PHYSICS

Chapter 4: Motion in a Plane

Motion in a Plane

Scalars Vs. Vectors

| Criteria	Scalar	Vector
Definition	A scalar is a quantity with magnitude only.	A vector is a quantity with magnitude and direction.
Direction	No	Yes
Specified by	A number (magnitude) and a unit	A number (magnitude), direction and a unit
Represented by	quantity's symbol	quantity's symbol in bold or an arrow sign above
Example	mass, temperature	velocity, acceleration

Position and Displacement Vectors

Position Vector: Position vector of an object at time t is the position of the object relative to the origin. It is represented by a straight line between the origin and the position at time t.
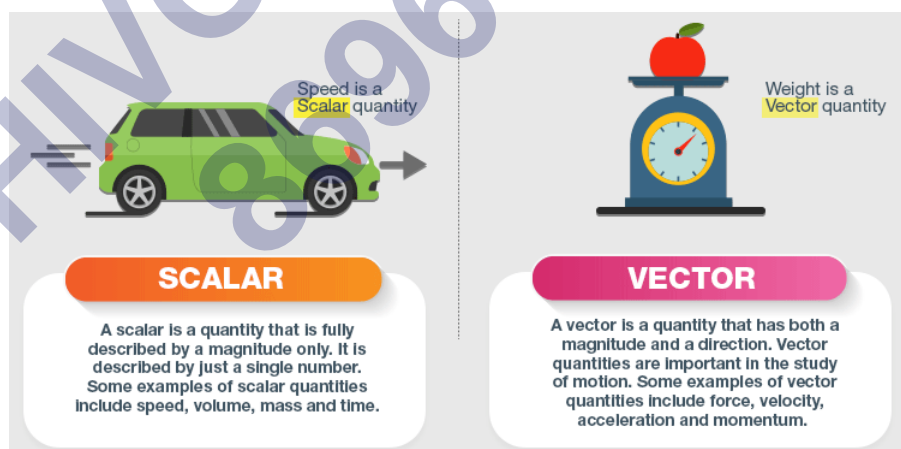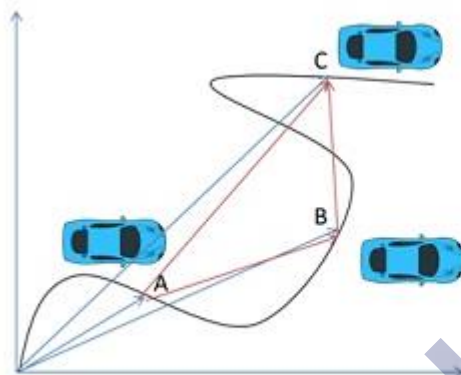

Displacement Vector: Displacement vector of an object between two points is the straight line between the two points irrespective of the path followed. The path length is always equal or greater than the displacement.
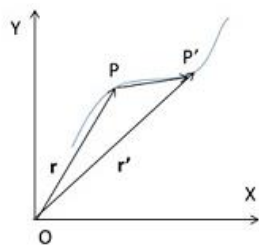
OP and OP' are position vectors represented by r and r'.
PP' is the displacement vector.
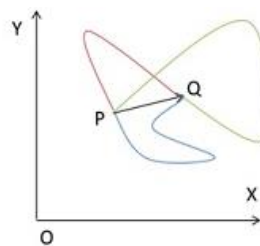
PQ is the displacement vector for any path followed (represented by green, blue and red paths).
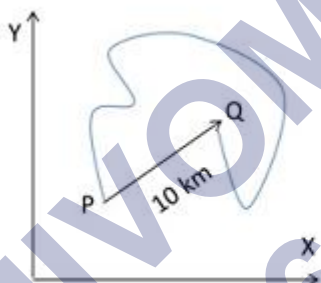
Example: Path taken by a car.

Path is represented by black line.

Blue lines represents Position vectors at point A, B and C.

Red lines represent Displacement vectors

If P to Q is a straight road of 10 km between station and hotel but the route taken is as shown in the figure of 23 km in 28 mins, then the average speed and average velocity of the car will be unequal.

Avg. Speed = Total Distance travelled/Total time
Avg. Speed = 23/(28/60) = 49.29 km/h

Avg. Velocity = Displacement/Total Time
Avg. Velocity = 10/(28/60) = 21.43 km/h

Free and Localized Vectors

A **free vector**(or non-localized vector) is a vector of which only the magnitude and direction are specified, not the position or line of action. Displacing it parallel to itself leaves it unchanged.

A **localized vector** is a vector where line of action and position are as important as magnitude and direction. These vectors change with change in position and direction.

Velocity vector of a car moving in a straight line is a free vector

Force vector is a localized vector as it depends upon position as well

Equality of Vectors

Two vectors are said to be equal only when they have same direction and magnitude. For example, two cars travelling with same speed in same direction. If they are travelling in opposite directions with same speed, then the vectors are unequal.

Equal Vectors   Unequal Vectors
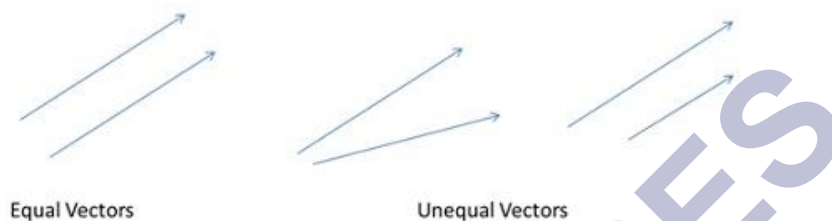
Multiplication of Vectors with real numbers

Multiplication Factor	Original vector	Magnitude of vector after multiplication	Direction of vector after multiplication
λ (>0)	A	λA	Same as that of A
-λ (<0)	A	λA	Opposite to that of A
λ (=0)	A	0 (null vector)	None. The initial and final positions coincide.

