15 = 30 cm

5. PA = PC + CA = PC + CQ [$\because$ CA = CQ (tangents drawn An from external point are equal)]

$\Rightarrow$ 12 = PC + 3 = PC = 9 cm

$\because$ PA = PB = PA – AC = PB – BD

$\Rightarrow$ PC = PD

$\therefore$ PD = 9 cm

Hence, PC + PD = 18 cm

6. Let the tangents to a circle with centre O be ABC and XYZ.

Construction : Join OB and OY.

Draw OP||AC

Since AB||PO

$\angle ABO + \angle POB = 180°$ (Adjacent interior angles)

$\angle ABO = 90°$ (A tangent to a circle is perpendicular to the radius through the point of contact)

$90° + \angle POB = 180° = \angle POB = 90°$

Similarly $\angle POY = 90°$

$\angle POB + \angle POY = 90° + 90° = 180°$

Hence, BOY is a straight line passing through the centre of the circle.

7. Given, $\angle QPR = 120°$

Radius is perpendicular to the tangent at the point of contact.

$\angle OQP = 90°$

$\Rightarrow \angle QPO = 60°$

(Tangents drawn to a circle from an external point are equally inclined to the segment, joining the centre to that point)

$$\text{In } \Delta QPO, \quad \cos 60° = \frac{PQ}{PO} \quad \Rightarrow \frac{1}{2} = \frac{PQ}{PO}$$

2PQ = PO

8. AE = CE and BE = ED [Tangents drawn from an external point are equal]

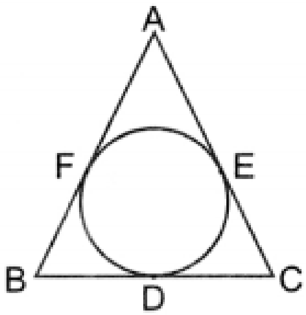
On addition, we get

AE + BE = CE + ED

$\angle QPO = 60°$

$\Rightarrow$ AB = CD

9.



Given, AB = AC

We have, BF + AF = AE + CE ....(i)

AB, BC and CA are tangents to the circle at F, D and E respectively.

∴ BF = BD, AE = AF and CE = CD ....(ii)

From (i) and (ii)

BD + AE = AE + CD (∵ AF = AE)

⇒ BD = CD

10. In the given figure,

AP = AR

BR = BQ

XP = XQ [Tangent to a circle from an external point are equal]

XA + AP = XB + BQ

XA + AR = XB + BR [AP = AR, BQ = BR]

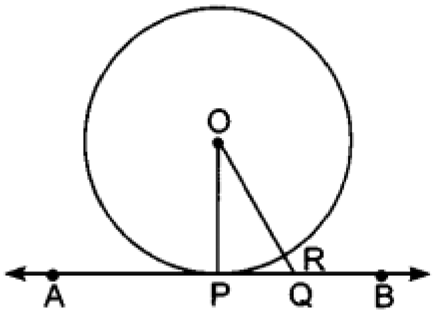
Long Answer :

1. Given: A circle C(O,r) and a tangent AB at a point P.

To Prove: OP ⊥ AB.

Construction: Take any point l, other than P, on the tangent AB. Join OQ. Suppose OQ meets the circle at R.

Proof: We know that among all line segments joining the point to a point on AB, the shortest one is perpendicular to AB. So, to prove that OP ⊥ AB it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.



Clearly, OP = OR [Radii of the same circle]

Now, OQ = OR + RQ

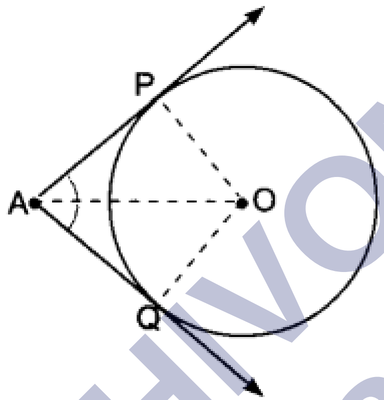
⇒ OQ > OR

⇒ OQ > OP [∵OP = OR]

Thus, OP is shorter than any other segment joining O to any point on AB.

Hence, OP ⊥ AB.

2.



Given: AP and AQ are two tangents from a point A to a circle C (O, r).

To Prove: AP = AQ

Construction: Join OP, OQ and OA.

Proof: In order to prove that AP = AQ, we shall first prove that ΔOPA ≅ ΔOQA.

Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

∴ OP ⊥ AP and OQ ⊥ AQ

⇒ ∠OPA = ∠OQA = 90°

Now, in right triangles OPA and OQA, we have

OP = OQ [Radii of a circle]

∠OPA = ∠OQA [Each 90°]

and OA = OA [Common]

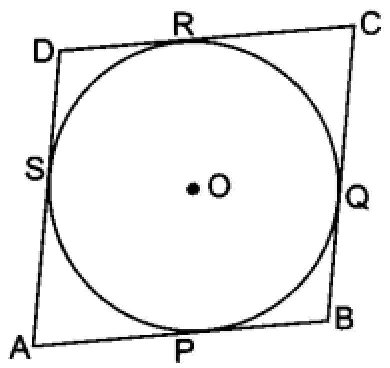
So, by RHS-criterion of congruence, we get

ΔOPA ≅ OQA

⇒ AP = AQ [CPCT]

Hence, lengths of two tangents from an external point are equal.

3.



Let ABCD be a parallelogram such that its sides touch a circle with centre O.

We know that the tangents to a circle from an exterior point are equal in length.

Therefore, we have

AP = AS [Tangents from A]

BP = BQ [Tangents from B] …. (ii)

CR = CQ [Tangents from C] …. (iii)

And DR = DS [Tangents from D] …. (iv)

Adding (i), (ii), (iii) and (iv), we have

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

AB + AB = BC + BC [∵ ABCD is a parallelogram ∴ AB = CD, BC = DA]

2AB = 2BC ⇒ AB = BC

Thus, AB = BC = CD = AD

Hence, ABCD is a rhombus.

4.

To find: TP

$$PR = RQ = \frac{16}{2} = 8 \text{ cm} \quad \text{[Perpendicular from the centre bisects the chord]}$$

In ΔOPR

$$OR = \sqrt{OP^2 - PR^2}$$

$$= \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$$

$$= \sqrt{36} = 6 \text{ cm}$$

Let $\angle POR$ be θ

In ΔPOR, $\tan\theta = \frac{PR}{RO} = \frac{8}{6}$

$$\tan\theta = \frac{4}{3}$$

We know, $OP \perp TP$ (Point of contact of a tangent is perpendicular to the line from the centre)

In ΔOTP, $\tan\theta = \frac{OP}{TP} \Rightarrow \frac{4}{3} = \frac{10}{TP}$

$$TP = \frac{10 \times 3}{4} = \frac{15}{2} = 7.5 \text{ cm.}$$

5. Let O be the centre and QOR = 8 cm is diameter of a circle. PQ is tangent such that $\angle POR = 120°$

Now, $OQ = OR = \frac{8}{2} = 4$ cm

$\angle POQ = 180 - 120° = 60°$ (Linear pair)

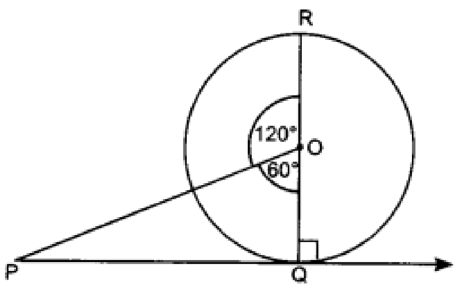
Also $OQ \perp PQ$

Now, in right ΔPOQ,

$$\cos 60° = \frac{OQ}{PO}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{OQ}{PO} \Rightarrow \frac{1}{2} = \frac{4}{PO}$$

$$\Rightarrow \quad PO = 8 \text{ cm.}$$



Again, $\tan 60° = \frac{PQ}{OQ} \Rightarrow \sqrt{3} = \frac{PQ}{4} \Rightarrow PQ = 4\sqrt{3}$ cm.

6. In right ΔPOT

$$PT = \sqrt{OT^2 - OP^2}$$

$$PT = \sqrt{169 - 25} = 12 \text{ cm and}$$

TE = 8 cm

Let PA = AE = x

(Tangents from an external point to a circle are equal)

