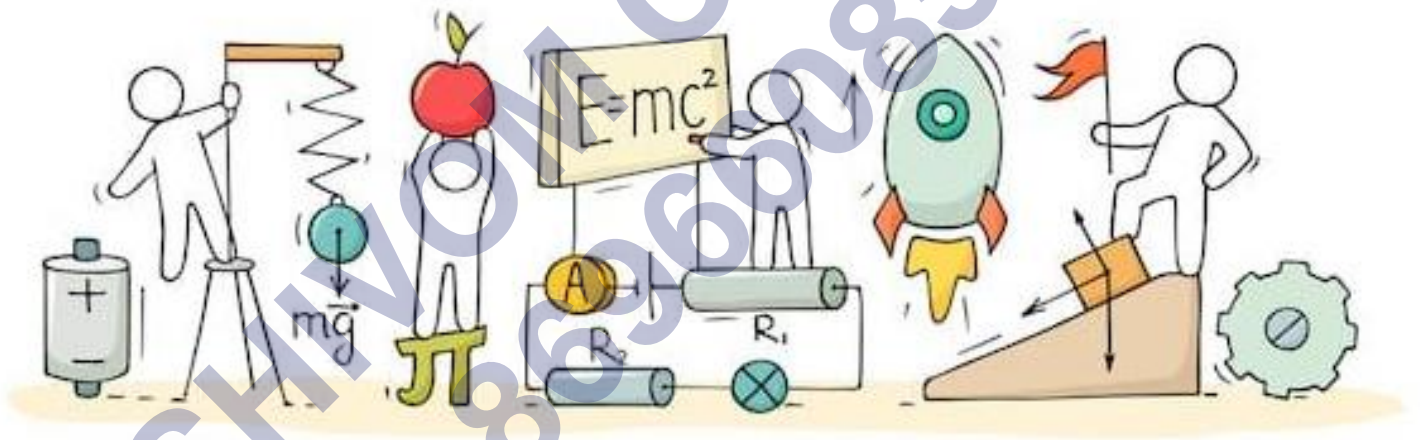# PHYSICS

Chapter 15: Waves

Waves

Introduction

In this chapter we will see the importance of waves in our life.

We will also study about the different properties of waves, some terms related to waves and also about different types of waves.We will also learn how waves propagate.

For example: -

1. Medium required by the waves to travel from one point to another:-
    - Consider a boy holding a thread and one end of thread is tied to the wall.
    - When a boy moves the thread, the thread moves in the form of a wave.
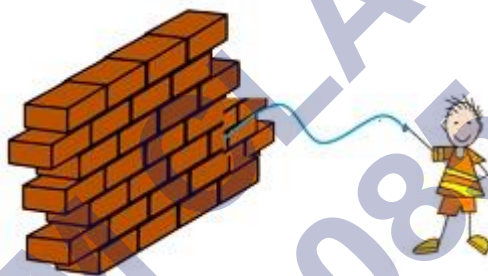
- Similarly a boat sailing over the sea,the boat is able to move because of waves.
- The ripples formed in a lake when we drop a stone in the lake.They are also waves.
- Earthquakes are caused due to the waves under the surface of the earth.
- The strings of the guitar when we play them are also waves again.
- Music system which we use to hear songs.This is due to sound waves.
- When 2 people talk they are able to hear each other because of the sound waves.

In the below Picture we can see waves need a medium to propagate.

2. Medium not required by some type of waves to move from one point to another:-
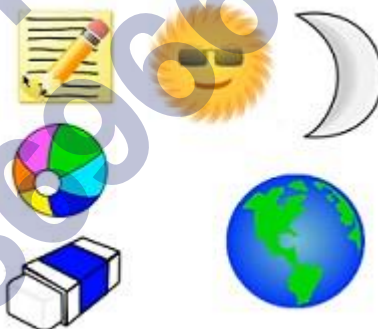
- TV remote waves play important part.
- Satelliteshelp ustouseTV, mobile phones, music system, the sun, the traffic lights, microwave,x-rays.

Some type of waves can propagate from one point to another without any medium.

3. Waves which are related to matter:-

- There are some set of waves which are inside the matter.
- For example: - whole of universe.

Waves propagating inside the matter

What is a wave?

A wave is s disturbance that propagates through spaceand time,usually with transference of energy.
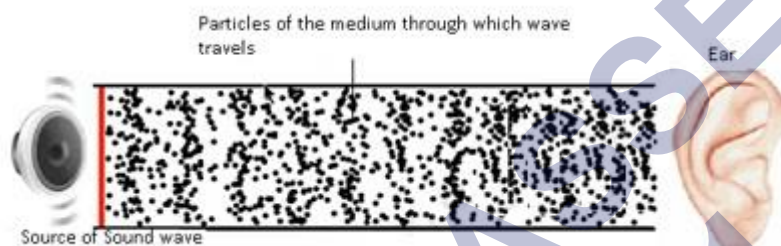
For example: -

- Consider the sound of the horn; this sound reaches our ear because of sound waves.
- There is transfer of energy from one point to another with the help of particles in the medium.

- These particles don't move they just move around their mean position,but the energy is getting transferred from one particle to another and it keeps on transferring till it reaches the destination.
- The movement of a particle is initiated by the disturbance.And this disturbance is transferred from one point to another through space and time.

Note:- Energy and not the matter is transferred from one point to another.

1. When a source of energy causes vibration to travel through the medium a wave is created.
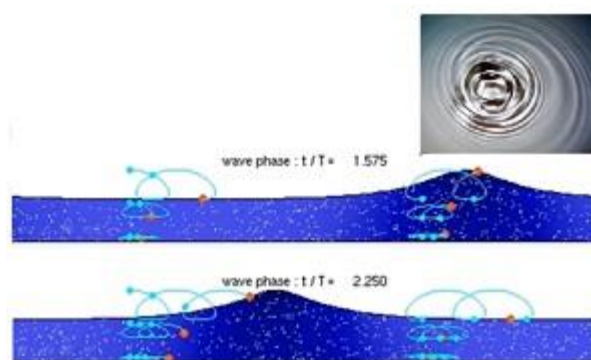

Types of Waves

1. Mechanical waves
2. Electromagnetic waves
3. Matter waves

Mechanical waves:-

1. The mechanical waves are governed by all the Newton's laws of motion.
2. Medium is needed for propagation of the wave.

For Example:- Water Waves, Sound Waves

Water waves: They are mechanical waves for which a medium is required to propagate.

Sound waves: A guitar or music system. Sound waves need a medium to propagate.They cannot travel in vacuum.

Electromagnetic waves:-

- Electromagnetic waves are related to electric and magnetic fields.
- An electromagnetic wave, does not need a medium to propagate, it carries no mass,does carry energy.
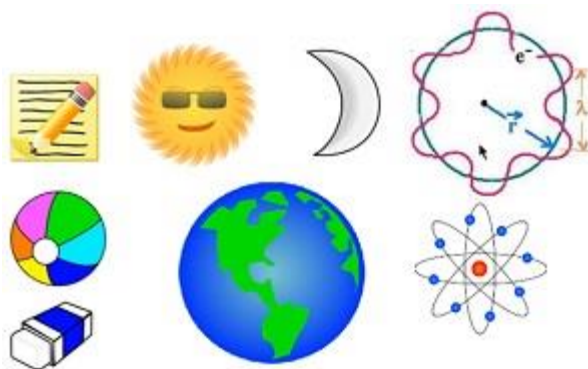
Examples:- Satellite system, mobile phones,radio, music player, x-rays and microwave.

Matter waves:-

- Waves related to matter. Matter consists of small particles.
- Matter waves are associated with moving electrons,protons,neutrons & other fundamental particlesetc.
- It is an abstract concept.

**Examples:-** pencil, sun, moon, earth, ball, atoms.

Transverse Waves

- The transverse waves are those in which direction of disturbance or displacement in the medium is perpendicular to that of the propagation of wave.
- The direction in which a wave propagates is perpendicular to the direction of disturbance.

Transverse Wave
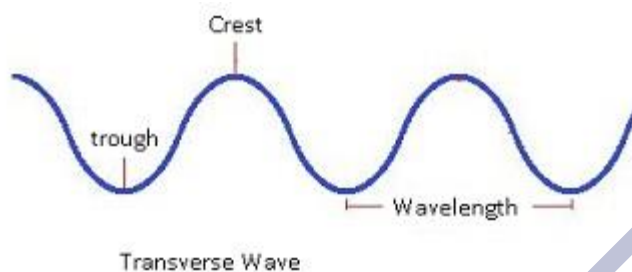
For example:-

- Consider a manholding one end of a thread and other end of the threadis fixed to wall.
- When a little jerk is given to the thread in the upward direction.The entire thread moves in a wavy manner.
- The jerk propagated along the entire length of the thread.
- The small disturbance which came from the source at one end, that disturbance getting propagatedand that is known as direction of propagation.
- Disturbance is vertically upward and wave is horizontal.They are perpendicular to each other.
- This type of wave is known as transverse wave.
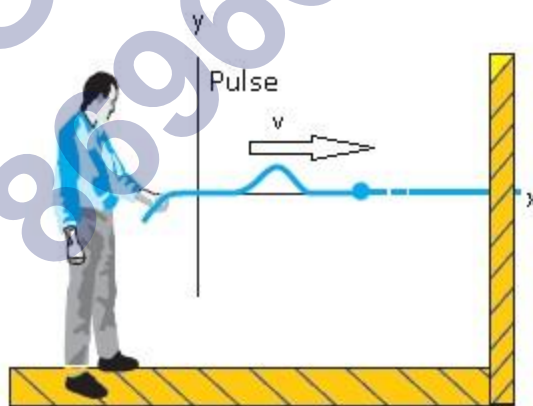
A single pulse is sent along a stretchedstring. A typical element of the string (suchas that marked with a dot) moves up andthen down as the pulse passes through.

The element's motion is perpendicular to thedirection in which the wave travels.

How are transverse waves caused?

- When we pull a thread in upward direction the formation and propagation of the waves are possible because entire thread is under tension.