Multiplication of vector with +2 and -2

Addition and Subtraction of Vectors – Triangle Method

The method of adding vectors graphically is by arranging them so that head of first is touching the tail of second vector and making a triangle by joining the open sides. This method is called head-to-tail method or triangle method of vector addition
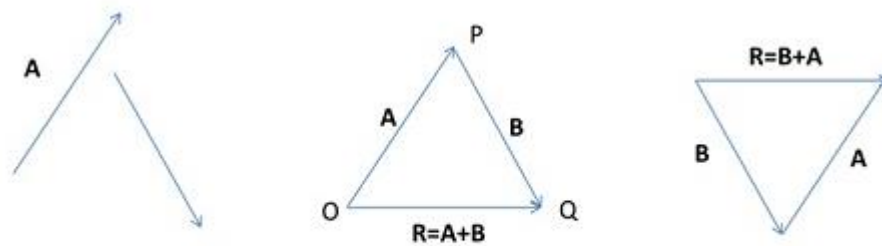
Head-to-Tail or Triangle Method of vector addition

- Vector addition is:
  - Commutative: A + B = B + A
  - Associative: (A + B) + C = A + (B+ C)
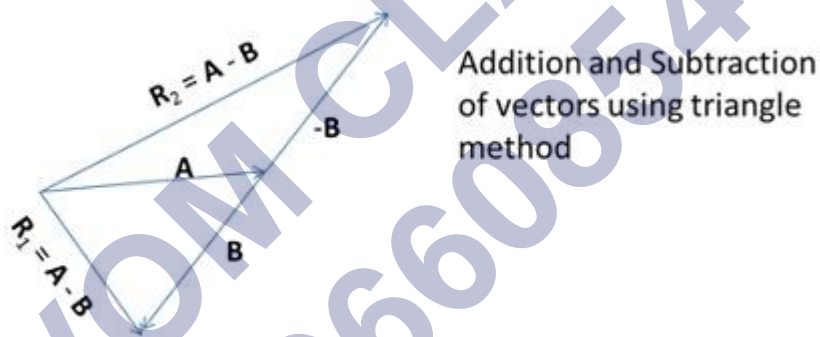- Adding two vectors with equal magnitudes and opposite directions results in null vector.
  - Null Vector: A + (-A) = 0
- Subtraction is adding a negative vector (opposite direction) to a positive vector.
  - A – B = A + (-B)

Addition and Subtraction of vectors using triangle method
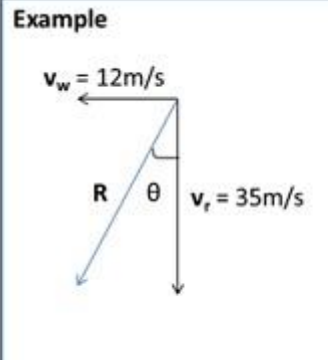
Addition of Vectors – Parallelogram Method

The method of adding vectors by parallelogram method is by:

- Touching the tail of the two vectors
- Complete a parallelogram by drawing lines from the heads of the two vectors.
- Vector resulting from the origin to the point of intersection of above lines gives the addition.
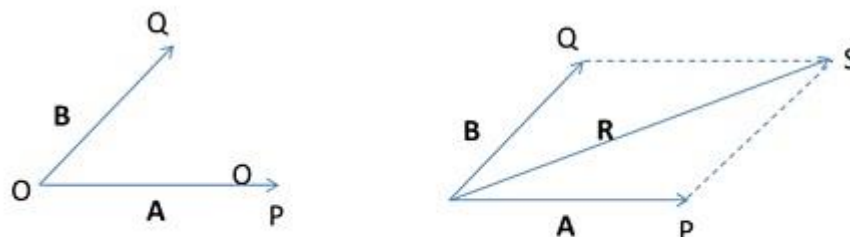
Parallelogram Method of vector addition

Example

If rain is falling vertically at a speed of 35 m/s and wind is blowing at 12 m/s (east to west), then the resultant vector **R** would be the actual path of rain.

$R = \sqrt{v_r^2 + v_w^2} = \sqrt{35^2 + 12^2} = 37m/s$

$\tan\theta = v_w/v_r = 12/35 = 0.343$

$\theta = 19°$

A person standing on ground must hold umbrella at an angle of 19° with vertical towards east to avoid rain.

Resolution of Vectors

A vector can be expressed in terms of other vectors in the same plane. If there are 3 vectors A, a and b, then A can be expressed as sum of a and b after multiplying them with some real numbers.

3 vectors in a plane   Joining the three to make a triangle

A can be resolved into two component vectors λa and μb. Hence, A = λa + μb. Here λ and μ are real numbers.

Unit Vectors

A unit vector is a vector of unit magnitude and a particular direction.

- They specify only direction. They do not have any dimension and unit.
- In a rectangular coordinate system, the x, y and z axes are represented by unit vectors, $\hat{i}, \hat{j}$ and $\hat{k}$
- These unit vectors are perpendicular to each other.
- $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$
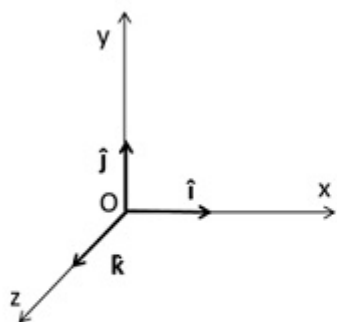
Unit vectors in the coordinate system along the three axes.

Vector **A** as combination of **A₁** and **A₂** which are expressed in terms of unit vectors.

In a 2-dimensional plane, a vector thus can be expressed as:

1. A = Ax î +Ay ĵ where, Ax = A cosθ and Ay = A sinθ
2. A = $\sqrt{A_x^2 + A_y^2}$

Analytical Method of Vector Addition

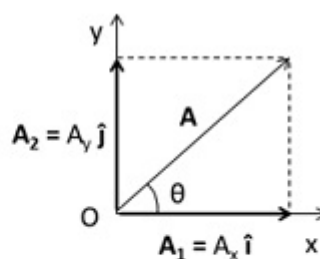| Vectors	Sum of the vectors	Subtraction of the vectors
A = Ax î +Ay ĵ and B = Bx î +By ĵ	R = A + B R = Rx î +Ry ĵwhere Rx = Ax + Bx and Ry = Ay + By	R = A - B R = Rx î +Ry ĵ where Rx = Ax – Bx and Ry = Ay - By
A = Ax î +Ay ĵ+Azk̂ B = Bx î +By ĵ+Bzk̂	R = A + B R = Rx î +Ry ĵ+Rzk̂ where Rx = Ax + Bx and Ry = Ay + By andRz = Az + Bz	R = A - B R = Rx î +Ry ĵ+Rzk̂ where Rx = Ax – Bx and Ry = Ay - By and Rz = Az - Bz
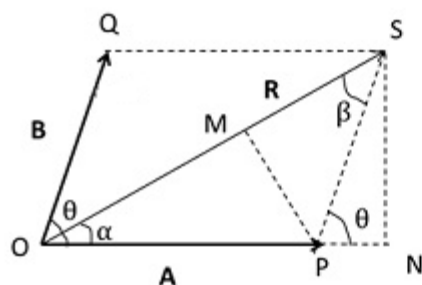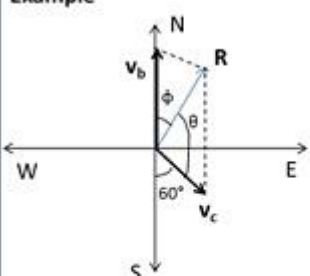
Law of Cosines

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Law of Sines

$$\frac{R}{\sin\theta} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha}$$

Example

If a motorboat is travelling at a speed of 25 km/h north and water current is 10 km/h at an angle of 60° (east to south), then the resultant velocity and direction of boat can be obtained using law of sines and cosines.