In right ΔAET

$TA^2 = TE^2 + EA^2$

$\Rightarrow (12 - x)^2 = 64 + x^2$

$\Rightarrow 144 + x^2 - 24x = 64 + x^2$

$\Rightarrow x = \frac{80}{24}$

$\Rightarrow x = 3.3$ cm

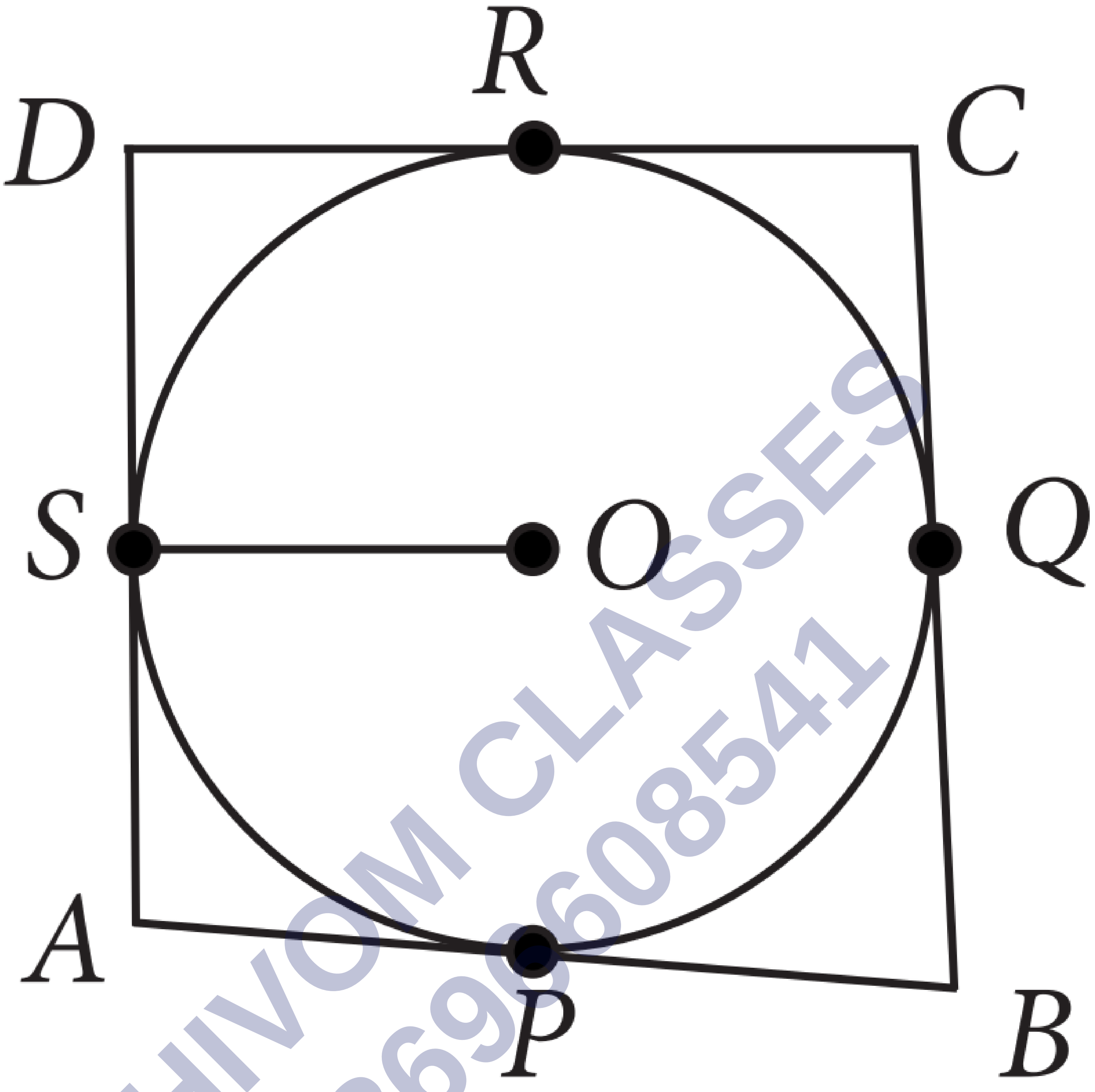
Thus, AB = 6.6 cm

Case Study Answer:

1. Answer :

i. (b) 90º

Solution:



Here, OS the is radius of circle.

Since radius at the point of contact is perpendicular to tangent So, $\angle OSA=90^{o}$

ii. (d) All of these

Solution:

Since, length of tangents drawn from an external point to a circle are equal.