- This tension is the small disturbance which is given at one end and it gets transferred to its neighbouring molecules.
- This will keep on continuing.So this small pulse will get propagated along the length of the thread.
- The movement of the particles is perpendicular to the propagation of the wave and the wave will propagate horizontally.
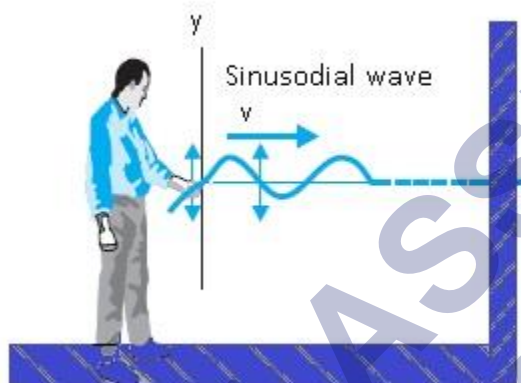
It is a transverse wave.

A sinusoidal wave is sent along the string.A typical element of the string moves up and down continuously as the wave passes.
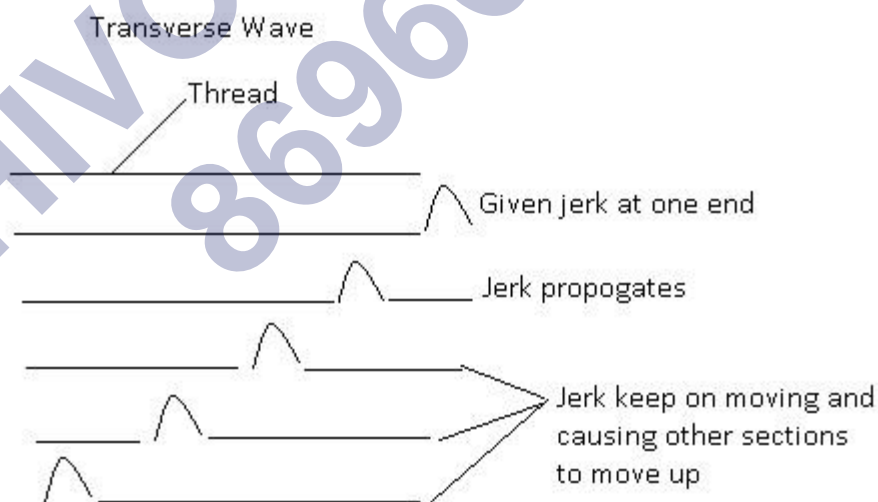
Conclusion:-

1. Transverse waves are those waves which propagates perpendicular to the direction of the disturbance.
2. Direction of disturbance is the direction of motion of particles of the medium.

Transverse Wave

Wave propogates

Movement of particle

Particle

Particles are moving up and down around their mean position.
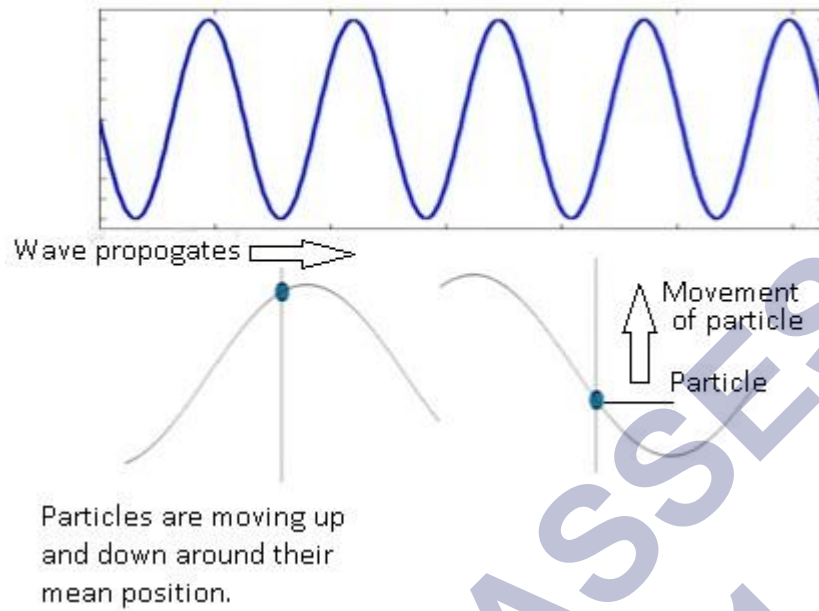
Longitudinal Waves

Longitudinal Wave

wavelength

Compression

Rarefaction

Movement
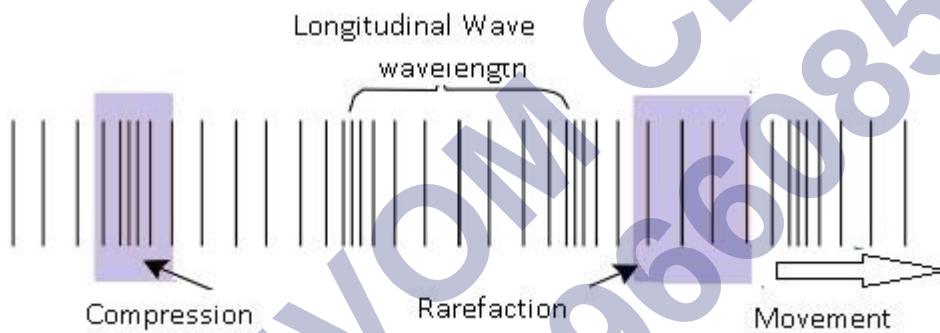
- Longitudinal means something related to length.
- In longitudinal waves direction of disturbance or displacement in the medium is along the propagation of the wave.

Compressions

Rarefactions

Compressed region

Stretched region
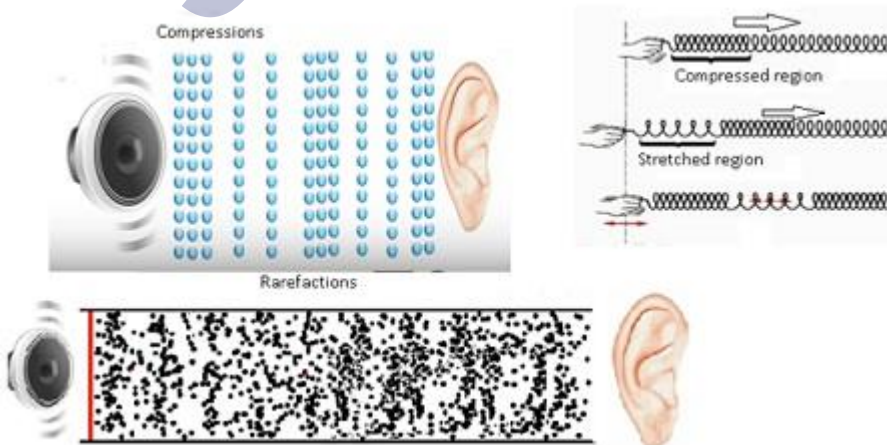
- For example: - Sound waves. Particles and wave moving along the horizontal direction. So both are in the same direction.
- In a Longitudinal wave there are regions where particles are very close to each other.These regions are known as compressions.
- In some regions the particles are far apart.Those regions are known as rarefactions.

Differentiate between transverse and longitudinal waves

| Transverse	Longitudinal
Constituents of the medium oscillate perpendicular to the direction of the wave propagation.	Constituents of the medium oscillate parallel to the direction of wave propagation.

Displacement in a progressive wave

- Amplitude and phase together describe the complete displacement of the wave.
- Displacement function is a periodic in space and time.
- Displacement of the particles in a medium takes place along the y-axis.
- Generally displacement is denoted as a function of X and T, but here it is denoted by y.
- In case of transverse wave displacement is given as:
- y(x,t) where x=propagation of the wave along x-axis, and particles oscillates along y-axis.
- Therefore $y(x,t) = A \sin(kx - \omega t + \phi)$.This is the expression for displacement.
- This expression is same as displacement equation which is used in oscillatory motion.
- As cosine function;$y(x,t) = B \cos(kx - \omega t + \phi)$,As both sine and cosine function)$y(x, t) = A \sin(kx - \omega t + \phi) + B \cos(kx - \omega t + \phi)$

Mathematically:

- Wave travelling along +X-axis: $y(x, t) = a \sin(kx - \omega t + \phi)$.
- Consider $y = a\sin(kx - \omega t + \phi) => y/a = \sin(kx - \omega t + \phi)$
- $\sin^{-1}(y/a) = kx - \omega t => kx = \sin^{-1}(y/a) + \omega t$
- $x = (1/k)\sin^{-1}(y/a) + (\omega t/k)$
- Wave travelling along –X-axis: $x = (1/k)\sin^{-1}(y/a) - (\omega t/k)$(only change in the sign of ωt)
- Conclusion:-
- As time t increases the value of x increases. This implies the x moves along x-axis.
- As time t decreases the value of x decrease. This implies the x moves along (-)ive x-axis.
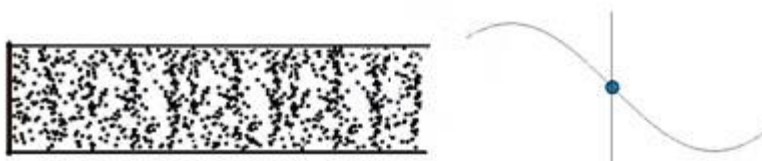
Amplitude and Phase of a wave

- Amplitude and phase together describes the position of the particle.
- Amplitude is the maximum displacement of the elements of the medium from their equilibrium positions as wave passes through them.
- It is denoted by A.