Magnitude of R using law of cosines,
$R = \sqrt{v_b^2 + v_c^2 + 2v_b v_c \cos 120°} = 22$ km/h

Direction of R using law of sines,
$R/\sin\theta = v_c/\sin\phi$, $\sin\phi = v_c \sin\theta / R$
$\phi = 23.4°$

Quantities related to motion of an object in a plane
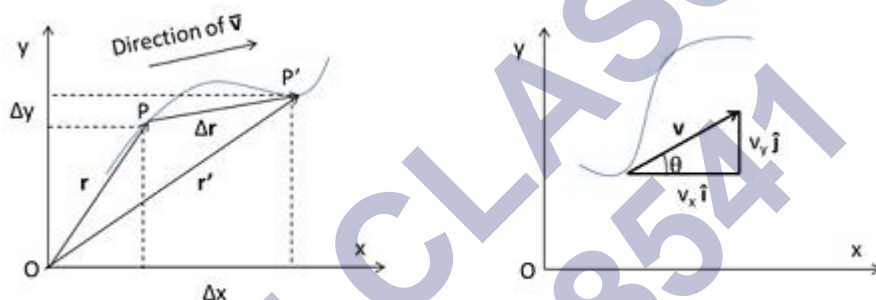
Particle moving in a plane from P to P' in time t to t' and
Velocity calculation of the particle in terms of unit vectors

Quantity	Value	Value in component form
Displacement Δr (Change in position)	r' - r	$\hat{i}\Delta x + \hat{j}\Delta y$
Average Velocity v̄ (ratio of displacement and corresponding time interval)	Δr/Δt	$v_x\hat{i} + v_y\hat{j}$ $v_x = \Delta x/\Delta t$, $v_y = \Delta y/\Delta t$
Instantaneous velocity v (limiting value of average velocity as the time interval approached zero)	dr/dt	$v_x\hat{i} + v_y\hat{j}$ $v_x = dx/dt$, $v_y = dy/dt$
Magnitude of v		

Direction of v, θ (direction of velocity at any point on the path is tangential to the path at that point and is in the direction of motion)	$\tan^{-1}(v_y/v_x)$	
Average Acceleration $\overline{a}$ (change in velocity divided by the time interval)	$\Delta v/\Delta t$	$a_x \hat{i} + a_y \hat{j}$ $a_x = \Delta v_x/\Delta t$, $a_y = \Delta v_y/\Delta t$
Instantaneous acceleration a (limiting value of the average acceleration as the time interval approaches zero)	dv/dt	$a_x \hat{i} + a_y \hat{j}$ $a_x = dv_x/dt$, $a_y = dv_y/dt$ $a_x = d^2x/dt^2$, $a_y = d^2y/dt^2$

Motion in a plane with constant acceleration

Motion in a plane (two dimensions) can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions. X and Y directions are hence independent of each other.

If v0 being the velocity at time 0, the displacement can be written as:

$x = x_0 + v_{0x}t + \frac{1}{2} a_x t^2$ and $y = y_0 + v_{0y}t + \frac{1}{2} a_y t^2$

Motion of an object in a plane with constant acceleration		
Velocity	Velocity in terms of components	Displacement
$v = v_0 + at$	$v_x = v_{0x} + a_x t$ $v_y = v_{0y} + a_y t$	$r = r_0 + v_0 t + \frac{1}{2} at^2$

Relative velocity in two dimensions

The concept of relative velocity in a plane is similar to the concept of relative velocity in a straight line.
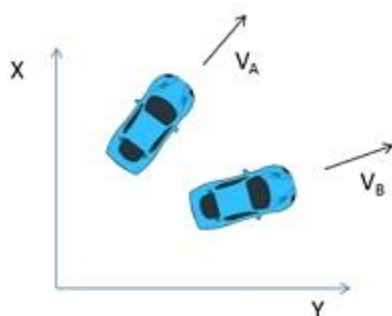
Velocity of A relative to B , $v_{AB} = v_A - v_B$
Velocity of B relative to A, $v_{BA} = v_B - v_A$

$$v_{AB} = - v_{BA}$$

$$|v_{AB}| = |v_{BA}|$$

Relative Velocity in a plane

Example

If rain is falling vertically at a speed of 35 m/s and a person is riding bicycle at 12 m/s (east to west), then the relative velocity of rain will be v_{rb}:

$v_{rb} = v_r - v_b = (35-12)$ m/s = 23 m/s
$\tan\theta = v_w/v_r = 12/35 = 0.343$
$\theta = 19°$

The person must hold umbrella at an angle of 19° with vertical towards west to avoid rain.

Projectile Motion

An object that becomes airborne after it is thrown or projected is called projectile. Example, football, javelin throw, etc.
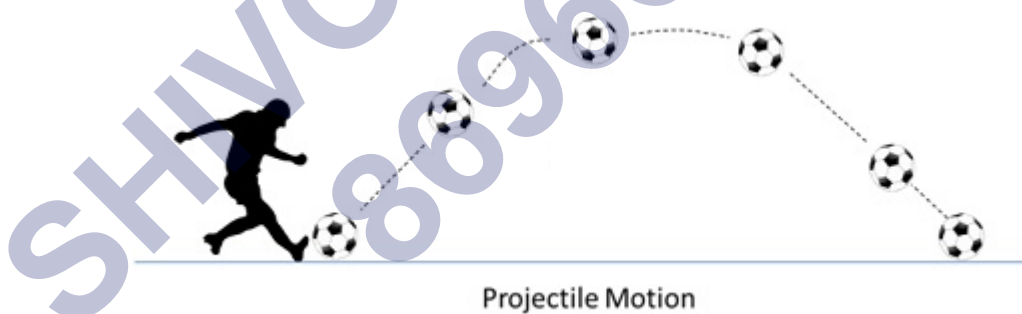
Projectile Motion

