

MATHEMATICS

Chapter 16: PROBABILITY

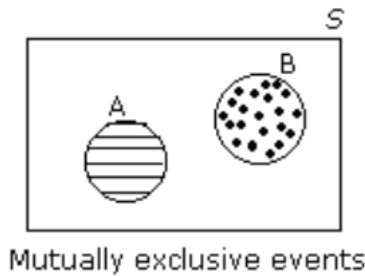


PROBABILITY

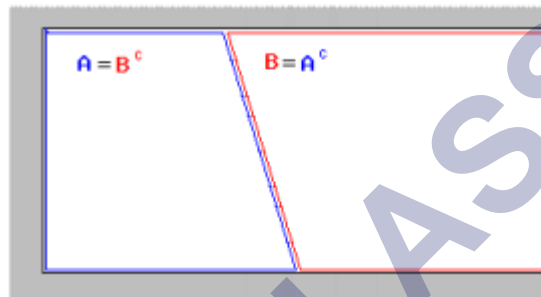
Key Concepts

1. The theory of probability is the branch of mathematics that deals with uncertain or unpredictable events. Probability is a concept which gives a numerical measurement for the likelihood of the occurrence of an event.
2. An act which gives some result is called an experiment.
3. A possible result of an experiment is called its **outcome**.
4. The sample space S of an experiment is the set of all its outcomes. Thus, each outcome is also called the sample point of the experiment.
5. An experiment repeated under essentially homogeneous and similar conditions may result in an outcome, which is either unique or not unique but one of the several possible outcomes.
6. An experiment is called a random experiment if it satisfies the following two conditions:
 - It has more than one possible outcome.
 - It is not possible to predict the outcome in advance.
7. The experiment that results in a unique outcome is called a deterministic experiment.
8. A sample space is a set consisting of all the outcomes, its cardinality is given by $n(S)$.
9. Any subset 'E' of a sample space for an experiment is called an **event**.
10. The empty set ϕ and sample space S describe events. In fact, ϕ is called an impossible event and S , i.e. the whole sample space, is called a sure event.
11. Whenever an outcome satisfies the conditions given in the event, we say that the **event has occurred**.
12. If an event E has only one sample point of a sample space, then it is called a simple (or elementary) event. In the experiment of tossing a coin, the sample space is $\{H, T\}$, and the event of getting a $\{H\}$ or a $\{T\}$ is a simple event.
13. A subset of the sample space which has more than one element is called a compound event. When a dice is thrown, the event of appearing of odd numbers is a compound event because $E = \{1, 3, 5\}$ has '3' sample points or elements in it.

14. Events are said to be equally likely if we have no reason to believe that one event is more likely to occur than the other. The outcomes of an unbiased coin are equally likely.
15. Let us define an event 'not A' corresponding to every event associated to a random variable, such that it occurs when and only when A does not occur. This is represented by the set \bar{A} and is called the negation of A or complementary event of A.
16. Certain event (sure event): If a random experiment occurs always, then the corresponding event is called a certain event.
17. Impossible event: If a random experiment never occurs, then the corresponding event is called an impossible event.
18. Mutually exclusive event: In a random experiment, if the occurrence of any one of the event prevents the occurrence of all other events, then the corresponding events are said to be mutually exclusive.
19. Exhaustive event: In a random experiment, if the union of two or more events is the sample space, then the associated events are said to be exhaustive events.
20. Probability of an event E is the ratio of happening of the number of elements in the event to the number of elements in the sample space.
- (i) $P(E) = \frac{n(E)}{n(S)}$ (ii) $0 \leq P(E) \leq 1$
21. Independent events: Two or more events are said to be independent if the occurrence or non-occurrence of any of them does not affect the probability of occurrence or non-occurrence of the other event.
22. The complement of an event A is the set of all outcomes which are not in A (or not favourable to A). It is denoted by A' .
23. Events A and B are said to be mutually exclusive **if and only if** they have no elements in common.



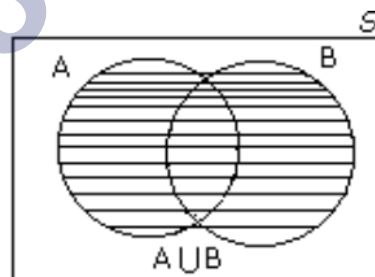
24. When every possible outcome of an experiment is considered, the events are called exhaustive events.



