MATHEMATICS

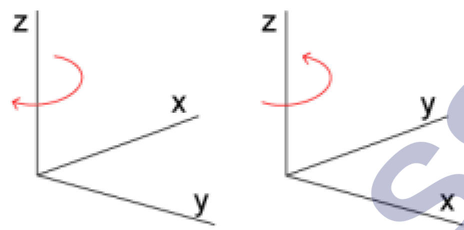
Chapter 12: Introduction to Three Dimensional Geometry

Introduction To Three Dimensional Geometry

Key Concepts

1. A point in space has three coordinates.

2. A three-dimensional system is an extension of the two-dimensional system. A third axis z is added to the XY plane. There are two possible orientations of the X- and Y-axis.These two orientations are known as the left-handed- and right-handed system.

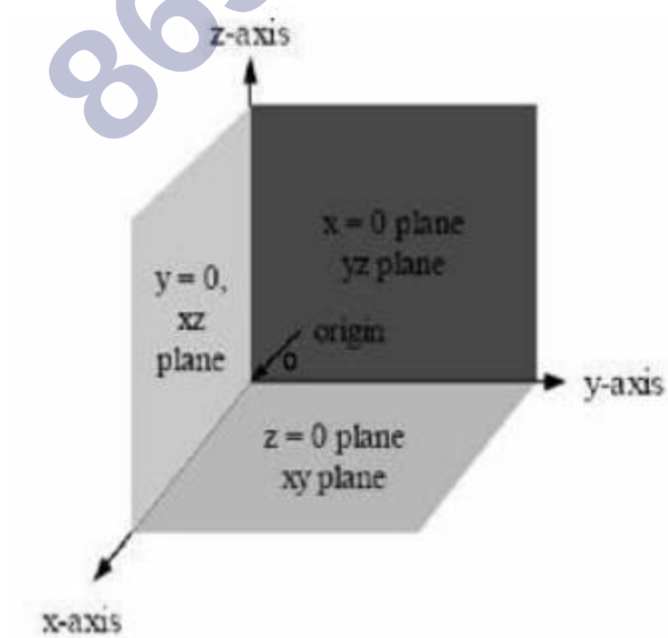


Right-handed system is mostly used.

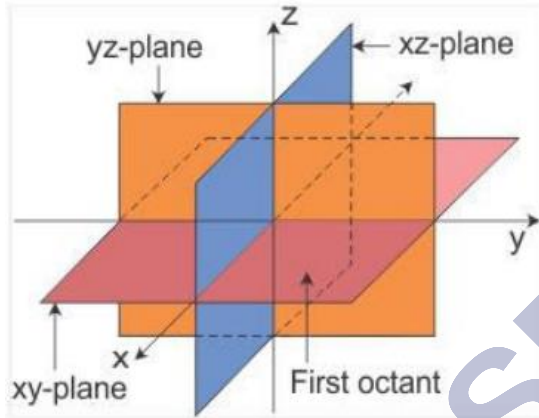
3. In three dimensions, the coordinate axes of a rectangular cartesian coordinate system are threemutually perpendicular lines. The axes are called the X-, Y- and Z-axis, respectively.

4. The three planes determined by the pair of axes are the coordinate planes called XY-, YZ- and ZX-planes.

5. There are 3 coordinate planes namely XOY, YOZ and ZOX also called the XY-, YZ- and ZX–planes,respectively.

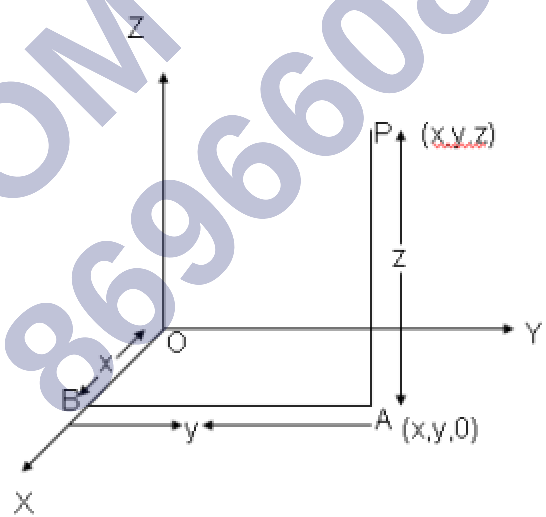


6. The three coordinate planes divide the whole space into 8 parts. Each of these parts is called an‘octant’. The octants are numbered as roman numerals I, II, III etc.