- Projectile motion comprises of two parts – horizontal motion of no acceleration and vertical motion of constant acceleration due to gravity.
- Projectile motion is in the form of a parabola, $y = ax + bx2$.
- Projectile motion is usually calculated by neglecting air resistance to simplify calculations.
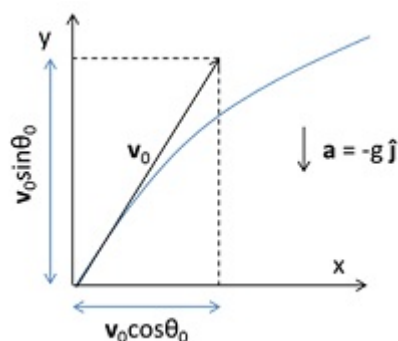
Motion of an object projected with velocity v_0 at an angle θ_0

Quantity	Value
Components of velocity at time t	$v_x = v_0 \cos\theta_0$ $v_y = v_0 \sin\theta_0 - gt$
Position at time t	$x = (v_0 \cos\theta_0)t$ $y = (v_0 \sin\theta_0)t - \frac{1}{2} gt^2$
Equation of path of projectile motion	$y = (\tan \theta_0)x - gx^2/2(v_0 \cos\theta_0)^2$
Time of maximum height	$t_m = v_0 \sin\theta_0 /g$
Time of flight	$2 t_m = 2 (v_0 \sin\theta_0 /g)$
Maximum height of projectile	$h_m = (v_0 \sin\theta_0)^2/2g$
Horizontal range of projectile	$R = v_0^2 \sin 2\theta_0/g$
Maximum horizontal range ($\theta_0=45°$)	$R_m = v_0^2/g$

Example

$h = 25$ m
$V = 40$ m/s

Motion of ball
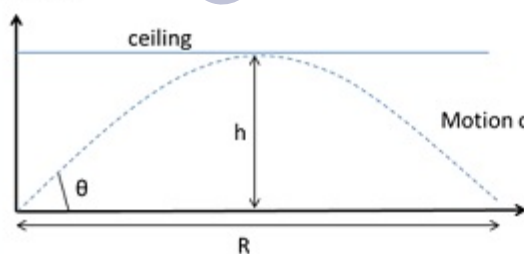
If a ball is projected at a speed of 40 m/s and the maximum height it can achieve is 25 m, then the angle 'θ' and maximum distance 'R' should be:

$h = (v_0 \sin\theta_0)^2/2g = (40 \sin\theta)^2/(2 \times 9.8)$
$\sin^2\theta = 0.30625$
$\sin\theta = 0.5534$
$\theta = 33.60°$

$R = v_0^2 \sin 2\theta_0/g$
$R = (40)^2 \times \sin (2 \times 33.6)/9.8$
$R = 1600 \times 0.922 / 9.8$
$R = 150.53$ m

Uniform circular motion

When an object follows a circular path at a constant speed, the motion is called uniform circular motion.

- Velocity at any point is along the tangent at that point in the direction of motion.
- Average velocity between two points is always perpendicular to Average displacement. Also, average acceleration is perpendicular to average displacement.
- For an infinitely small time interval, Δ tà 0, the average acceleration becomes instantaneous acceleration which means that in uniform circular motion the acceleration of an object is always directed towards the center. This is called centripetal acceleration.
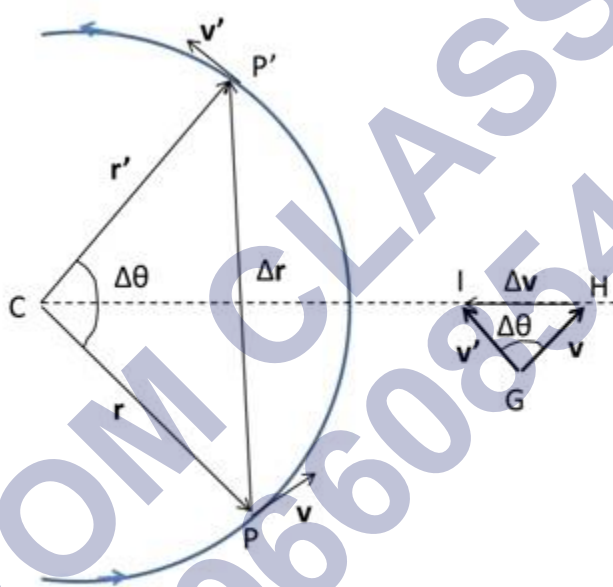
Velocity and Acceleration of an object in uniform circular motion

Quantity	Values
Centripetal Acceleration	$a_c = v^2/R$, R – radius of the circle $a_c = \omega^2 R$, ω – angular speed $a_c = 4\pi^2 v^2 R$, ν – frequency
Angular Distance	$\Delta\theta = \omega \, \Delta t$
Speed	$v = R\omega$

Example

A stone tied to one end of a string is whirled at constant speed in a circle. If it makes 14 revolutions in 25 sec, the magnitude and direction of acceleration can be calculated as:
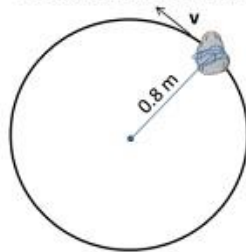
Frequency, v = revolutions/time taken
$v = 14/25$ Hz

Angular frequency, $\omega = 2\pi v$
$\omega = 2 \times 22/7 \times 14/25 = 88/25$ rad/sec

Centripetal acceleration, $a_c = \omega^2 r$
$a_c = (88/25)^2 \times 0.8$
$a_c = 9.91$ m/s^2

The direction of centripetal acceleration is always directed towards the center at all points.

Class : 11th Physics
Chapter- 4 : Motion in a plane

Unit Vector

It has magnitude as one or unity

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

Negative Vector

Vectors having same magnitude but opposite direction

eg. $\vec{A}$ is a negative of $\vec{B}$

If, $\vec{A} = -\vec{B}$

Acceleration vector

$$\vec{a} = a_x\hat{i} + a_y\hat{j}; \quad a_x = dv_x/dt \ \& \ a_y = dv_y/dt$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

Position of displacement vector

Position vector , $\vec{r} = x\hat{i} + y\hat{j}$

Displacement vector , $\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$

Vectors Types

Equal Vectors

Vectors having same direction and magnitude

Additional vector

Law of Triangle   Law of Parallelogram

$$\vec{R} = \vec{P} + \vec{Q}$$

Co-initial vector

Vectors having common starting point.

Motion in a plane

Zero or Null vector

It has zero magnitude and orbitary direction

Velocity vector

$$\vec{v} = v_x\hat{i} + v_y\hat{j},$$

$$\text{magnitude } |\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

Multiplication of a vector by a real number

$$\lambda \times \vec{A} = \lambda\vec{A}$$

Substraction of two vectors

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

Relative velocity

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

$$\vec{V}_{AB} = -\vec{V}_{BA} \text{ and}$$

$$|\vec{V}_{AB}| = |\vec{V}_{BA}|$$

Motion in a plane

Motion of Body under 2 dimensional frame.