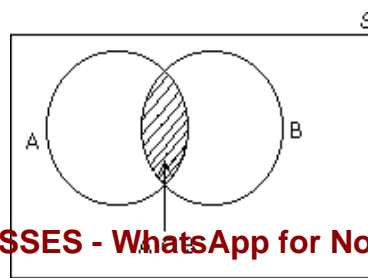
25. Events E_1, E_2, \dots, E_n are mutually exclusive and exhaustive if $E_1 \cup E_2 \cup \dots \cup E_n = S$ and $E_i \cap E_j = \varnothing$, for every distinct pair of events.

26. When the sets A and B are two events associated with a sample space, ' $A \cup B$ ' is the event 'either A or B or both'.

Therefore, event ' A or B ' = $A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$



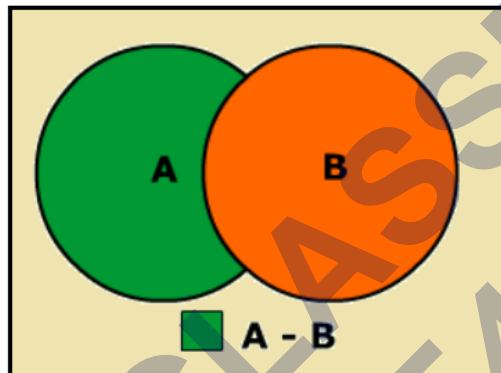
27. If A and B are events, then the event ' A and B ' is defined as the set of all the outcomes



which are favourable to both A and B, i.e. 'A and B' is the event $A \cap B$. This is represented diagrammatically as follows

28. If A and B are events, then the event ' $A - B$ ' is defined to be the set of all outcomes which are favourable to A but not to B. $A - B = A \cap B' = \{x: x \in A \text{ and } x \notin B\}$

This is diagrammatically represented as



29. If S is the sample space of an experiment with n equally likely outcomes $S = \{\omega_1, \omega_2, \omega_3, \dots, \omega_n\}$, then $P(\omega_1) = P(\omega_2) = \dots = P(\omega_n) = 1/n$.

$$\sum_{i=1}^n P(\omega_i) = 1$$

So $P(\omega_n) = 1/n$

30. Let S be the sample space of a random experiment. The probability P is a real valued function with domain the power set of S and range the interval [0, 1] satisfying the axioms that

(i) For any event E, $P(E)$ is greater than or equal to 0.

(ii) $P(S) = 1$.

(iii) Number $P(\omega_i)$ associated with sample point ω_i such that $0 \leq P(\omega_i) \leq 1$.

31. Addition theorem of probability: If 'A' and 'B' be any two events, then the probability of the occurrence of at least one of the events 'A' and 'B' is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive events then

$$P(A \cup B) = P(A) + P(B)$$

32. Addition theorem for 3 events: If A, B and C are three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

33. If 'E' is any event and E' is the complement of event 'E', then $P(E') = 1 - P(E)$

34. Probability of the difference of events: Let A and B be events.

$$\text{Then, } P(A - B) = P(A) - P(A \cap B).$$

35. Addition theorem in terms of the difference of events:

$$P(A \cup B) = P(A - B) + P(B - A) + P(A \cap B)$$

36. If A and B are two events, then the probability of occurrence of A only is

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

37. If A and B are two events, then the probability of occurrence of B only is

$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

38. If A and B are two events, then the probability of occurrence of exactly one of A and B is

$$P(A \cap \bar{B}) + P(\bar{A} \cap B) = P(A) + P(B) - 2P(A \cap B) = P(A \cup B) - P(A \cap B)$$