In case of transverse wave,

- The distance between the point P and Q (in the Figure) is maximum displacement. This maximum displacement of the particles is known as amplitude.
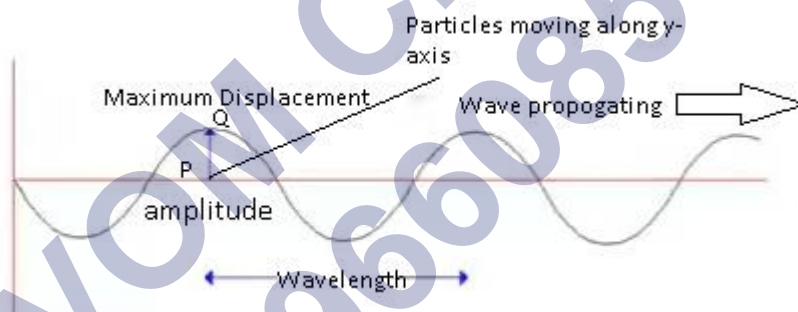
In case of longitudinal wave,

- There are regions of compressions (particles are closely packed) and rarefactions (particles are far apart).
- In compressions density of the wave medium is highest and in rarefactions density of the wave is lowest.
- Consider when the particle is at rarefaction, in that region as particle gets more space as a result the particles oscillates to the maximum displacement.
- Whereas in compressed region the particles oscillates very less as the space is not very much.
- The peak or the maximum amplitude is the centre of two compressed regions. Because at the center of the two compressed region the particle is most free to displace to maximum displaced position.

Conclusion:-

- In case of longitudinal wave the particles will not oscillate to a very large distance. This displacement won't represent the amplitude as it is not maximum possible displacement.

Amplitude is represented basically by the centre of the rarefaction region where the particle is most free to oscillate to its maximum displacement.

Phase

Phase of a wave describes the state of motion as the wave sweeps through an element at a particular position.

In-phase– Two points are said to be in-phase with each other when these two points are at the same position and they both are doing the same thing i.e. both the two points are exhibiting the same behaviour.
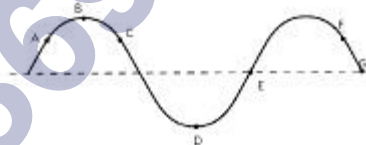
Points C and F are in phase with each other.

Out-of-phase –

- Two points are said to be out of phase even though they are at the same points but they are doing opposite thing i.e. both the points are exhibiting the different behaviour.
- Out of phase means which is not in phase.
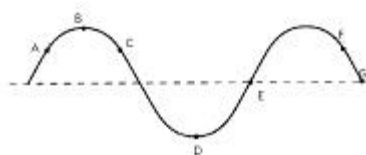
Points B and D,E and G are out of phase by 1800

- Two waves can be completely in-phase or out of phase with each other. They can be partially in phase or out of phase with each other.
- Observations made from the figure (1).
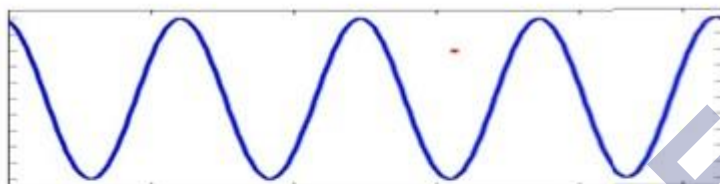
Figure(1)
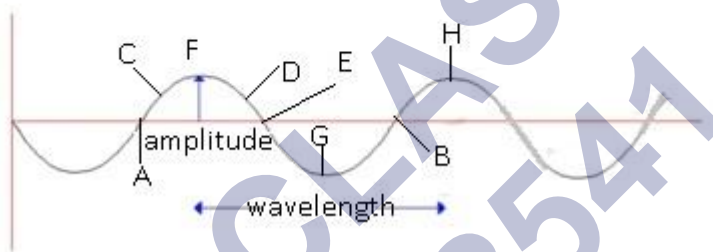
- Consider two points A and B on a wave. Their positions as well as their behaviour are same. Therefore points A and B are in phase.
- Consider points A and C on a wave. They are not in phase with each other as their position is not same.
- Similarly the points C and D are not in phase with each other as their positions are same but the behaviour is different. Therefore they are not in phase with each other.
- Consider the points F and G their positions are same but the behaviour is totally opposite. So F and G are out of phase.
- Consider the points F and H; they are in phase with each other as their position is same as well as their behaviour.

Wave Number

Wave number describes the number of wavelengths per unit distance.

Denoted by 'k'.

$y(x,t) = a \sin(kx - \omega t + \phi)$ assuming $\phi = 0$.

- At initial time t=0:-
  - $y(x,0) = a\sin kx$ (i)
  - When $x = x + \lambda$ then $y(x+\lambda,0) = a \sin k(x+\lambda)$ (ii)

- When x=x+2λ then y(x+2λ,0)=a sink(x+2λ)

Value of y is equal at all points because all the points' λ, 2λ are in phase with each other. Therefore,

From(i) and (ii) asin kx= a sink(x+λ) =asin(kx+k λ)

This is true if and only if: -k λ =2 πn, where n=1, 2, 3…

k=(2 π n)/ λ. This is the expression for wave number.

k is also known as propagation constant because it tells about the propagation of the wave.

Wave number is an indirect way of describing the propagation of wave.

Travelling Waves

- Travelling waves are those waves which travel from one medium to another.
- They are also known as progressive wave. Because they progress from one point to another.
- Both longitudinal and transverse waves can be travelling wave.
- Wave as a whole moves along one direction.
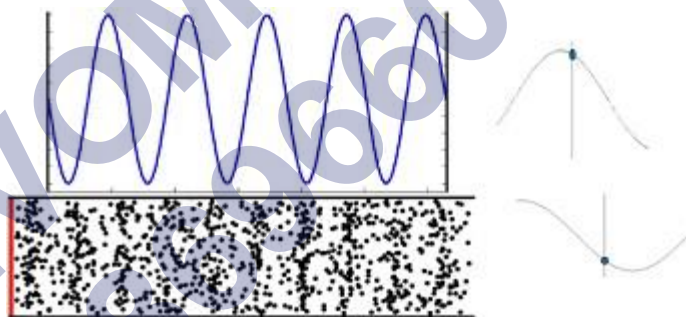
Standing (Stationary) Waves

- A stationary wave is a wave which is not moving, i.e. it is at rest.
- When two waves with the same frequency, wavelength and amplitude travelling in opposite directions will interfere they produce a standing wave.
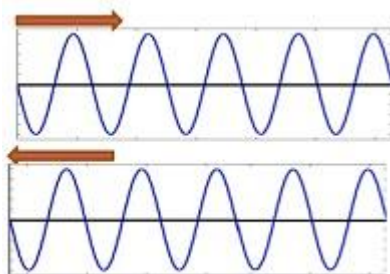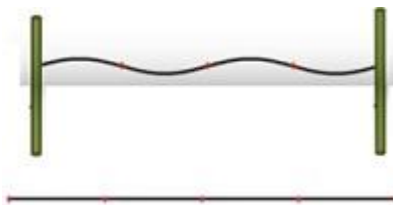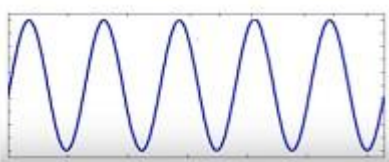
- Conditions to have a standing wave:- Two travelling waves can produce a standing wave, if the waves are moving in opposite directions and they have the same amplitude and frequency.
- At certain instances when the peaks of both the waves will overlap. Then both the peaks will add up to form the resultant wave.
- At certain instances when the peak of the one wave combine with the negative of the second wave .Then the net amplitude will become 0.
- As a result a standing wave is produced. In case of stationary wave the wave form does not move.
- **Explanation:-**
  - o Consider Its wave in the figure and suppose we have a rigid wall which does not move. When an incident wave hits the rigid wall it reflects back with a phase difference of π.
  - o Consider IInd wave in the figure, when the reflected wave travels towards the left there is another incident wave which is coming towards right.
  - o The incident wave is continuously coming come from left to right and the reflected wave will keep continuing from right to left.
  - o At some instant of time there will be two waves one going towards right and one going towards left as a result these two waves will overlap and form a standing wave.
- **Mathematically:**
- Wave travelling towards left $yl(x,t) = a \sin(kx - \omega t)$ and towards right $yr(x,t) = a \sin(kx + \omega t)$
- The principle of superposition gives, for the combined wave
- $y(x, t) = yl(x, t) + yr(x, t) = a \sin(kx - \omega t) + a \sin(kx + \omega t)$
- $y(x,t) = (2a \sin kx) \cos \omega t$ (By calculating and simplifying)
- The above equation represents the standing wave expression.
- Amplitude = 2a sin kx.
  - o The amplitude is dependent on the position of the particle.
  - o The cos ωt represents the time dependent variation or the phase of the standing wave.

Difference between the travelling wave and stationary wave

Travelling Wave(Progressive Wave)	Stationary Wave (Standing wave)

Waveform moves. Movement of the waveform is always indicated by the movement of the peaks of the wave.	Waveform doesn't move. Peaks don't move.
Wave amplitude is same for all the elements in the medium. Denoted by 'A'.	Wave amplitude is different for different elements. Denoted by asinkx.
Amplitude is not dependent on the position of the elements of the medium.	Amplitude is dependent on the position of the elements of the medium.
$y(x,t)=a\sin(kx - \omega t + \phi)$	$y(x,t)=2a\sin(kx)\cos(\omega t)$

Nodes and Antinodes: system closed at both ends

- System closed at both ends means both the ends are rigid boundaries.
- Whenever there is rigid body there is no displacement at the boundary. This implies at boundary amplitude is always 0. Nodes are formed at boundary.
- Standing waves on a string of length L fixed at both ends have restricted wavelength.
- This means wave will vibrate for certain specific values of wavelength.
- At both ends, nodes will be formed.=>Amplitude=0.
- Expression for node $x = (n\lambda)/2$.This value is true when x is 0 and L.
- When x=L:- $L=(n\lambda)/2$ =>$\lambda=(2L)/n$; n=1,2,3,4,.....
- λ cannot take any value but it can take values which satisfy $\lambda=(2L)/n$ this expression.
- That is why we can say that the standing wave on a string which is tied on both ends has the restricted wavelength.
- As wavelength is restricted therefore wavenumber is also restricted.
- $v = v/\lambda$ (relation between wavelength and frequency)
- Corresponding frequencies which a standing wave can have is given as: -$v= (vn)/2L$ where v= speed of the travelling wave.
- These frequencies are known as natural frequency or modes of oscillations.