Total time of flight, $T_f = \dfrac{2u\sin\theta}{g}$

Maximum height, $H_{max} = \dfrac{u^2\sin^2\theta}{2g}$

Horizontal range, $R = \dfrac{u^2\sin2\theta}{g}$

Projectile motion

Motion of an object that is in flight after being thrown or projected.

Uniform circular motion

When an object follows a circular path at a constant speed , the motion of the object is called uniform circular motion.

Angular velocity , $\omega = \theta/t$ Angular acceleration, $d = \Delta\omega/\Delta t$ eg- merry go around.

Equation of path of projectile

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$

Components of velocity (in velocity)

$$u_x = u\cos\theta, u_y = u\sin\theta$$

Position after time t

$$x = (u\cos\theta)t;$$

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

Components of acceleration

at any instant

$$a_x = 0, a_y = g$$

Centripetal Acceleration

A body in a circular motion acted upon by an acceleration directed towards center of the circular motion.

$$a_c = v^2/r = rw^2 = 4\pi^2 rv^2$$

Important Questions

Multiple Choice questions-

1. A body of mass 500 gram is rotating in a vertical circle of radius 1 m. What is the difference in its kinetic energies at the top and the bottom of the circle?

(a) 4.9 J

(b) 19.8 J

(c) 2.8 J

(d) 9.8 J

2. A particle has a displacement of 2 units along the x -axis, 1 unit along the y – axis and 2 units along the z – axis. Then the resultant displacement of the particle is

(a) 3 units

(b) 5 units

(c) 4 units

(d) 1 units

3. A car is moving on a circular path and takes a turn. If R^1 and R^2 are the reactions on the inner and outer wheels respectively, then

(a) $R^1 = > R^2$

(b) $R^1 = R^2$

(c) $R^1 < R^2$

(d) $R^1 > R^2$

4. The angle between centripetal acceleration and tangential acceleration is?

(a) 180°

(b) 0°

(c) 90°

(d) 45°

5. Large angle produces?

(a) high trajectory

(b) curve trajectory

(c) flat trajectory

(d) straight trajectory

6. He dimensional formula for normal acceleration is

(a) LT^{-1}

(b) L^2T^2

(c) L^3T^{-2}

(d) LT^{-2}

7. A book is pushed with an initial horizontal velocity of 5.0 meters per second off the top of a desk. What is the initial vertical velocity of the book?

(a) 10. m/s

(b) 0 m/s

(c) 50 m/s

(d) 2.5 m/s

8. One radian is defined to be the angle subtended where the arc length S is exactly equal to the?

(a) radius of the circle.

(b) diameter of the circle.

(c) circumference of the circle.

(d) half of radius of the circle.

9. A body travels along the circumference of a circle of radius 2 m with a linear velocity of 6 m/s. Then its angular velocity is

(a) 6 rad /s

(b) 3 rad /s

(c) 2 rad / s

(d) 4 rad / s

10. One° (1°) is equal to?

(a) 0.1745 rad

(b) 0.01745 rad

(c) 0.001745 rad

(d) 7.1745 rad

Very Short:

1. Under what condition $|a + b| = |a| + |b|$ holds good?

2. Under what condition $|a - b| = |a| - |b|$ holds good?

3. The sum and difference of the two vectors are equal in magnitude

   i. e. $|a + b| = |a - b|$. What conclusion do you draw from this?

4. What is the angle between $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$?

5. What is the minimum number of coplanar vectors of different magnitudes which can give zero resultant?

6. When a – b = a + b condition holds good than what can you say about b?

7. What is the magnitude of the component of the $9\hat{i} - 9\hat{j} + 19\hat{k}$ vector along the x-axis?

8. Can displacement vector be added to force vector?

9. What is the effect on the dimensions of a vector if it is multiplied by a non-dimensional scalar?

10. (a) What is the angle between $\hat{i} + \hat{j}$ and $\hat{i}$ vectors?

    (b) What is the angle between $\hat{i} - \hat{j}$ and the x-axis?

    (c) What is the angle between $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?ss

Short Questions:

1. Name two quantities that are the largest when the maximum height attained by the projectile is largest.

2. A stone dropped from the window of a stationary railway carriage takes 2 seconds to reach the ground. At what time the stone will reach the ground when the carriage is moving with

    (a) the constant velocity of $80 kmh^{-1}$

    (b) constant acceleration of $2 ms^{-2}$?

3. Can a particle accelerate when its speed is constant? Explain.

4. (a) Is circular motion possible at a constant speed or at constant velocity? Explain.

    (b) Define frequency and time period.

5. When the component of a vector A along the direction of vector B is zero, what can you conclude about the two vectors?

6. Comment on the statement whether it is true or false "Displacement vector is fundamentally a position vector.'' Why?

7. Does the nature of a vector changes when it is multiplied by a scalar?

8. Can the walk of a man be an example of the resolution of vectors? Explain.

Long Questions:

1. Discuss the problem of a swimmer who wants to cross the river in the shortest time.

2. State and prove parallelogram law of vector addition. Discuss some special cases.

3. Derive the relation between linear velocity and angular velocity. Also, deduce its direction.

Assertion Reason Questions:

1. *Directions:* Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

   Assertion: In projectile motion, the angle between the instantaneous velocity and acceleration at the highest point is 180°.
   Reason: At the highest point, velocity of projectile will be in horizontal direction only.

2. *Directions:* Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

   Assertion: Two particles of different mass, projected with same velocity at same angles. The maximum height attained by both the particle will be same.
   **Reason:** The maximum height of projectile is independent of particle mass.

Case Study Questions:

1. Vectors are the physical quantities which have both magnitudes and directions and obey the triangle/parallelogram laws of addition and subtraction. It is specified by giving its magnitude by a number and its direction. e.g., Displacement, acceleration, velocity, momentum, force, etc. A vector is represented by a bold face type and also by an arrow placed over a letter, i.e.

F, a, b or $\vec{F}, \vec{a}, \vec{b}$.

The length of the line gives the magnitude, and the arrowhead gives the direction. The point P is called head or terminal point and point O is called tail or initial point of the vector **OP**.
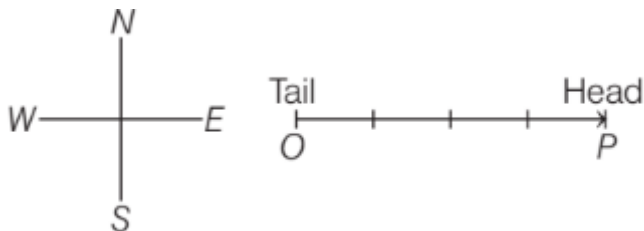
i. Amongst the following quantities, which is not a vector quantity?