39. If A and B are two events associated to a random experiment such that $A \subset B$, then

$$\bar{A} \cap B \neq \phi.$$

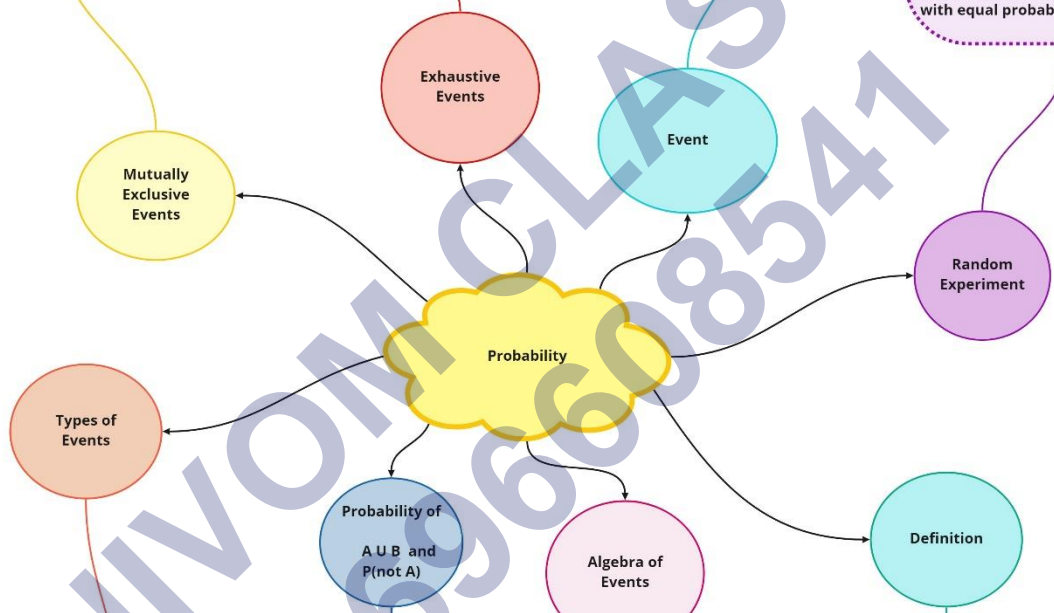
40. If A and B are two events, then $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$.

Events A & B are called mutually exclusive events if occurrence of any one of them excludes occurrence of other event, i.e. they cannot occur simultaneously.
Eg: A die is thrown. Event A = All even outcomes & event B = All odd outcomes. Then A & B are mutually exclusive events, they cannot occur simultaneously.
Note: Simple events of a sample space are always mutually exclusive.

Many events that together form sample space are called exhaustive events.
Eg: A die is thrown. Event A = All even outcomes and event B = All odd outcomes. Event A & B together forms exhaustive events as it forms sample space

It is the set of favourable outcomes. Any subset E of a sample space S is called an event.
Eg: Event of getting an even number (outcome) in a throw of a die.
Occurance of event: The event E of a sample space 'S' is said to have occurred if the outcome w of the experiment is such that $w \in E$. If the outcome w is such that $w \notin E$, we say that event E has not occurred.

An Experiment is called random experiment if it satisfies the following two conditions:
• It has more than one possible outcome.
• It is not possible to predict the outcome in advance.
Outcome: A possible result of a random experiment is called its outcome.
Sample Space: Set of all possible outcomes of a random experiment is called sample space. It is denoted by symbol 'S'. Eg: In a toss of a coin, sample space is Head & Tail. i.e., $S = \{H, T\}$
Sample Point: Each element of the Sample Space is called a sample point.
Eg: In a toss of a coin, head is a sample point
Equally Likely Outcomes: All outcome with equal probability.



• Impossible and Sure Event: The empty set ϕ is called an Impossible event, where as the whole sample space 'S' is called 'Sure event'.
Eg: In a rolling of a die, impossible event is that number more than 6 and event of getting number less than or equal to 6 is sure event.
• Simple Event: If an event has only one sample point of a sample space, it is called a 'simple event'.
Eg: In rolling of a die, simple event could be the event of getting number 4.
• Compound Event: If an event has more than one sample point, it is called a 'compound event'.
Eg: In rolling of a die, compound event could be event of getting an even number.
• Complementary Event: Complement event to A = 'not A'
Eg: If an event A = Event of getting odd number in a throw of a die i.e., {1, 3, 5} then, complementary event to A = Event of getting an even number in a throw of a die, i.e. {2, 4, 6} $A' = \{W: W \in S \text{ and } W \notin A\} = S - A$ (where S is the sample space)

- Event A or B or $(A \cup B)$
 $A \cup B = \{w: w \in A \text{ or } w \in B\}$
- Event A and B or $(A \cap B)$
 $A \cap B = \{w: w \in A \text{ and } w \in B\}$
- Event A but not B or $(A - B)$
 $A - B = A \cap B'$

If A and B are any two events, then
• $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
• $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
If A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$

• Probability of the event 'not A'
 $P(A') = P(\text{not } A) = 1 - P(A)$

Probability is the measure of uncertainty of various phenomenon, numerically. It can have positive value from 0 to 1.