Modes of Oscillations:-

- v= (vn)/2L where v=speed of the travelling wave, L=length of the string, n=any natural number.

- **First Harmonic:-**
    - For n=1, mode of oscillation is known as Fundamental mode.
    - Therefore v1=v/(2L).This is the lowest possible value of frequency.
    - Therefore v1is the lowest possible mode of the frequency.
    - 2 nodes at the ends and 1 antinode.
- **Second Harmonic:-**
    - For n=2, v2=(2v)/ (2L) =v/L
    - This is second harmonic mode of oscillation.
    - 3 nodes at the ends and 2 antinodes.
- **Third Harmonic:-**
    - For n=3,v3 = (3v)/ (2L).
    - This is third harmonic mode of oscillation.
    - 4 nodes and 3 antinodes.

Problem:- Find the frequency of note emitted (fundamental note) by a string 1m long and stretched by a load of 20 kg, if this string weighs 4.9 g. Given, g = 980 cm s–2?

Answer:-

L = 100 cm T = 20 kg = 20 × 1000 × 980 dyne

m= 4.9/100 = 0.049 g cm-1

Now the frequency of fundamental note produced,

v= (1/2L) √ (T/m)

v = 1/ (2x100) √(20x1000x980)/(0.049)

=100Hz

Problem:- A pipe 20 cm long is closed at one end, which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will this same source can be in resonance with the pipe, if both ends are open? Speed of sound = 340ms–1?

Answer:-

The frequency of nth mode of vibration of a pipe closed at one end is given by

vn=(2n 1)v/4L

v=340ms-1; L=20cm=0.2m;vn=430Hz.

Therefore 430= ((2n-1) x 340)/ (4x0.2)

=>n=1

Therefore, first mode of vibration of the pipe is excited, for open pipe since n must be an integer, the same source cannot be in resonance with the pipe with both ends open.
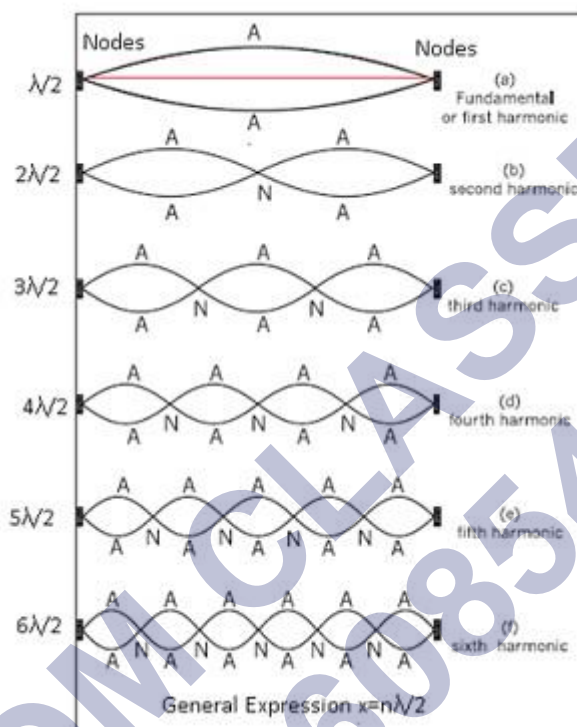
Stationary waves in a stretched string fixed at both ends. Various modes of vibration are shown.

Beats

Beats is the phenomenon caused by two sound waves of nearly same frequencies and amplitudes travelling in the same direction.

For example:-

- Tuning of musical instruments like piano, harmonium etc. Before we start playing on these musical instruments they are set against the standard frequency. If it is not set a striking noise will keep on coming till it is set.

Mathematically

- Consider only the time dependent and not the position dependent part of the wave.
- s1=a cos ω1t and s2=a cos ω2t; where amplitude and phase of the waves are same, but the frequencies are varying. Also considering ω1> ω2.
- When these 2 waves superimpose s= s1+ s2=a[cos ω1t + cos ω2t]

- By simplifying , 2a (cos(ω1 - ω2)/2)t cos(ω1 + ω2)/2)t)
- =>ω1 - ω2 is very small as ω1> ω2.Let (ω1 - ω2)=ωb
- =>ω1 + ω2 is very large. Let (ω1 + ω2)=ωa
- s= 2a cos ωbt cos ωat
- cosωat will vary rapidly with time and 2acosωbt will change slowly with time.
- Therefore we can say 2acosωbt = constant. As a result 2acosωbt = amplitude as it has small angular variation.
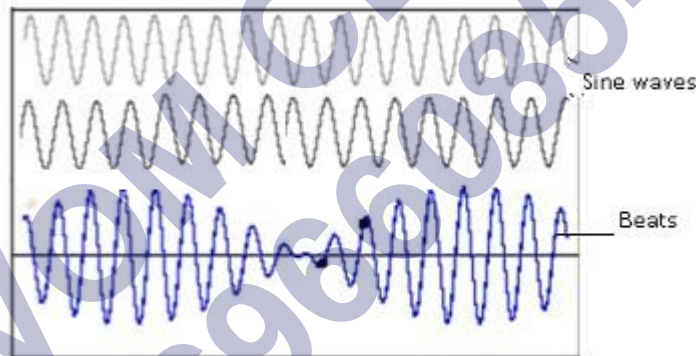
Beat Frequency

- Beat frequency can be defined as the difference in the frequencies of two waves.
- Consider if there is a wave of frequency ω1 and another wave of frequency ω2.Then the beat frequency will be ω1- ω2.
- It is denoted by ω
- Also ω = 2πv
- Therefore v beat = v1 – v2
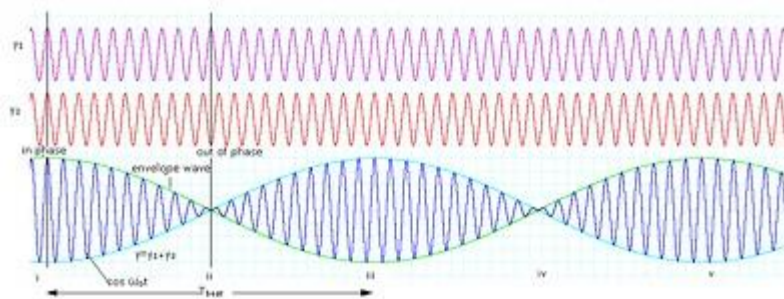
Problem:- Two sitar strings A and B playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease to 3 Hz. What is the original frequency of B if the frequency of A is 427 Hz?

Answer:- Increase in the tension of a string increases its frequency. If the original frequency of B (vB) were greater than that of A (vA), furtherincrease in vB should have resulted in an increase in the beat frequency. But the beat frequency is found to decrease. This shows thatvB< vA. Since vA – vB = 5 Hz, and vA = 427 Hz, weget vB = 422 Hz.

Doppler's Effect

- Doppler Effect is the phenomenon of motion-related frequency change.
- Consider if a truck is coming from very far off location as it approaches near our house, the sound increases and when it passes our house the sound will be maximum. And when it goes away from our house sound decreases.
- This effect is known as Doppler Effect.
- A person who is observing is known as Observer and object from where the sound wave is getting generated it is known as Source.
- When the observer and source come nearer to each other as a result waves get compressed. Therefore wavelength decreases and frequency increases.
- Case 1:- stationary observer and moving source
- Let the source is located at a distance L from the observer.
- At any time t1, the source is at position P1.
- Time taken by the wave to reach observer =L/v where v=speed of the sound wave.
- After some time source moves to position P0 in time T0.
- Distance between P1 and P0 =vsTo where vs is the velocity of the source.
- Let t2be the time taken by the second wave to reach the observer
  - Total time taken by the for the second wave to be sent to the observer = To +(L+vsTo)/v
  - Total time taken by the for the third wave to be sent to the observer=2To +(L+2vsTo)/v
  - Therefore for nth point tn+1 =nTo +(L+nvsTo)/v
  - =>In time tn+1the observer captures n waves.
- Total time taken by the waves to travel Time period T= (tn+1 - t1)/n

- =To +(vsTo)/v =>T=To(1+vs/v)
- Or v= 1/T
- =>v = v0(1+vs/v)-1
- By using binomial Theorem, v= v0 (1- vs/v)
- If the source is moving towards the observer the expression will become v= v0 (1+ vs/v)
- Case 2:- moving observer and stationary source
- As the source is not moving therefore vs is replaced by -v0.
- Therefore v= v0 (1+ v0/v)
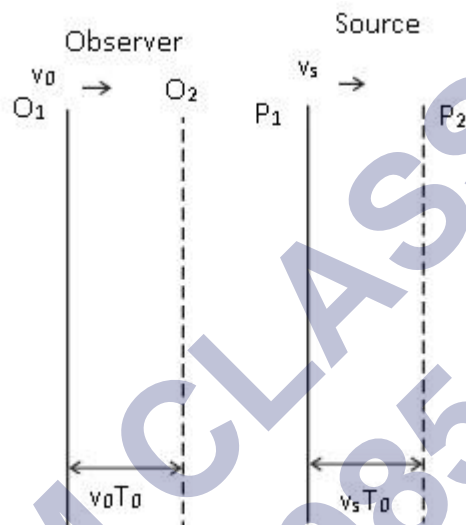
The observer O and the source S, both moving respectively with velocities v0 and vs . They are at position O1 and P1 at time t = 0, when the source emits the first crest of a sound, whose velocity is v with respect to the medium. After one period, t = T0, they have moved to O2 and P2,respectively through distances v0 T0 and vs T0, when the source emits the next crest.

Problem:- A metre-long tube opens at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.