7. To each point in space, there corresponds an ordered triplet (x, y, z) of real numbers. There is a one toone correspondence between the points in space and ordered triplet (x, y, z) of real numbers.

8. If P(x, y, z) is any point in space, then x, y and z are perpendicular distances from YZ-, ZX- and XY–planes, respectively.



9. The coordinates of the origin O are (0, 0, 0).

10. The coordinates of any point on the X-axis are of the type (x, 0, 0)

The coordinates of any point on the Y-axis are of the type (0, y, 0).

The coordinates of any point on the Z-axis are of the type (0, 0, z).

11. The x coordinate of the point in the YZ-plane must be zero.

A point in the XY-plane will have its z coordinate zero.

A point in the XZ-plane will have its y coordinate zero.

12. Three points are said to be collinear if the sum of distances between any two pairs of the points isequal to the distance between the third pair of points. The distance formula can be used to prove collinearity.

13. If we were dealing in one dimension, then x = a is a single point, and if it is in two dimensions, then itwill be a straight line and in 3D it is a plane || to YZ-plane and passing through point a.

14. The distance of any point from the XY-plane = | z coordinate | and is similarly obtained for the other 2planes.

15. When a line segment is trisected, it means it is divided into three equal parts by two points R and S.This is equivalent to saying that either R or S divides the line segment in the ratio 2:1 or 1:2.

Key Formulae

1.

Octants Coordinates	OXYZ	OX'YZ	OXY'Z	OX'Y'Z	OXYZ'	OX'YZ'	OXY'Z'	OX'Y'Z'
x	+	−	+	−	+	−	+	−
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

2. Distance between two points P(x_1, y_1,z_1) and Q (x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

3. Distance between two points P(x_1, y_1, z_1) and Q (0, 0, 0) is given by $PQ = \sqrt{x_1^2 + y_1^2 + z_1^2}$.

4. The coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2,y_2, z_2) internally and externally in the ratio m:n are given by

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + na_1}{m+n}\right) \text{and} \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

5. The coordinates of the mid-point of the line segment joining two points P(x_1, y_1, z_1) and

$Q(x_2, y_2, z_2)$ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

6. The coordinates of the centroid of the triangle, whose vertices are

   (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$.

7. The coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) in the ratio k:1 are

   $$\left(\frac{kx_2 + x_1}{1 + k}, \frac{ky_2 + y_1}{1 + k}, \frac{kz_2 + z_1}{1 + k}\right).$$

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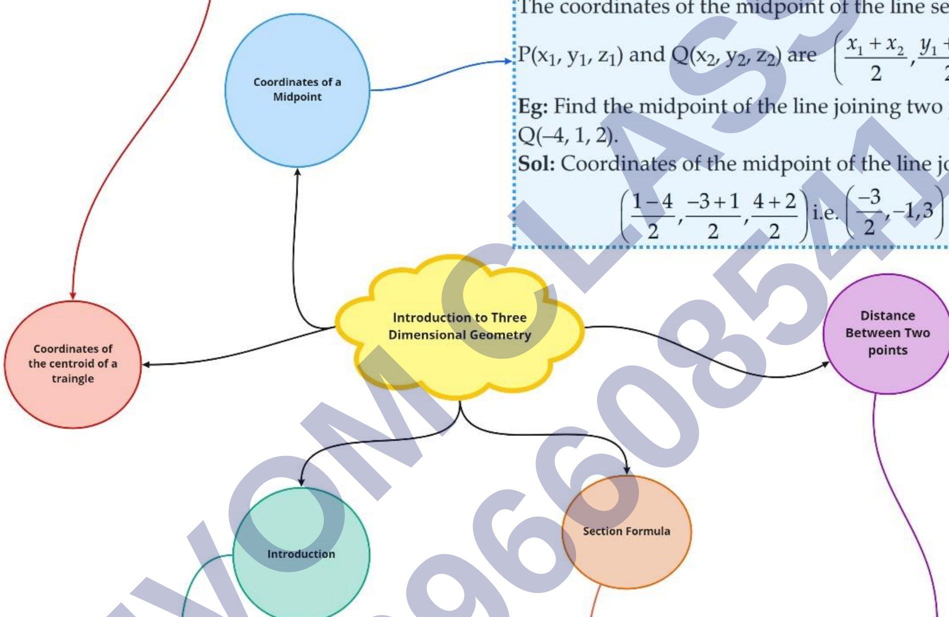
The coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

Eg: The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, –5, 7) and (–1, 7, –6), respectively, find the coordinates of the point C.