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$$

Eg: Probability of getting an even no. in a throw of a die.

Sol. Here, favourable outcomes = {2, 4, 6}
Total no. of outcomes = {1, 2, 3, 4, 5, 6}

$$\text{Probability} = \frac{3}{6} = \frac{1}{2}$$

Important Questions

Multiple Choice questions-

Question 1. Events A and B are independent if

- (a) $P(A \cap B) = P(A/B) P(B)$
- (b) $P(A \cap B) = P(B/A) P(A)$
- (c) $P(A \cap B) = P(A) + P(B)$
- (d) $P(A \cap B) = P(A) \times P(B)$

Question 2. A single letter is selected at random from the word PROBABILITY. The probability that it is a vowel is

- (a) $2/11$
- (b) $3/11$
- (c) $4/11$
- (d) $5/11$

Question 3. A die is rolled, find the probability that an even prime number is obtained

- (a) $1/2$
- (b) $1/3$
- (c) $1/4$
- (d) $1/6$

Question 4. When a coin is tossed 8 times getting a head is a success. Then the probability that at least 2 heads will occur is

- (a) $247/265$
- (b) $73/256$
- (c) $247/256$
- (d) $27/256$

Question 5. The probability that the leap year will have 53 sundays and 53 monday is

- (a) $2/3$
- (b) $1/2$
- (c) $2/7$

(d) $1/7$

Question 6. Let A and B are two mutually exclusive events and if $P(A) = 0.5$ and $P(B) = 0.6$ then $P(A \cup B)$ is

(a) 0

(b) 1

(c) 0.6

(d) 0.9

Question 7. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals

(a) $1/2$

(b) $7/15$

(c) $2/15$

(d) $1/3$

Question 8. The events A, B, C are mutually exclusive events such that $P(A) = (3x + 1)/3$, $P(B) = (x - 1)/4$ and $P(C) = (1 - 2x)/4$. The set of possible values of x are in the interval

(a) $[1/3, 1/2]$

(b) $[1/3, 2/3]$

(c) $[1/3, 13/3]$

(d) $[0, 1]$

Question 9. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. The probability that none of the balls drawn is blue is

(a) $10/21$

(b) $11/21$

(c) $2/7$

(d) $5/7$

Question 10. If 4-digit numbers greater than 5000 are randomly formed from the digits 0, 1, 3, 5 and 7, then the probability of forming a number divisible by 5 when the digits are repeated is

(a) $1/5$

(b) $2/5$

(c) $3/5$

(d) $\frac{4}{5}$

Very Short Questions:

- Three coins are tossed simultaneously list the sample space for the event.
- Two dice are thrown simultaneously. Find the prob. of getting doublet.
- 20 cards are numbered from 1 to 20. One card is then drawn at random. What is the prob. of a prime no.
- If $\frac{3}{10}$ is the prob. that an event will happen, what is the prob. that it will not happen?
- If A and B are two mutually exclusive events such that
 $P(A) = \frac{1}{2}$ and
 $P(B) = \frac{1}{3}$ find $P(A \text{ or } B)$
- If E and F are events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$ find $P(\text{not } E \text{ and not } F)$
- A letter is chosen at random from the word 'ASSASSINATION'. Find the prob. that letter is a consonant.
- There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?
- 4 cards are drawn from a well shuffled deck of 52 cards what is the prob. of obtaining 3 diamonds and one spade.
- Describe the sample space. A coin is tossed and a die is thrown.