Answer:-

Frequency of the turning fork, v = 340 Hz

Since the given pipe is attached with a piston at one end, it will behave as a pipe with one end closed and the other end open, as shown in the given figure.
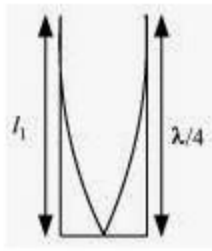
Such a system produces odd harmonics. The fundamental note in a closed pipe is given by the relation

l1=λ/4 Where,

Length of the pipe,l1=25.5cm=0.255m

Therefore, λ=4l1 =4x0.255 = 1.02m

The speed of sound is given by the relation:

v=vλ =340x1.02 =346.8m/s

Class : 11th Physics
Chapter- 15 : Waves

Relation b/w wave velocity (v) Wavelength (y) & frequency (v)

$v = v\lambda$

Time period $(T) = \dfrac{1}{\text{Frequency}} = \dfrac{1}{v}$

Angular Frequency $(\omega) = 2\pi v$

Wave number, $v = \dfrac{1}{\text{wavelength}} = \dfrac{1}{\lambda}$

Beats
Difference in frequencies of two superposing waves,

$v_{\text{beat}} = v_1 \sim v_2$

Displacement relation in a plane progressive wave

$y(x, t) = A\sin(-kx + t)\,\omega_\phi$

$y(x,t) = A\sin 2\pi\left(\dfrac{t}{T} - \dfrac{x}{\lambda}\right)$

Velocity amplitude $v_0 = A\omega^2$

Acceleration Amplitude $a_0 = A\omega^2$

Principle of superposition of waves
When two or more waves are propagating with different displacements then the net displacement of collective wave is given as $Y = y_1 + y_2 + y_3 + ... + y_n$

Waves

Stationary Waves

Stationary waves
(i) In string: Fundamental Frequency, $v = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$

(ii) In organ pipe:
   (a) Open at both ends,
   Fundamental freq. or I harmonic $v = \dfrac{v}{2l}$

   (b) Closed at one end $v = \dfrac{v}{4l}$

(iii) • In open organ pipe, higher harmonics, both odd & even.
   • In closed organ pipe, higher harmonics, odd only.

Dopoler Effect

General Formula, Apparent Frequency

$n^1 = n\left(\dfrac{v \pm v_s \pm v_m}{v \mp v_0 \pm v_m}\right)$

Here, v = speed of sound,
v_0 = observer speed
v_s = source speed
v_m = medium speed

Types

Matter Waves
Waves associated with particles like-electrons, protons, neutrons, atoms, molecules etc.

Mechanical Waves
Require material medium for their propagation

Electromagnetic Waves
Do not require any material medium for their propagation e.g., light waves

Transverse Waves
Individual particles of the medium oscillate perpendicular to the direction of propagation of wave

Longitudinal Waves
Individual particles of the medium oscillate along the direction of propagation of wave.

Properties
Essential properties for propagation
- Elasticity
- Inertia
- Minimum friction

Speed of transverse wave in a stretched string

$V = \sqrt{\dfrac{T}{m}}$, T = tension & M = mass/length

Speed of a longitudinal wave speed of sound, $V = \sqrt{\dfrac{B}{\rho}}$

$= \sqrt{\dfrac{Y}{\rho}} = \sqrt{\dfrac{P}{\rho}} = \sqrt{\dfrac{\gamma P}{\rho}}$ for air, $\gamma = 7/5$

Important Questions

Multiple Choice questions-

Q.1) Which of the following are mechanical waves

a) Water waves

b) Sound waves

c) Seismic waves

d) All

Q.2) Which of the following are electromagnetic waves

a) Light

b) Radio waves

c) X rays

d) All

Q.3) Electromagnetic waves

a) Requires material medium for their propagation

b) Do not require material medium for their propagation

c) Both a and b

d) None

Q.4) _____ waves can travel through vacuum.

a) Sound waves

b) Light waves

c) Radio waves

d) Both b and c

Q.5) The speed of electromagnetic waves is

a) 299,792,458 m/s

b) 299, 792, 458 km/s

c) 299, 792, 458 cm/s

d) None

Q.6) Pressure is given by

a) F/m

b) F/V

c) F/A

d) FA

Q.7) Sound waves are the

a) Transverse waves

b) Longitudinal waves

c) Both a and b

d) None

Q.8) Which of the following are progressive waves

a) Transverse waves

b) Longitudinal waves

c) Both a and b

d) None

Q.9) In case of transvers waves, the direction of motion of particles is

a) Parallel to the direction of propagation

b) Perpendicular to the direction of propagation

c) Normal to the direction of propagation

d) Both b and c

Q.10) In case of longitudinal waves, the direction of motion of particles is

a) Parallel to direction of propagation

b) Perpendicular to direction of propagation

c) Normal to direction of propagation

d) None

Q.11) The waves in an ocean are the combination of

a) Longitudinal waves

b) Transverse waves

c) Both a and b

d) None

Q.12) The maximum displacement of the particle of the wave from its mean or equilibrium position is called as

a) Phase

b) Epoch

c) Distance

d) Amplitude

Q.13) The minimum distance between two points having the same phase is called as

a) Frequency of the wave

b) Amplitude

c) Wavelength of the wave

d) None

Q.14) K = 2π/ wavelength, then k is called as

a) Wave number

b) Propagation constant

c) Force constant

d) Both a and b

Q.15) The SI unit of propagation constant is given by

a) rad/m

b) rad

c) m/rad

d) rad m

Assertion Reason Questions:

1. *Directions:*

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not the correct

explanation of the assertion.
(c) If assertion is true but reason is false.
(d) If the assertion and reason both are false

Assertion: Sine and cosine functions are periodic functions.
Reason: Sinusoidal functions repeats it values after a definite interval of time.

2. *Directions:*

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
(c) If assertion is true but reason is false.
(d) If the assertion and reason both are false

Assertion: Simple harmonic motion is a uniform motion.
Reason: Simple harmonic motion is not the projection of uniform circular motion.

MCQ Answers-

1. Ans: d) all

2. Ans: d) all

3. Ans: b) do not require material medium for their propagation

4. Ans: d) both b and c

5. Ans: a) 299, 792, 458 m/s

6. Ans: c) F/A

7. Ans: b) longitudinal waves

8. Ans: c) both a and b

9. Ans: d) both b and c

10. Ans: a) parallel to direction of propagation

11. Ans: c) both a and b

12. Ans: d) amplitude

13. Ans: c) wavelength of the wave

14. Ans: d) both a and b

15. Ans: a) rad/m

Very Short Questions-

1. How is the time period effected, if the amplitude of a simple pendulum is in Creased?

2. Define force constant of a spring.

3. At what distance from the mean position, is the kinetic energy in simple harmonic oscillator equal to potential energy?

4. How is the frequency of oscillation related with the frequency of change in the of K. E and PE of the body in S.H.M.?

5. What is the frequency of total energy of a particle in S.H.M.?

6. How is the length of seconds pendulum related with acceleration due gravity of any planet?

7. If the bob of a simple pendulum is made to oscillate in some fluid of density greater than the density of air (density of the bob density of the fluid), then time period of the pendulum increased or decrease.

8. How is the time period of the pendulum effected when pendulum is taken to hills Or in mines?

9. Define angular frequency. Give its S.I. unit.

10. Does the direction of acceleration at various points during the oscillation of a simple pendulum remain towards mean position?

Very Short Answers-

1. **Ans.** No effect on time period when amplitude of pendulum is increased or decreased.

2. **Ans.** The spring constant of a spring is the change in the force it exerts, divided by the change in deflection of the spring.

3. **Ans.** Not at the mid-point, between mean and extreme position. it will be at $x = a\sqrt{2}$.

4. **Ans.** P.E. or K.E. completes two vibrations in a time during which S.H.M completes one vibration or the frequency of P.E. or K.E. is double than that of

S.H.M

5. **Ans.** The frequency of total energy of particle is S.H.M is zero because it retain constant.

6. **Ans.** Length of the seconds pendulum proportional to acceleration due to gravity)

7. **Ans.** Increased

8. **Ans.** As $T \alpha \dfrac{1}{\sqrt{g}}$, T will increase.

9. **Ans.** It is the angle covered per unit time or it is the quantity obtained by multiplying frequency by a factor of 2π.
$\omega = 2\pi v$, S.I. unit is rads s^{-1}

10. **Ans.** No, the resultant of Tension in the string and weight of bob is not always towards the mean position.

Short Questions-

1. A mass = m suspend separately from two springs of spring constant k_1 and k_2 gives time period t_1 and t_2 respectively. If the same mass is connected to both the springs as shown in figure. Calculate the time period 't' of the combined system?
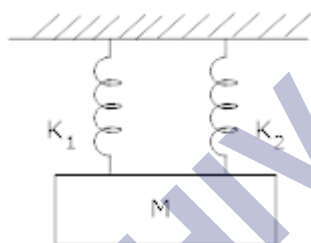
2. Show that the total energy of a body executing SHN is independent of time?

3. A particles moves such that its acceleration 'a' is given by a = -b x where x = displacement from equilibrium position and b is a constant. Find the period of oscillation? 2

4. A particle is S.H.N. is described by the displacement function: →

$$x = A \cos\left(wt + \Phi\right); w = \dfrac{2\pi}{T}$$

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is π cm | s, What are its amplitude and phase angle?

5. Determine the time period of a simple pendulum of length = l when mass of bob = m Kg?   3

6. Which of the following examples represent periodic motion?

(a) A swimmer completing one (return) trip from one bank of a river to the other and back.

(b) A freely suspended bar magnet displaced from its N-S direction and released.

(c) A hydrogen molecule rotating about its center of mass.

(d) An arrow released from a bow.