Sol: Let the coordinates of C be (x, y, z) and the coordinates of the centroid G be (1, 1, 1). Then $\frac{x+3-1}{3}=1$, i.e., $x=1$;

$\frac{y-5+7}{3} = 1$, i.e., $y=1$;

$\frac{z+7-6}{3}=1$, i.e., $z=2$. So, $C(x,y,z) = (1,1,2)$



The coordinates of the midpoint of the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

Eg: Find the midpoint of the line joining two points P(1, –3, 4) and Q(–4, 1, 2).

Sol: Coordinates of the midpoint of the line joining the points P & Q are

$$\left(\frac{1-4}{2}, \frac{-3+1}{2}, \frac{4+2}{2}\right) \text{ i.e. } \left(\frac{-3}{2}, -1, 3\right)$$

- In three dimensions, the coordinate axes of a rectangular cartesian coordinate system are three mutually perpendicular lines. The axes are called x, y and z axes.
- The three planes determined by the pair of axes are the coordinate planes, called xy, yz and zx-planes.
- The three coordinate planes divide the space into eight parts known as octants.
- The coordinates of a point P in 3D Geometry is always written in the form of triplet like (x,y,z). Here, x, y and z are the distances from yz, zx and *yx* planes, respectively.

Eg:
- Any point on x-axis is : (x, 0, 0)
- Any point on y-axis is : (0, y, 0)
- Any point on z-axis is : (0, 0, z)

Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Eg: Find the distance between the points P(1, –3, 4) and (–4, 1, 2).

Sol: The distance PQ between the points P & Q is given by

$$PQ = \sqrt{(-4-1)^2+(1+3)^2+(2-4)^2}$$
$$= \sqrt{25+16+4} = \sqrt{45} = 3\sqrt{5} \text{ units}$$

The coordinates of the point R which divides the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally and externally in the ratio m : n are given by

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right) \ \& \ \left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$$

respectively.

Eg: Find the coordinates of the point which divides the line segment joining the points (1,–2, 3)and (3, 4, –5) in the ratio 2:3 internally.

Sol : Let P(x, y, z) be the point which divides line segment joining A (1,–2, 3) and B (3, 4, –5) internally in the ratio 2:3. Therefore,

$$x = \frac{2(3)+3(1)}{2+3} = \frac{9}{5} \quad y = \frac{2(4)+3(-2)}{2+3} = \frac{2}{5} \quad z = \frac{2(-5)+3(3)}{2+3} = \frac{-1}{5}$$

Thus, the required point is $\left(\frac{9}{5}, \frac{2}{5}, \frac{-1}{5}\right)$.

Important Questions

Multiple Choice questions-

Question 1. The projections of a directed line segment on the coordinate axes are 12, 4, 3. The DCS of the line are:

(a) 12/13, -4/13, 3/13

(b) -12/13, -4/13, 3/13

(c) 12/13, 4/13, 3/13

(d) None of these

Question 2. The angle between the planes $r . n_1 = d_1$ and $r . n_1 = d_2$ is:

(a) $\cos\theta = \{|n_1| \times |n_2|\}/ (n_1 . n_2)$

(b) $\cos\theta = (n_1 . n_2)/\{|n_1| \times |n_2|\}^2$

(c) $\cos\theta = (n_1 . n_2)/\{|n_1| \times |n_2|\}$

(d) $\cos\theta = (n_1 . n_2)^2 /\{|n_1| \times |n_2|\}$

Question 3. For every point P(x, y, z) on the xy-plane

(a) x = 0

(b) y = 0

(c) z = 0

(d) None of these

Question 4. The locus of a point P(x, y, z) which moves in such a way that x = a and y = b, is a.