Short Questions:

- A coin is tossed three times consider the following event A : No head appears, B : Exactly one head appears and C : At least two heads appears do they form a set of mutually exclusive and exhaustive events.
- A and B are events such that $P(A) = 0.42$, $P(B) = 0.48$, and $P(A \text{ and } B) = 0.16$ determine
 (i) $P(\text{not } A)$ (ii) $P(\text{not } B)$ (iii) $P(A \text{ or } B)$
- Find the prob. that when a hand of 7 cards is drawn from a well shuffled deck of 52 cards, it contains (i) all king (ii) 3 kings (iii) at least 3 kings
- From a group of 2 boys and 3 girls, two children are selected at random. Describe the sample space associated with
 (i) E_1 : both the selected children are boys
 (ii) E_2 : at least one selected child is a boy

- (iii) E_3 : one boy and one girl is selected
 (iv) E_4 : both the selected children are girls
5. A book contains 100 pages. A page is chosen at random. What is the chance that the sum of the digit on the page is equal to 9.

Long Questions:

- Three letters are dictated to three persons and an envelope is addressed to each of them, those letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the prob. that at least one letter is in its proper envelope.
- If 4 digit no. greater than 5,000 are randomly formed the digits 0,1,3,5 and 7 what is the probability of forming a no. divisible by 5 when
 - The digits are repeated
 - The repetition of digits is not allowed.
- 20 cards are numbered from 1 to 20. One card is drawn at random what is the prob. that the no. on the card drawn is
 - A prime no.
 - An odd no.
 - A multiple of 5
 - Not divisible by 3.
- In a single throw of three dice, find the prob. of getting
 - A total of 5
 - A total of at most 5.

Assertion Reason Questions:

- In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

Assertion (A) : The probability of a sure event is 1.

Reason (R) : Let E be an event. Then, $0 \leq P(E) \leq 1$.

- Both Assertion and Reason are true and Reason is a correct explanation of Assertion.
 - Both Assertion and Reason are true but Reason is not a correct explanation of Assertion.
 - Assertion is true and Reason is false.
 - Assertion is false and Reason is true.
- In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

Assertion (A) : If the probability of winning a game is $\frac{8}{18}$, then the probability of losing the game is $\frac{7}{15}$.

Reason (R) For any event E, we have $P(E) + P(\text{not } E) = 1$.

(i) Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

(ii) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

(iii) Assertion (A) is true and Reason (R) is false.

(iv) Assertion (A) is false and Reason (R) is true.

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Answer Key:**MCQ:**

1. (d) $P(A \cap B) = P(A) \times P(B)$
2. (b) $3/11$
3. (d) $1/6$
4. (c) $247/256$
5. (d) $1/7$
6. (d) 0.9
7. (b) $7/15$
8. (a) $[1/3, 1/2]$
9. (a) $10/21$
10. (b) $2/5$

Very Short Answer:

1. $S = HHH, HHT, HTH, THH, HTT, TTH, THT, TTT.$
2. $n = (s) = 36$ S be the sample space
let E be the event of getting doublet

$$P(E) = \frac{6}{36} [\because E = ((1,1), (2,2), (3,3), (4,4), (5,5), (6,6))] \\ = \frac{1}{6}$$

3. Let S be the sample space and E be the event of prime no.

$$n(S) = \{1, 2, 3, \dots, 20\}$$

$$n(E) = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

4. Let E be the event

$$P(E) = \frac{3}{10}$$

$$P(\bar{E}) = 1 - P(E)$$

$$= 1 - \frac{3}{10}$$

$$= \frac{7}{10}$$

5.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{3} - \phi \quad [P(A \cap B) = \phi]$$

$$= \frac{5}{6}$$

6.

$$P(E' \cap F') = P(E \cup F)'$$

$$= 1 - P(E \cup F) \left[\because P(E \cup F) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8} \right]$$

$$= 1 - \frac{5}{8}$$

$$= \frac{3}{8}$$

7. $P(\text{consonant}) = \frac{7}{13}$

8. $P(\text{a woman member is selected}) = \frac{6}{10} = \frac{3}{5}$

9.

$$\frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4} = \frac{286}{20825} \left[\because \begin{array}{l} 3 \text{ Spades out of } 13 \\ \text{and one ace out of } 13 \end{array} \right]$$

10. {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

Short Answer:1. Observations are 2, 7, 4, 6, 8 and p which are 6 in numbers $n = 6$

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$A = \{TTT\}, B = \{HTT, THT, TTH\}, C = \{HHT, HTH, THH, HHH\}$$

$$A \cup B \cup C = S$$

Therefore A, B and C are exhaustive events.