7. Figure 14.27 depicts four *x-t* plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?
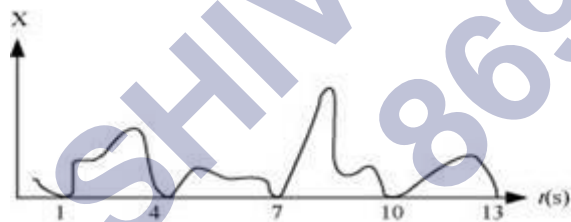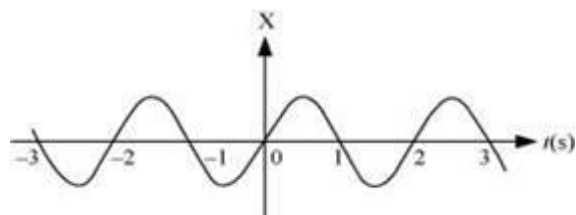
(a)
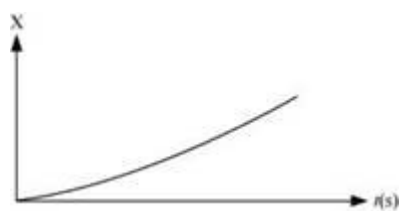
(b)
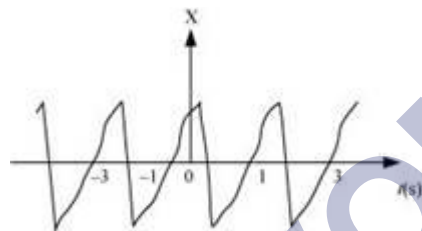
(c)

(d)

8. Which of the following relationships between the acceleration *a* and the displacement *x* of a particle involve simple harmonic motion?

(a) $a = 0.7x$

(b) $a = -200 x^2$

(c) $a = -10x$

(d) $a = 100 x^3$

9. The acceleration due to gravity on the surface of moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g on the surface of earth is 9.8 ms^{-2})

10. A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

Short Answers-

1. **Ans.** If T = Time Period of simple pendulum

m = Mass

k = Spring constant

then, $T = 2\pi\sqrt{\dfrac{m}{k}}$

or k = $\dfrac{4\pi^2 m}{T^2}$

For first spring : $\rightarrow k_1 = \dfrac{4\pi^2 m}{t_1^2}$ let $T = t_1$

For second spring : $\rightarrow k_2 = \dfrac{4\pi^2 m}{t_2^2}$ let $T = t_2$

When springs is connected in parallel, effective spring constant, k = k = $k_1 + k_2$

or k = $\dfrac{4\pi^2 m}{t_1^2} + \dfrac{4\pi^2 m}{t_2^2}$

If t = total time period

$\dfrac{4\pi^2 m}{t^2} = \dfrac{4\pi^2 m}{t_1^2} + \dfrac{4\pi^2 m}{t_2^2}$

$\dfrac{1}{t^2} = \dfrac{1}{t_1^2} + \dfrac{1}{t_2^2}$

Or $t^{-2} = t_1^{-2} + t_2^{-2}$

2. **Ans.** Let y = displacement at any time 't'

a = amplitude

w = Angular frequency

v = velocity,

y = a Sin wt

$$v = \frac{dy}{dt} = \frac{d}{dt} \left(a \, Sin \, wt \right)$$

So, $v = a \, w \, Cos \, wt$

Now, kinetic energy = K. E. = $\frac{1}{2} m \, v^2$

$$K.E. = \frac{1}{2} m \, w^2 a^2 \, Cos^2 \, wt \rightarrow 1)$$

Potential energy = $\frac{1}{2} ky^2$

$$P.E. = \frac{1}{2} ka^2 \, Sin^2 \, wt \rightarrow 2)$$

Adding equation 1) & 2)

Total energy = K.E.+ P.E

$$= \frac{1}{2} mw^2 a^2 \, Cos^2 \, wt + \frac{1}{2} ka^2 \, Sin^2 \, wt$$

Since $w = \sqrt{\frac{k}{m}} \Rightarrow w^2 m = k^2$

Total energy $= \frac{1}{2} mw^2 a^2 Cos^2 wt + \frac{1}{2} ka^2 Sin^2 wt$

$$= \frac{1}{2} ka^2 \, Cos^2 \, wt + \frac{1}{2} ka^2 \, Sin_2 \, wt$$

$$= \frac{1}{2} ka^2 \left(Cos^2 \, wt + Sin^2 \, wt \right)$$

Total energy $= \frac{1}{2} ka$

Thus total mechanical energy is always constant is equal to $\frac{1}{2} ka^2$. The total energy is independent to time. The potential energy oscillates with time and has a maximum value of $\frac{ka^2}{2}$. Similarly K. E. oscillates with time and has a maximum value of $\frac{ka^2}{2}$. At any instant = constant = $\frac{ka^2}{2}$. The K. E or P.E. oscillates at double the frequency of S.H.M.

3. Ans. Given that a = -b x, Since a $\propto$ x and is directed apposite to x, the particle do moves in S. H. M.

a = b x (in magnitude)

$$or \ \frac{x}{a} = \frac{1}{b}$$

$$or \ \frac{Displacement}{Accleration} = \frac{1}{b} \rightarrow 1)$$

$$Time \ period = T = 2\pi\sqrt{\frac{Displacement}{Accleration}}$$

Using equation 1)

$$T = 2\pi\sqrt{\frac{1}{b}}$$

4. Ans. At t = 0; x = 1cm; w = π|s

t = time

x = Postition

w = Argular frequency

$\therefore$ x = A Cos (Wt + ϕ)

1 = A Cos ($\pi \times 0 + \phi$)

1 = A Cos ϕ

$$Now, v = \frac{dx}{dt} = \frac{d}{dt}(A \ Cos \ (wt+\phi)$$

At t = 0 v = π cm|s; w = π|s

$\cancel{\pi}$ = -A$\cancel{\pi}$ Sin ($\pi \times 0 + \phi$)

$\Rightarrow$ -1 = A Sin ϕ $\rightarrow$ 2)

Squaring and adding 1) & 2)

A2 Cos^2 ϕ + A2 Sin$^2\phi$ = 1+1

A2$\left(Cos^2\phi + Sin^2\phi\right)$ = 2

$A^2 = 2$

$A = \sqrt{2}cm$

Dividing 2) by 1), we have :-

$$\frac{\cancel{A} \ Sin \ \phi}{\cancel{A} \ Cos \ \phi} = -1$$

tan $\phi = -1$

$or\ \phi \equiv \tan^{-1}(-1)$
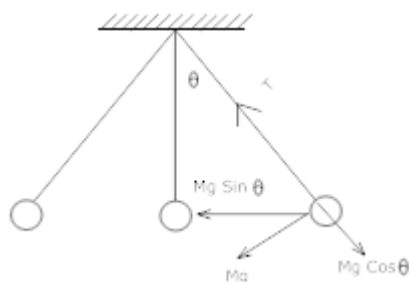
$\phi = \dfrac{3\pi}{4}$

5. Ans. It consist of a heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support which is free to oscillate.

The distance between point of suspension and point of oscillation is effective length of pendulum.

M = Mass of B ob

x = Displacement = OB

l = length of simple pendulum

Let the bob is displaced through a small angle θ the forces acting on it:-

1) weight = Mg acting vertically downwards.

2) Tension = T acting upwards.

Divide Mg into its components → Mg Cos θ & Mg Sin θ

T = Mg Cos θ

F = Mg Sin θ

- ve sign shows force is divested towards the ocean positions. If θ = Small,

Sin θ $\cong \theta = \dfrac{Arc\ \text{OB}}{l} = \dfrac{x}{l}$

$F = -Mg\dfrac{x}{l}$

In S.H.M., vestoring fore, $F = -mg\ \theta\ F = -mg\ \dfrac{x}{l} \rightarrow 1)$

Also, if k = spring constant

$F = -k\,x$

$mg\ \dfrac{x}{l} = -k\,x\left(\text{equating } F = -mg\ \dfrac{x}{l}\right)$

$$k = \frac{mg}{l}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{m \times l}{m g}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

i.e.1.) Time period depends on length of pendulum and 'g' of place where experiment is done.

2) T is independent of amplitude of vibration provided and it is small and also of the mass of bob.

6. Ans. (b) and **(c)**

(a) The swimmer"s motion is not periodic. The motion of the swimmer between the banks of a river is back and forth. However, it does not have a definite period. This is because the time taken by the swimmer during his back and forth journey may not be the same.

(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic. This is because the magnet oscillates about its position with a definite period of time.

(c) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such motion is periodic.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

7. Ans. (b) and **(d)** are periodic

(a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.

(b) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.

(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

8. Ans. (c) A motion represents simple harmonic motion if it is governed by the force law:

$F = -kx$

$ma = -k$

$$\therefore a = -\frac{k}{m}x$$

Where,

F is the force

m is the mass (a constant for a body)

x is the displacement

a is the acceleration

k is a constant

Among the given equations, only equation $a = -10\,x$ is written in the above form with $\frac{k}{m} = 10$ Hence, this relation represents SHM.

9. Ans. Acceleration due to gravity on the surface of moon, $g' = 1.7\ m\ s^{-2}$

Acceleration due to gravity on the surface of earth, $g = 9.8$ m s^{-2}

Time period of a simple pendulum on earth, $T = 3.5$ s

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where,

l is the length of the pendulum

$$\therefore l = \frac{T^2}{(2\pi)^2} \times g$$

$$= \frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8\,m$$

The length of the pendulum remains constant.

On moon"s surface, time period, $T' = 2\pi\sqrt{\frac{l}{g'}}$

$$= 2\pi\sqrt{\frac{\dfrac{(3.5)^2}{4 \times (3.14)^2} \times 9.8}{1.7}} = 8.4\,s$$

Hence, the time period of the simple pendulum on the surface of moon is 8.4 s.

10. Ans. The bob of the simple pendulum will experience the acceleration due to gravity and the centripetal acceleration provided by the circular motion of the car.

Acceleration due to gravity = g

Centripetal acceleration $= \dfrac{v^2}{R}$

Where,

v is the uniform speed of the car

R is the radius of the track

Effective acceleration (a_{eff}) is given as:

$$a_{eff} = \sqrt{g^2 + \left(\dfrac{v^2}{R}\right)^2}$$

Time period, $T = 2\pi\sqrt{\dfrac{l}{a_{eff}}}$

Where, l is the length of the pendulum

$\therefore$ Time period, $T = 2\pi\sqrt{\dfrac{l}{\sqrt{g^2 + \dfrac{v^4}{R^2}}}}$

Long questions-

1. What is Simple pendulum? Find an expression for the time period and frequency of a simple pendulum?

2. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(a) at the end A,

(b) at the end B,

(c) at the mid-point of AB going towards A,

(d) at 2 cm away from B going towards A,

(e) at 3 cm away from A going towards B, and

(f) at 4 cm away from B going towards A.