   (a) Force

   (b) Acceleration

   (c) Temperature

   (d) Velocity

ii. Set of vectors A and B, P and Q are as shown below
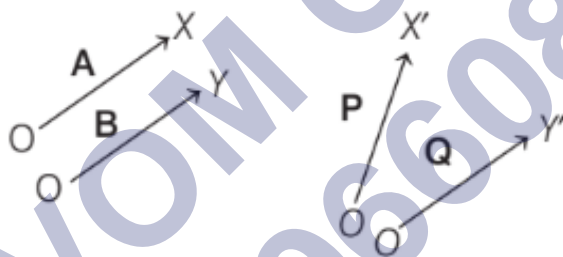

   Length of A and B is equal, similarly length of P and Q is equal. Then, the vectors which are equal, are

   (a) A and P

   (b) P and Q

   (c) A and B

   (d) B and Q

iii. $|\lambda A| = \lambda |A|$, if

   (a) $\lambda > 0$

   (b) $\lambda < 0$

   (c) $\lambda = 0$

   (d) $\lambda \neq 0$

iv. Among the following properties regarding null vector which is incorrect?

(a) A + 0 = A

(b) $\lambda 0 = \lambda$

(c) 0A = 0

(d) A − A = 0

v. The x and y components of a position vector P have numerical values 5 and 6, respectively. Direction and magnitude of vector P are

(a) $\tan^{-1}\left(\frac{6}{5}\right)$ and $\sqrt{61}$

(b) $\tan^{-1}\left(\frac{5}{6}\right)$ and $\sqrt{61}$

(c) 60° and 8

(d) 30° and 9

2. Projectile motion is a form of motion in which an object or particle is thrown with some initial velocity near the earth's surface, and it moves along a curved path under the action of gravity alone. The path followed by a projectile is called its trajectory, which is shown below. When a projectile is projected obliquely, then its trajectory is as shown in the figure below
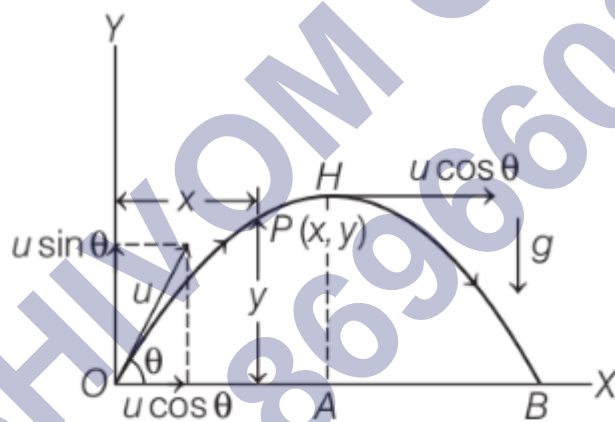
Here velocity u is resolved into two components, we get (a) u cos θ along OX and (b) u sin θ along OY

i. The example of such type of motion is

(a) Motion of car on a banked road

(b) Motion of boat in sea

(c) A javelin thrown by an athlete

(d) Motion of ball thrown vertically upward

ii. The acceleration of the object in horizontal direction is

(a) Constant

(b) Decreasing

(c) Increasing

(d) Zero

iii. The vertical component of velocity at point H is

(a) Maximum

(b) Zero

(c) Double to that at O

(d) Equal to horizontal component

iv. A cricket ball is thrown at a speed of 28 m/s in a direction 30° with the horizontal. The time   taken by the ball to return to the same level will be

(a) 2.0 s

(b) 3.0 s

(c) 4.0 s

(d) 2.9 s

v. In above case, the distance from the thrower to the point where the ball returns to the same level will be

(a) 39 m

(b) 69 m

(c) 68 m

(d) 72 m

✓ Answer Key:

Multiple Choice Answers-

1. Answer: (d) 9.8 J

2. Answer: (a) 3 units

3. Answer: (c) $R^1 < R^2$

4. Answer: (c) 90°

5. Answer: (a) high trajectory

6. Answer: (d) LT^{-2}

7. Answer: (b) 0 m/s

8. Answer: (a) radius of the circle.

9. Answer: (b) 3 rad /s

10.Answer: (b) 0.01745 rad

Very Short Answers:

1. Answer: When a and b act in the same direction i. e. when 0 = 0 between • them, then |a + b|=|a| + |b|.

2. Answer: The condition |a − b|=|a| − |b| holds goods when a and b act in the opposite direction.

3. Answer: The two vectors are equal in magnitude and are perpendicular to each other.

4. Answer: The given vectors act along two parallel lines in opposite directions i.e. they are anti-parallel, so the angle between them is 180°.

5. Answer: 3, If three vectors can be represented completely by the three sides of a triangle taken in the same order, then their resultant is zero.

6. Answer: For a − b = a + b condition to hold good, b must be a null vector.

7. Answer: 9.

8. Answer: No.

9. Answer: There is no effect on the dimensions of a vector if it is multiplied by a non-dimensional scalar.

10.Answer:

   (a) 45°

   (b) 45°

   (c) 90°

Short Questions Answers:

1. Answer: Time of flight and the vertical component of velocity are the two quantities that are the largest when the maximum height attained by the projectile is the largest.

2. Answer: The time taken by the freely falling stone to reach the ground is given by

    $$t = \sqrt{\frac{2h}{g}}$$

    In both cases, the stone will fall through the same height as it is falling when the railway carriage is stationary. Hence the stone will reach the ground after 2 seconds.

3. Answer: Yes. A particle can be accelerated if its velocity changes. A particle having uniform circular motion has constant speed but its direction of motion

changes continuously. Due to this, there is a change in velocity and hence the particle is moving with variable velocity. Thus, particle is accelerating.

4. Answer:

    (a) Circular motion is possible at a constant speed because, in a circular motion, the magnitude of the velocity i.e. speed remains constant while the direction of motion changes continuously.

    (b) Frequency is defined as the number of rotations completed by a body in one second and the time period is defined as the time taken by an object to complete one rotation.

5. Answer:

    The two vectors A and B are perpendicular to each other.

    Explanation: Let θ = angle between the two vectors A and B component of vector A along the direction of B is obtained by resolving A i.e. A cos θ.
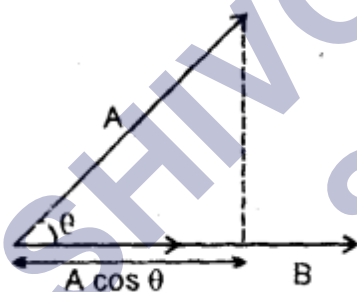
    Now according to the statement

    A cos θ = 0

    or

    cos θ = 0 = cos 90°

    θ = 90°

    i.e. A $\perp$ B


    Hence proved.