Also $A \cap B = \phi$, $A \cap C = \phi$, $C \cap C = \phi$, disjoint i.e. they are mutually exclusive.

2.

$$P(\text{not } A) = 1 - p(A) = 1 - 0.42 = 0.58$$

$$P(\text{not } B) = 1 - p(B) = 1 - 0.48 = 0.52$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.42 + 0.48 - 0.16$$

$$= 0.74$$

3.

$$P(\text{all king}) = \frac{{}^4C_4 \times {}^{48}C_3}{{}^{52}C_7} = \frac{1}{7735}$$

$$P(3 \text{ king}) = \frac{{}^4C_3 \times {}^{48}C_4}{{}^{52}C_7} = \frac{9}{1547}$$

$$P(\text{at least 3 king}) = p(3 \text{ king}) + p(4 \text{ king})$$

$$= \frac{9}{1547} + \frac{1}{7735} = \frac{46}{7735}$$

4.

$$S = \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, G_1G_2, G_1G_3, G_2G_3\}$$

$$E_1 = \{B_1B_2\}$$

$$E_2 = \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$$

$$E_3 = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$$

$$E_4 = \{G_1G_2, G_1G_3, G_2G_3\}$$

5.

$$E = \{9, 18, 27, 36, 45, 54, 63, 72, 81, 90\}$$

$$S = 100$$

$$P(E) = \frac{10}{100}$$

$$= \frac{1}{10}$$

Long Answer:

1. Let the tree letters be denoted by A_1 A_2 and A_3 and three envelopes by E_1 E_2 and E_3 .

Total No. of ways to putting the letter into three envelopes is $3P_3 = 6$

No. of ways in which none of the letters is put into proper envelope = 2

Req. prob.

P (at least one letters is put into proper envelope) = $1 - P$ (none letters is put into proper envelopes)

$$= 1 - \frac{2}{6}$$

$$= \frac{2}{3}$$

2. (i)

Thousand	H	T	U
5,7			

For a digit greatest then 5000 Thousand Place filled in 2 ways and remaining three place be filled in 5 ways

No. 40. can be formed = $2 \times 5 \times 5 \times 5 - 1 = 249$

ATQ

Thousand	H	T	U
5,7			0,5

If no. is divisible by 5

Unit place filled in 2 ways and thousand place also by 2 ways (5, 7)

No. formed = $2 \times 5 \times 5 \times 2 - 1 = 99$

Req. prob. $\frac{99}{249}$

(ii) Digit not repeated

Thousand	H	T	U
5,7			

Thousand place filled in 2 ways

4 digit no. greater than 5 thousand = $2 \times 4 \times 3 \times 2 = 48$

Thousand	H	T	U
5			0

7			5,0
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$$\text{Favorable case} = 1 \times 3 \times 2 \times 2 + 1 \times 3 \times 2 \times 1$$

7 at thousand place 5 at thousand places

$$= 12 + 6 = 18$$

$$\text{Req. prob.} = \frac{18}{48} = \frac{3}{8}$$

3. Let S be the sample space

$$S = \{1, 2, 3, 4, 5, \dots, 20\}$$

Let E_1 , E_2 and E_3 , E_4 are the event of getting prime no., an odd no, multiple of 5 and not divisible by 3 respectively

$$P(E_1) = \frac{8}{20} = \frac{2}{5}, E_1 = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

$$P(E_2) = \frac{10}{20} = \frac{1}{2}, E_2 = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$$

$$P(E_3) = \frac{4}{20} = \frac{1}{5}, E_3 = \{5, 10, 15, 20\}$$

$$P(E_4) = \frac{14}{20} = \frac{7}{10}, E_4 = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\}$$

4. Let S be the sample space E_1 be the event of total of 5.

$$(i) E_1 = \{(1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1)\}$$

$$S = 6 \times 6 \times 6 = 216$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{216} = \frac{1}{36}$$

$$(ii) E_2 = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1)\}$$

$$P(E_2) = \frac{10}{216} = \frac{5}{108}$$

Assertion Reason Answer:

- (ii) Both Assertion and Reason are true but Reason is not a correct explanation of Assertion.
- (i) Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).