$\therefore$ AS = AP, BP = BQ,

CQ = CR and DR = DS

iii. (a) 4cm

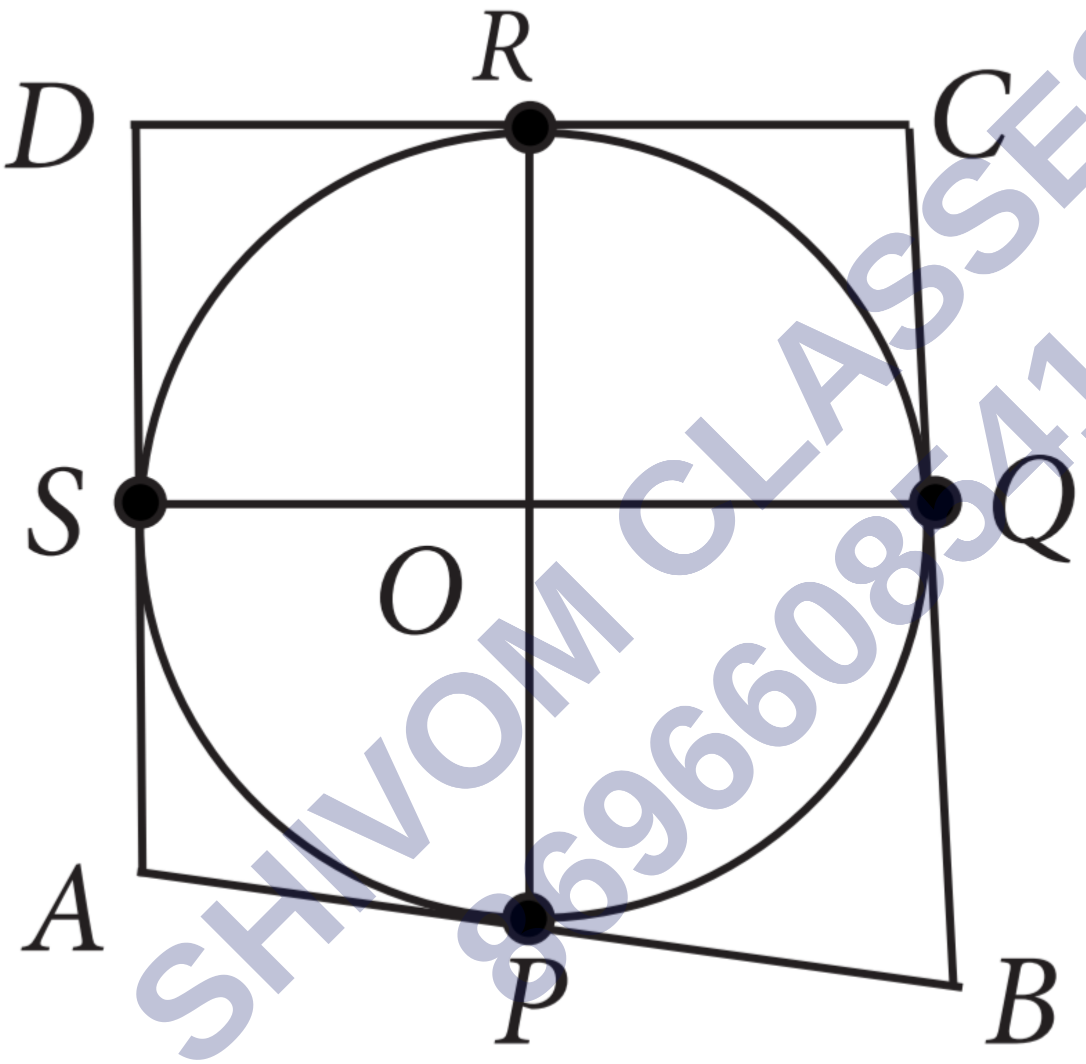
Solution:

AP = AS = AD - DS = AD - DR = 11 - 7 = 4cm.

iv. (b) 120º

Solution:

In quadrilateral OQCR,



$\angle QCR = 60^o$, (Given)

And $\angle OQC = \angle ORC = 90^o$

[Since, radius at the point of contact is perpendicular to tangent.]

$\therefore \angle QCR = 360^o - 90^o - 90^o - 60^o = 120$

v. (c) AB + CD = AD + BC

Solution:

From (I), we have AS = AP, DS = DR, BQ = BP and CQ = CR

Adding all above equations, we get

AS + DS + BQ + CQ = AP + DR + BP + CR

$\Rightarrow$ AD + BC = AB + CD

2. Answer :

i. (a) Secant
ii. (d) Infinite
iii. (b) 2
iv. (a) Infinite
v. (c) Tangent

Assertion Reason Answer-

1. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
2. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).