6. Answer: The given statement is true. The displacement vector gives the position of a point just like the position vector. The only difference between the displacement and the position vector is that the displacement vector gives the position of a point with reference to a point other than the origin, while the position vector gives the position of a point with reference to the origin. Since the choice of origin is quite arbitrary, so the given statement.

7. Answer: The nature of a vector may or may not be changed when it is multiplied.

For example, when a vector is multiplied by a pure number like 1, 2, 3, …. etc., then the nature of the vector does not change. On the other hand, when a vector is multiplied by a scalar physical quantity, then the nature of the vector changes.

For example, when acceleration (vector) is multiplied by a mass (scalar) of a body, then it gives force (a vector quantity) whose nature is different than acceleration.

8. Answer: Yes, when a man walks, he pushes the ground with his foot. In return, an equal and opposite reaction acts on his foot. The reaction is resolved into two components: horizontal and vertical components. The horizontal component of the reaction helps the man to move forward while the vertical component balances the weight of the man.
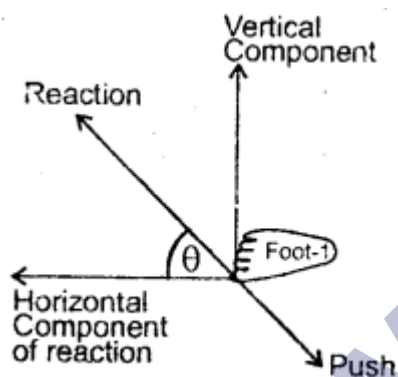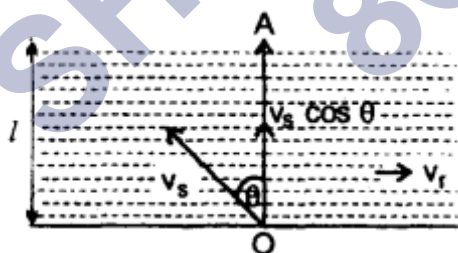

Long Questions Answers:

1. Answer:

Let v_s and v_r be the velocities of swimmer and river respectively.

Let v = resultant velocity of vs and vr

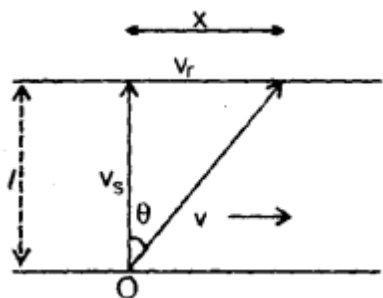1. Let the swimmer begins to swim at an angle θ with the line OA where OA is ⊥ to the flow of the river.

If t = time taken to cross the river, then

$$t = \frac{l}{v_s \cos \theta} \quad \text{...(i)}$$

where l = breadth of the river

For t to be minimum, cos 0 should be maximum.

i.e., cos θ = 1

This is possible if θ = 0

Thus, we conclude that the swimmer should swim in a direction perpendicular to the direction of the flow of the river.

2. $v = \sqrt{v_s^2 + v_r^2}$

where v$\vec{v}$ is the resultant velocity of v_s and v_r.

3. tan θ = $\frac{v_r}{v_s} = \frac{x}{l}$

or

X = l$\frac{v_r}{v_s}$

4. t = $\frac{l}{v_s}$

2. Answer:

It states that if two vectors can be represented completely (i.e. both in magnitude and direction) by the two adjacent sides of a parallelogram drawn from a point then their resultant is represented completely by its diagonal drawn from the same point.

Proof: Let P and Q be the two vectors represented completely by the adjacent sides OA and OB of the parallelogram OACB s.t.

$\overrightarrow{OA}$ = P, $\overrightarrow{OB}$ = Q

or

$|\overrightarrow{OA}|$ = |P|, $|\overrightarrow{OB}|$ = |Q|
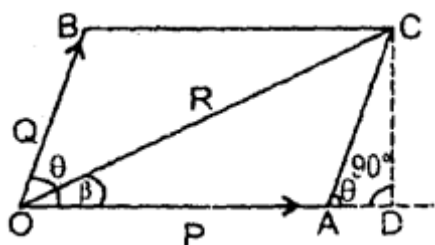
θ = angle between them = ∠AOB

If R be their resultant, then it will be represented completely by the diagonal OC through point O s.t. OC = R

The magnitude of R: Draw CD $\perp$ to OA produced,

$\therefore$ $\angle DAC = \angle AOB = \theta$

Now in right angled triangle ODC,

$$OC^2 = OD^3 + DC^2$$
$$= (OA + AD)^2 + DC^2$$
$$= OA^2 + AD^2 + 2.OA.AD + DC^2$$
$$= OA^2 + (AD^2 + DC^2) + 2OA.AD \quad \ldots (i)$$

Also in r.t. $\angle$d $\triangle ADC$,

$$AC^2 = AD^2 + DC^2 \quad \ldots (ii)$$

Also $\dfrac{AD}{AC} = \cos\theta$

or $AD = AC\cos\theta \quad \ldots (iii)$

and $\dfrac{DC}{AC} = \sin\theta$

or $DC = AC\sin\theta \quad \ldots (iv)$

$\therefore$ from equation (i), (ii), (iii), we get

$$OC^2 = OA^2 + AC^2 + 2.OA.AC\cos\theta$$

or $OC = \sqrt{OA^2 + AC^2 + 2.OA.AC\cos\theta} \quad \ldots (v)$

As OC = R, OA = P, AC = OB = Q $\ldots (vi)$

$\therefore$ from (v) and (vi), we get

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \quad \ldots (vii)$$

eqn. (vii) gives the magnitude of R.

The direction of R: Let β be the angle made by R with P

$\therefore$ in rt. $\angle$d $\triangle ODC$,

$$\tan\beta = \frac{DC}{OD} = \frac{DC}{OA + AD}$$

$$= \frac{AC\sin\theta}{OA + AC\cos\theta} \quad \text{[by using (iii) and (iv)]}$$

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta} \quad \ldots (viii)$$

Special cases: (a) When two vectors are acting in the same direction:

Then $\theta = 0°$

$$\therefore \qquad R = \sqrt{(P+Q)^2} = P + Q$$

and $\qquad \tan \beta = \dfrac{Q.0}{P+Q} = 0$ or $\beta = 0°$

Thus, the magnitude of the resultant vector is equal to the sum of the magnitudes of the two vectors acting in the same direction, and their resultant acts in the direction of P and Q.