3. The motion of a particle executing simple harmonic motion is described by the displacement function,

x (t) = A cos (ω t + ω).

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is π s-1. If instead of the cosine function, we choose the sine function to describe the SHM: x = B sin (ω t + α), what are the amplitude and initial phase of the particle with the above initial conditions.

4. In Exercise 14.9, let us take the position of mass when the spring is unstreched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is

(a) at the mean position,

(b) at the maximum stretched position, and

(c) at the maximum compressed position.

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

5. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

(a) x = -2 sin (3t + π/3)

(b) x = cos (π/6 - t)

(c) x = 3 sin (2πt + π/4)

(d) x = 2 cos πt

6. Figure 14.30 (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure 14.30 (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. 14.30(b) is stretched by the same force F.
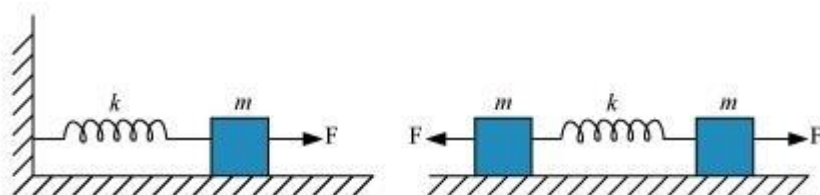
(a) What is the maximum extension of the spring in the two cases?

(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?

7. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

8. An air chamber of volume V has a neck area of cross section a into which a ball of mass m just fits and can move up and down without any friction (Fig.14.33). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal seeFig.14.33.
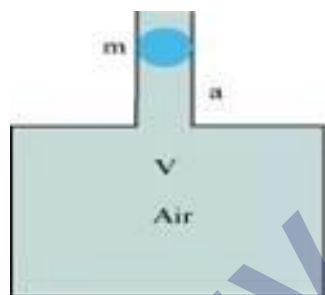
Long Answers-

1. Ans.A simple pendulum is the most common example of the body executing S.H.M, it consist of heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support, which is free to oscillate.

Let m = mass of bob

l = length of pendulum

Let O is the equilibrium position, OP = X

Let θ = small angle through which the bob is displaced.

The forces acting on the bob are:-

1) The weight = M g acting vertically downwards.

2) The tension = T in string acting along Ps.

Resolving Mg into 2 components as Mg Cos θ and Mg Sin θ,

Now, T = Mg Cos θ

Restoring force F = - Mg Sin θ

- ve sign shows force is directed towards mean position.

Let θ = Small, so Sin θ ≈ θ = $\dfrac{Arc(op)}{1} = \dfrac{x}{1}$

Hence F = - mg θ

F = - mg $\dfrac{x}{l}$ →3)

Now, In S.H.M, F = k x →4) k = Spring constant

Equating equation3) & 4) for F

- k x = - m g $\dfrac{x}{l}$

Spring factor = k = $\dfrac{mg}{l}$

Inertia factor = Mass of bob = m

Now, Time period = T

= $2\pi \sqrt{\dfrac{Inertia\ factor}{Spring\ factor}}$

= $2\pi \sqrt{\dfrac{m}{mg/l}}$

$T = 2\pi \sqrt{\dfrac{l}{g}}$

---

2. Ans.(a) Zero, Positive, Positive

(b) Zero, Negative, Negative
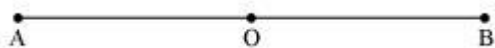
(c) Negative, Zero, Zero

(d) Negative, Negative, Negative

(e) Zero, Positive, Positive

(f) Negative, Negative, Negative

Explanation:
The given situation is shown in the following figure. Points A and B are the two end points, with AB = 10 cm. O is the midpoint of the path.

A particle is in linear simple harmonic motion between the end points

(a) At the extreme point A, the particle is at rest momentarily. Hence, its velocity is zero at this point.

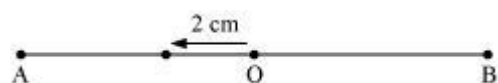Its acceleration is positive as it is directed along AO.

Force is also positive in this case as the particle is directed rightward.

(b) At the extreme point B, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is negative as it is directed along B.

Force is also negative in this case as the particle is directed leftward.

(c)

The particle is executing a simple harmonic motion. O is the mean position of the particle. Its velocity at the mean position O is the maximum. The value for velocity is negative as the particle is directed leftward. The acceleration and force of a particle executing SHM is zero at the mean position.

(d)

The particle is moving toward point O from the end B. This direction of motion is opposite to the conventional positive direction, which is from A to B. Hence, the particle"s velocity and acceleration, and the force on it are all negative.

(e)
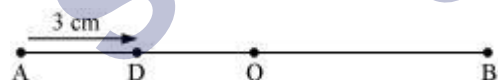
The particle is moving toward point O from the end A. This direction of motion is from A to B, which is the conventional positive direction. Hence, the values for velocity, acceleration, and force are all positive.

(f)

This case is similar to the one given in **(d)**.

3. Ans. Initially, at $t = 0$:

Displacement, *x* = 1 cm

Initial velocity, *v* = ω cm/sec.

Angular frequency, ω = π rad/s^{-1}

It is given that:

$$x(x) = A\cos(\omega t + \phi)$$

$$1 = A\cos(\omega \times 0 + \phi) = A\cos\phi$$

$$A\cos\phi = 1 \quad \text{.....(i)}$$

Velocity, $v = \dfrac{dx}{dt}$

$$\omega = -A\omega \sin(\omega t + \phi)$$

$$1 = A\sin(\omega \times 0 + \phi) = A\sin\phi$$

$$A\sin\phi = -1 \quad \text{........(ii)}$$

Squaring and adding equations (*i*) and (*ii*), we get:

$$A^2\left(\sin^2\phi + \cos^2\phi\right) = 1 + 1$$

$$A^2 = 2$$

$$\therefore A = \sqrt{2} \, cm$$

Dividing equation (*ii*) by equation (*i*), we get:

$$\tan\phi = -1$$

$$\therefore \phi = \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$$

SHM is given as:

$$x = B\sin(\omega t + a)$$

Putting the given values in this equation, we get:

$$1 = B\sin[\omega \times 0 + a]$$

$$B\sin a = 1 \quad \text{....(iii)}$$

Velocity, $v = \omega B\cos(\omega t + a)$

Substituting the given values, we get:

$$\pi = \pi B\sin a$$

$$B\sin a = 1 \quad \text{........(iv)}$$

Squaring and adding equations (*iii*) and (*iv*), we get:

$$B^2 \left[\sin^2 a + \cos^2 a \right] = 1 + 1$$

$$B^2 = 2$$

$$\therefore B = \sqrt{2} \, cm$$

Dividing equation (*iii*) by equation (*iv*), we get:

$$\frac{B \sin a}{B \cos a} = \frac{1}{1}$$

$$\tan a = 1 = \tan \frac{\pi}{4}$$

$$a \, \frac{\pi}{4}, \frac{5\pi}{4}, \ldots \ldots$$

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4. Ans. (a) $x = 2\sin 20t$

(b) $x = 2\cos 20t$

(c) $x = -2\cos 20t$

The functions have the same frequency and amplitude, but different initial phases.

Distance travelled by the mass sideways, $A = 2.0$ cm

Force constant of the spring, $k = 1200$ N $\mathbf{m^{-1}}$

Mass, $m = 3$ kg

Angular frequency of oscillation:

$$\omega = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{1200}{3}} = \sqrt{400} = 20 \, rad \, s^{-1}$$

(a) When the mass is at the mean position, initial phase is 0.

Displacement, $x = A\sin \omega t$

$= 2\sin 20t$

(b) At the maximum stretched position, the mass is toward the extreme right. Hence, the initial phase is $\frac{\pi}{2}$.

Displacement, $x = A\sin \left(\omega t + \frac{\pi}{2} \right)$

$$= 2\sin\left(20t + \frac{\pi}{2}\right)$$

= 2cos 20*t*

(c) At the maximum compressed position, the mass is toward the extreme left. Hence, the initial phase is $\frac{3\pi}{2}$.

Displacement, $x = A\sin\left(\omega t + \frac{3\pi}{2}\right)$

$$= 2\sin\left(20t + \frac{3\pi}{2}\right)$$

= −2cos 20*t*

The functions have the same frequency $\left(\frac{20}{2\pi} Hz\right)$ and amplitude (2 cm), but different initial phases $\left(0, \frac{\pi}{2}, \frac{3\pi}{2}\right)$.

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5. Ans.(a) $x = -2\sin\left(3t + \frac{\pi}{3}\right) = +2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$

$$= 2\cos\left(3t + \frac{5\pi}{6}\right)$$

If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 2 cm

Phase angle, $\phi = \frac{5\pi}{6} = 150°$

Angular velocity, $\omega = \frac{2\pi}{T} = 3\,rad\,/\,sec.$

The motion of the particle can be plotted as shown in the following figure.

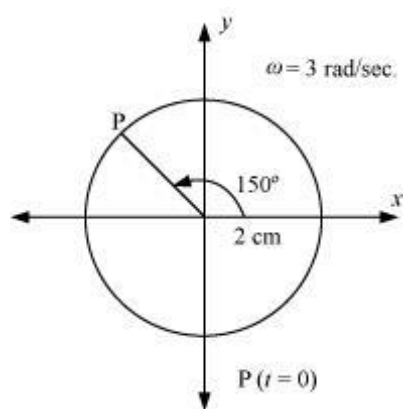**(b)** $x = \cos\left(\dfrac{\pi}{6} - t\right) = \cos\left(t - \dfrac{\pi}{6}\right)$

If this equation is compared with the standard SHM equation $x = A\cos\left(\dfrac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A=2

Phase angle, $\phi = \dfrac{\pi}{6} = 30°$

Angular velocity, $\omega = \dfrac{2\pi}{T} = 1\,rad\,/\,sec.$

The motion of the particle can be plotted as shown in the following figure.