(a) Plane parallel to xy-plane

(b) Line parallel to x-axis

(c) Line parallel to y-axis

(d) Line parallel to z-axis

Question 5. The equation of the plane containing the line 2x – 5y + z = 3, x + y + 4z = 5 and parallel to the plane x + 3y + 6z = 1 is

(a) x + 3y + 6z + 7 = 0

(b) x + 3y – 6z – 7 = 0

(c) x – 3y + 6z – 7 = 0

(d) x + 3y + 6z – 7 = 0

Question 6. The coordinate of foot of perpendicular drawn from the point A(1, 0, 3) to the join of the point B(4, 7, 1) and C(3, 5, 3) are

(a) (5/3, 7/3, 17/3)

(b) (5, 7, 17)

(c) (5/3, -7/3, 17/3)

(d) (5/7, -7/3, -17/3)

Question 7. The coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ plane is

(a) (0, 17/2, 13/2)

(b) (0, -17/2, -13/2)

(c) (0, 17/2, -13/2)

(d) None of these

Question 8. If P is a point in space such that OP = 12 and OP inclined at angles 45 and 60 degrees with OX and OY respectively, then the position vector of P is

(a) $6i + 6j \pm 6\sqrt{2}k$

(b) $6i + 6\sqrt{2}j \pm 6k$

(c) $6\sqrt{2}i + 6j \pm 6k$

(d) None of these

Question 9. The image of the point P(1,3,4) in the plane $2x - y + z = 0$ is

(a) (-3, 5, 2)

(b) (3, 5, 2)

(c) (3, -5, 2)

(d) (3, 5, -2)

Question 10. There is one and only one sphere through

(a) 4 points not in the same plane

(b) 4 points not lie in the same straight line

(c) none of these

(d) 3 points not lie in the same line

Very Short Questions:

1. Name the octants in which the following lie. (5,2,3)

2. Name the octants in which the following lie. (-5,4,3)

3. Find the image of (-2,3,4) in the y z plane.
4. Find the image of (5,2,-7) in the plane xy.
5. A point lie on X –axis what are co ordinate of the point
6. Write the name of plane in which x axis and y- axis taken together.
7. The point (4, -3, 6) lie in which octants.
8. The point (2, 0, 8) lie in which palne.
9. A point is in the XZ plane. What is the value of y co-ordinates?
10. What is the coordinates of XY plane?

Short Questions:

1. Given that P(3,2,-4), Q(5,4,-6) and R(9,8,-10) are collinear. Find the ratio in which Q divides PR.
2. Determine the points in xy plane which is equidistant from these point A (2,0,3) B(0,3,2) and C(0,0,1).
3. Find the locus of the point which is equidistant from the point A(0,2,3) and B(2,-2, 1)
4. Show that the points A(0,1,2) B(2,-1,3) and C(1,-3,1) are vertices of an isosceles right angled triangle.
5. Using section formula, prove that the three points A(-2,3,5), B(1,2,3), and C(7,0,-1) are collinear.

Long Questions:

1. Prove that the lines joining the vertices of a tetrahedron to the centroids of the opposite faces are concurrent.
2. The mid points of the sides of a triangle are (1,5,-1), (0,4,-2) and (2,3,4). Find its vertices.
3. Let P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) be two points in space find coordinate of point R which divides P and Q in the ratio $m_1 : m_2$ by geometrically.
4. **Show that the plane ax + by + cz + d = 0 divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratios** $\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}$.
5. Prove that the points 0(0, 0, 0), A(2, 0, 0), B(1, $\sqrt{3}$, 0), and c $\left(1, \frac{1}{\sqrt{3}}, \frac{2\sqrt{2}}{\sqrt{3}}\right)$ are the vertices of a regular tetrahedron.

Answer Key:

MCQ:

1. (c) 12/13, 4/13, 3/13
2. (c) $\cos\theta = (n_1 . n_2)/\{|n_1| \times |n_2|\}$
3. (c) z = 0
4. (b) Line parallel to x-axis
5. (d) x + 3y + 6z – 7 = 0
6. (a) (5/3, 7/3, 17/3)
7. (c) (0, 17/2, -13/2)
8. (c) 6√2i + 6j ± 6k
9. (a) (-3, 5, 2)
10. (a) 4 points not in the same plane

Very Short Answer:

1. I
2. II
3. (2, 3, 4)
4. (5, 2, 7)
5. (a, 0, 0)
6. XY Plane
7. VIII
8. XZ
9. Zero
10. (x, y, 0)

Short Answer:

1. Suppose Q divides PR in the ratio λ:1. Then coordinator of Q are.

$$\left(\frac{9\lambda+3}{\lambda+1}, \frac{8\lambda+2}{\lambda+1}, \frac{-10\lambda-4}{\lambda+1}\right)$$