(b) When two vectors act in the opposite directions:

Then $\theta = 180°$

$$\therefore \qquad \cos \theta = -1 \text{ and } \sin \theta = 0$$

$$\therefore \qquad R = \sqrt{P^2 + Q^2 + 2PQ(-1)}$$

$$= \sqrt{P^2 + Q^2 - 2PQ}$$

$$= \sqrt{(P-Q)^2} \text{ or } \sqrt{(Q-P)^2}$$

$$= (P-Q) \text{ or } (Q-P)$$

and $\qquad \tan \beta = \dfrac{Q \times 0}{P + Q(-1)} = 0 = \tan 0°$ or $\tan 180°$

$$\therefore \qquad \beta = 0° \text{ or } 180°$$

Thus, the magnitude of the resultant of two vectors acting in the opposite direction is equal to the difference of the magnitude of two vectors and it acts in the direction of the bigger vector.

(c) If $\theta = 90°$ i.e. if $P \perp Q$,
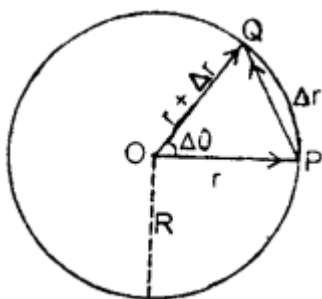
then $\cos 90° = 0$

and

$\sin 90° = 1$

$R = \sqrt{P^2 + Q^2}$

and

$\tan \beta = \dfrac{Q}{P}$

3. Answer:

Let R be the radius of the circular path of centre O on which an object is moving with uniform angular velocity co. Let v = its linear velocity. Let the object move from point P at time t to point Q at time t + Δt. If r and r + Δr be its position vectors at point P and Q respectively, then

$$\overrightarrow{OP} = \mathbf{r}$$

and $\overrightarrow{OQ} = \mathbf{r} + \Delta\mathbf{r}$

Also $|\mathbf{r}| = |\mathbf{r} + \Delta\mathbf{r}| = R$

$$= \text{radius of circle.}$$

$\therefore$ Linear displacement of the particle from P to Q in small time interval $\Delta t = \Delta r$.

Let $\Delta\theta$ = its angular displacement

$\therefore \omega = \frac{\Delta\theta}{\Delta t}$

or

$\Delta\theta = \omega\Delta t$ ....(1)

Also we know that $\Delta\theta = \frac{\widehat{PQ}}{R}$ ...(2)

$\therefore$ from (1) and (2), we get

$$\omega\Delta t = \frac{\widehat{PQ}}{R} \text{ or } \frac{\widehat{PQ}}{\Delta t} = \omega R \qquad\qquad .... (3)$$

Now when $\Delta t \to 0$, then from eqn. (1) $\Delta\theta \to 0$

so arc PQ = $\widehat{PQ}$ = chord PQ

Thus eqn. (3) reduces to

$$\frac{PQ}{\Delta t} = \omega R$$

or $\mathbf{v} = R\omega$

where $v = \frac{PQ}{\Delta t}$ is the linear velocity of the object.

Direction of velocity vector: In isosceles ΔOPQ,

$$\angle PQO + \angle OPQ + \angle QOP = 180°$$

As

$$\angle Q = \angle P$$

∴

$$2\angle P + \Delta\theta = \pi$$

or

$$\angle QPO = \frac{\pi}{2} - \frac{\Delta\theta}{2}$$

$$= \frac{\pi}{2} - \frac{\omega\Delta t}{2} \qquad \dots (4)$$

when $\Delta t \to 0$, $\angle QPO \to \frac{\pi}{2}$

i.e. $\overrightarrow{OP}$ tends to become $\perp$ to $\overrightarrow{OP}$

or

$\overrightarrow{OP}$ tends to lie along the tangent at P. Hence velocity vector at P is directed along the tangent to the circle in the direction of motion

Assertion Reason Answer:

1. (d) Assertion is incorrect, reason is correct.

2. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

Case Study Answer:

1. i   (c) Temperature

Explanation:

Temperature is not a vector quantity because it has magnitude only. However, force, acceleration and velocity have both a magnitude and a direction. So, these are vectors in nature.

ii (c) A and B

Explanation:

Two vectors are said to be equal, if and only if they have the same magnitude and direction. Among the given vectors A and B are equal vectors as they have same magnitude (length) and direction. However, P and Q are not equal even though they are of same magnitude because their directions are different.

iii (a) $\lambda > 0$

Explanation:

$|\lambda A| = \lambda |A|$, if $\lambda > 0$ as multiplication of vector A with a positive number λ gives a vector whose magnitude is changed by the factor λ but the direction is same as that of A.

iv (b) $\lambda 0 = \lambda$

Explanation:

Null vector 0 is a vector, whose magnitude is zero and its direction cannot be specified. So, it means, $|0| = 0$. Thus, $\lambda 0 = 0$. Hence, property given in option (b) is incorrect.

v (a) $\tan^{-1}\left(\frac{6}{5}\right)$ and $\sqrt{61}$

Explanation:

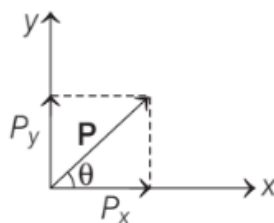Let **P** be as shown in the figure, then according to the given information

$$P_x = 5, \; P_y = 6$$

$$\therefore \quad |\mathbf{P}| = \sqrt{P_x^2 + P_y^2}$$

$$= \sqrt{25 + 36}$$

$$\Rightarrow \quad |\mathbf{P}| = \sqrt{61}$$

and $\tan \theta = \dfrac{P_y}{P_x} = \dfrac{6}{5} \Rightarrow \theta = \tan^{-1}\left(\dfrac{6}{5}\right)$

2. i (c) a javelin thrown by an athlete

Explanation:

A javelin thrown by an athlete is an example of projectile motion.

ii (d) zero

Explanation:

The horizontal component of velocity (u cos θ) is constant throughout the motion, so there will be no acceleration in horizontal direction.

iii (b) zero

Explanation:

As the vertical components of velocity (u sin θ) decreases continuously with height, from O to H, due to downward force of gravity and becomes zero at H

iv (d) 2.9 s

Explanation:

The time taken by the ball to return to the same level,

$$T = \frac{2v_0 \sin \theta}{g} = \frac{2 \times 28 \times \sin 30°}{9.8} \approx 2.9 \text{ s}$$

v (b) 69 m

Explanation:

The distance from the thrower to the point where the ball returns to the same level is

$$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{28 \times 28 \times \sin 60°}{9.8} \approx 69 \text{ m}$$