(c) $x = 3\sin\left(2\pi t + \dfrac{\pi}{4}\right)$

$= -3\cos\left[\left(2\pi t + \dfrac{\pi}{4}\right) + \dfrac{\pi}{2}\right] = -3\cos\left(2\pi t + \dfrac{3\pi}{4}\right)$
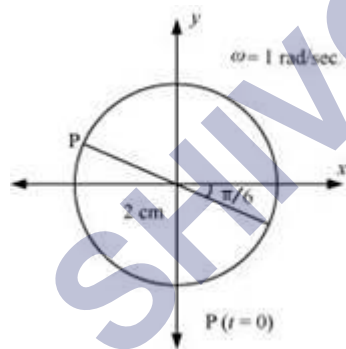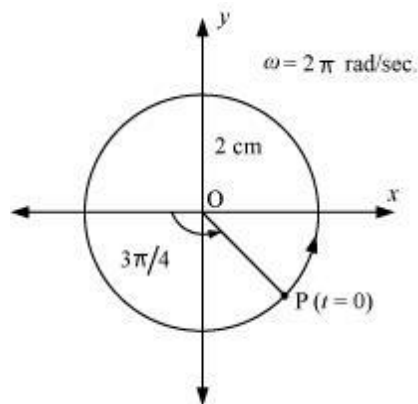
If this equation is compared with the standard SHM equation $x = A\cos\left(\dfrac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, $A = 3$ cm

Phase angle, $\phi = \dfrac{3\pi}{4} = 135°$

Angular velocity, $\omega = \dfrac{2\pi}{T} = 2\pi\,rad\,/\,sec.$

The motion of the particle can be plotted as shown in the following figure.

(d) $x = 2\cos\pi t$

If this equation is compared with the standard SHM equation $x = A\cos\left(\dfrac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 2 cm

Phase angle, $\phi = 0$

Angular velocity, $\omega = \pi$ rad/s

The motion of the particle can be plotted as shown in the following figure.

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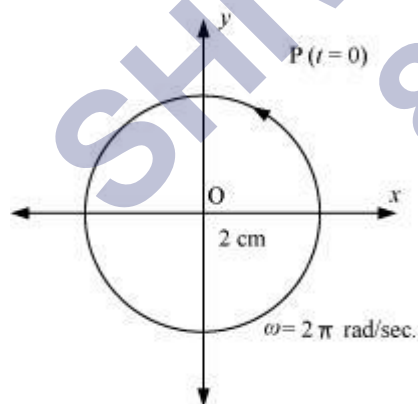
6. Ans.(a) For the one block system:

When a force F, is applied to the free end of the spring, an extension l, is produced. For the maximum extension, it can be written as:

$F = kl$

Where, k is the spring constant

Hence, the maximum extension produced in the spring, $l = \dfrac{F}{k}$

For the two block system:

The displacement (x) produced in this case is:

$$x = \dfrac{1}{2}$$

Net force, $F = +2\,kx \quad 2k\dfrac{1}{2}$

$$\therefore l = \dfrac{F}{k}$$

(b) For the one block system:

For mass (m) of the block, force is written as:

$$F = ma = m\dfrac{d^2x}{dt^2}$$

Where, x is the displacement of the block in time t

$$\therefore m\dfrac{d^2x}{dt^2} = -kx$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\dfrac{d^2x}{dt^2} = -\left(\dfrac{k}{m}\right)x = -\omega^2 x$$

Where, $\omega^2 = \dfrac{k}{m}$

$$\omega = \sqrt{\dfrac{k}{m}}$$

Where,

ω is angular frequency of the oscillation

$\therefore$Time period of the oscillation, $T = \dfrac{2\pi}{\omega}$

$$= \dfrac{2\pi}{\sqrt{\dfrac{k}{m}}} = 2\pi\sqrt{\dfrac{m}{k}}$$

For the two block system:

$$F = m\dfrac{d^2x}{dr^2}$$

$$m\frac{d^2x}{dr^2} = -2kr$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\frac{d^2x}{dr^2} = -\left[\frac{2k}{m}\right]x = -\omega^2 x$$

Where,

Angular frequency, $\omega = \sqrt{\frac{2k}{m}}$

∴Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$

---

7. Ans. Area of cross-section of the U-tube = A

Density of the mercury column = ρ

Acceleration due to gravity = g

Restoring force, F = Weight of the mercury column of a certain height

F = –(Volume × Density × g)

F = –(A × $2h$ × ρ × g) = –2ρgh = –k × Displacement in one of the arms (h)

Where,

$2h$ is the height of the mercury column in the two arms

k is a constant, given by $k = -\frac{F}{h} = 2A\rho g$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2A\rho g}}$

Where,

m is the mass of the mercury column

Let l be the length of the total mercury in the U-tube.

Mass of mercury, m = Volume of mercury × Density of mercury

= $Al\rho$

∴ $T = 2\pi\sqrt{\frac{m}{2A\rho g}} = 2\pi\sqrt{\frac{l}{2g}}$

Hence, the mercury column executes simple harmonic motion with time period $2\pi\sqrt{\dfrac{l}{2g}}$.

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8. Ans. Volume of the air chamber = V

Area of cross-section of the neck = a

Mass of the ball = m

The pressure inside the chamber is equal to the atmospheric pressure.

Let the ball be depressed by x units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber.

Decrease in the volume of the air chamber, $\Delta V = ax$

Volumetric strain $= \dfrac{Change\ in\ volume}{Original\ valume}$

$\Rightarrow \dfrac{\Delta V}{V} = \dfrac{ax}{V}$

Bulk Modulus of air, $B = \dfrac{Stress}{Strain} = \dfrac{-p}{\dfrac{ax}{V}}$

In this case, stress is the increase in pressure. The negative sign indicates that pressure increases with a decrease in volume.

$p = \dfrac{-Bax}{V}$

The restoring force acting on the ball, $F = p \times a$

$\dfrac{-Bax}{V} \cdot a$

$= \dfrac{-Ba^2x}{V}$

In simple harmonic motion, the equation for restoring force is:

$F = -kx \ldots (ii)$

Where, k is the spring constant

Comparing equations (i) and (ii), we get:

$= \dfrac{Ba^2}{V}$

Time period, $T = 2\pi\sqrt{\dfrac{m}{k}}$

$$= 2\pi \sqrt{\frac{Vm}{Ba^2}}$$

Assertion Reason Answer:

1. (a) If both assertion and reason are true but reason is not the correct explanation of the assertion.

2. (d) If the assertion and reason both are false.

Case Study Questions-

1. A motion that repeats itself at regular intervals of time is called periodic motion. Very often, the body undergoing periodic motion has an equilibrium position somewhere inside its path. When the body is at this position no net external force acts on it. Therefore, if it is left there at rest, it remains there forever. If the body is given a small displacement from the position, a force comes into play which tries to bring the body periodic motion need not be oscillatory. Circular motion is a periodic motion, but it is not oscillatory. The smallest interval of time after which the motion is repeated is called its period. Let us denote the period by the symbol T. Its SI unit is second. The reciprocal of T gives the number of repetitions that occur per unit time. This quantity is called the frequency of the periodic motion. It is represented by the symbol n. The waves, Heinrich Rudolph Hertz (1857–1894), a special name has been given to the unit of frequency. It is called hertz (abbreviated as Hz). Answer the following.a)

    i. Every oscillatory motion is periodic motion true or false?

        a. True

        b. False

    ii. Circular motion is

        a. Oscillatory motion

        b. Periodic motion

        c. Rotational motion

        d. None of these

    iii. Define period. Give its SI unit and dimensions

    iv. Define frequency of periodic motion. How it is related to time period

    v. What is oscillatory motion

2. When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω, and the oscillations are called free oscillations. All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations. These are called forced or driven oscillations. We consider the case when the

external force is itself fact of forced periodic oscillations is that the system oscillates not with its natural frequency ω, but at familiar example of forced oscillation is when a child in a garden swing periodically presses his feet against the ground (or someone else periodically gives the child a push) to maintain the oscillations. The maximum possible amplitude for a given driving frequency is governed by the driving frequency and the damping, and is never infinity. The phenomenon of increase in amplitude when the driving force is close to the natural frequency of the oscillator is experience with swings is a good example of resonance. You might have realized that the skill in swinging to greater heights lies in the synchronization of the rhythm of pushing against the ground with the natural frequency of the swing.

i. When a system oscillates with its natural frequency ω, and the oscillations are called

   a. Free oscillations

   b. Forced oscillations

ii. All free oscillations eventually die out because of

   a. Damping force

   b. electromagnetic force

   c. None of these

iii. What is free oscillation?

iv. What is forced oscillations?

v. What is resonance?

Case Study Answer-

1. **Answer**

   i. (a) True

   ii. (b) Periodic motion

   iii. The smallest interval of time after which the motion is repeated is called its period. Its SI unit is second and dimensions are [T1].

   iv. Reciprocal of Time period (T) gives the number of repetitions that occur per unit time. This quantity is called the frequency of the periodic motion. It is represented by the symbol n. The relation between n and T is n = 1/T i.e. they are inversely proportional to each other. The unit of n is thus s-1 or hertz.

   v. Oscillatory motion is type of periodic motion in which body performs periodic to and fro motion about some mean position. Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.

2. **Answer**

   i. (a) Free oscillations

ii. (b) Damping force

iii. When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω, and the oscillations are called free oscillations.

iv. Forced oscillations are oscillations where external force drives the oscillations with frequency given by external force.

v. The phenomenon of increase in amplitude when the driving force is close to encounter phenomena which involve resonance. Your experience with swings is a good example of resonance. You might have realized that the skill in swinging to greater heights lies in the synchronization of the rhythm of pushing against the ground with the natural frequency of the swing.