But, coordinates of Q are (5,4,-6). Therefore

$$\frac{9\lambda+3}{\lambda+1}=5, \frac{8\lambda+2}{\lambda+1}=4, \frac{-10\lambda-4}{\lambda+1}=6$$

These three equations give

$= \frac{1}{2}$

So Q divides PR in the ratio $\frac{1}{2}$: 1 or 1:2

2. We know that Z- coordinate of every point on xy-plane is zero. So, let P(x, y, 0) be a point in xy-plane such that PA = PB = PC

Now, PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-2)^2+(y-0)^2+(0-3)^2=(x-0)^2+(y-3)^2+(0-2)^2$

$\Rightarrow 4x-6y=0$ or $2x-3y=0$.......(i)

$PB = PC$

$\Rightarrow PB^2 = PC^2$

$\Rightarrow (x-0)^2+(y-3)^2+(0-2)^2=(x-0)^2+(y-0)^2+(0-1)^2$

$\Rightarrow -6y+12=0 \Rightarrow y=2$.........(ii)

Putting y = 2 in (i) we obtain x = 3

Hence the required points (3,2,0).

3. Let P(x, y, z) be any point which is equidistant from A(0,2,3) and B(2,-2,1). Then

PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow \sqrt{(x-0)^2+(y-2)^2+(2-3)^2}=\sqrt{(x-2)^2+(y+2)^2+(z-1)^2}$

$\Rightarrow 4x-8y-42+4=0$ or $x-2y-2+1=0$

4. We have

$AB=\sqrt{(2-0)^2+(-1-1)^2(+3-2)^2}=\sqrt{4+4+1}=3$

$BC=\sqrt{(1-2)^2+(-3+1)^2+(1-3)^2}=\sqrt{1+4+4}=3$

And $CA=\sqrt{(1-0)^2+(-3-1)^2+(1-2)^2}=\sqrt{1+16+1}=3\sqrt{2}$

Clearly AB = BC and $AB^2 + BC^2 = AC^2$

Hence, triangle ABC is an isosceles right angled triangle.

5. Suppose the given points are collinear and C divides AB in the ratio λ : 1.

Then coordinates of C are

$$\left(\frac{\lambda-2}{\lambda+1}, \frac{2\lambda+3}{\lambda+1}, \frac{3\lambda+5}{\lambda+1}\right)$$

But, coordinates of C are (3,0,-1) from each of there equations, we get $\lambda = \frac{3}{2}$

Since each of there equation give the same value of V. therefore, the given points are collinear and C divides AB externally in the ratio 3:2.

Long Answer:

1. Let ABCD be tetrahedron such that the coordinates of its vertices are A (x_1, y_1, z_1), B(x_2, y_2, z_2) C (x_3, y_3, z_3) and D (x_4, y_4, z_4)

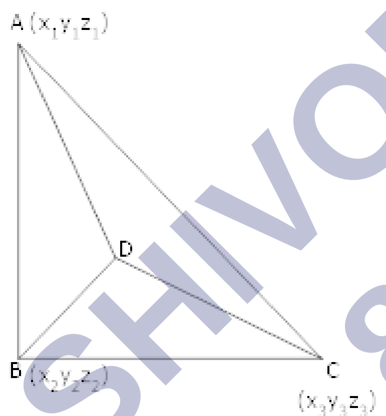
The coordinates of the centroids of faces ABC, DAB, DBC and DCA respectively

$$G_1\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right]$$

$$G_2\left[\frac{x_1+x_2+x_4}{3}, \frac{y_1+y_2+y_4}{3}, \frac{z_1+z_2+z_4}{3}\right]$$

$$G_3\left[\frac{x_2+x_3+x_4}{3}, \frac{y_2+y_3+y_4}{3}, \frac{z_2+z_3+z_4}{3}\right]$$

$$G_4\left[\frac{x_4+x_3+x_1}{3}, \frac{y_4+y_3+y_1}{3}, \frac{z_4+z_3+z_1}{3}\right]$$



Now, coordinates of point G dividing DG1 in the ratio 3:1 are

$$\left[\frac{1.x_4+3\left(\frac{x_1+x_2+x_3}{3}\right)}{1+3}, \frac{1.y_4+3\left(\frac{y_1+y_2+y_3}{3}\right)}{1+3}, \frac{1.z_4+3\left(\frac{z_1+z_2+z_3}{3}\right)}{1+3}\right]$$

$$=\left[\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right]$$

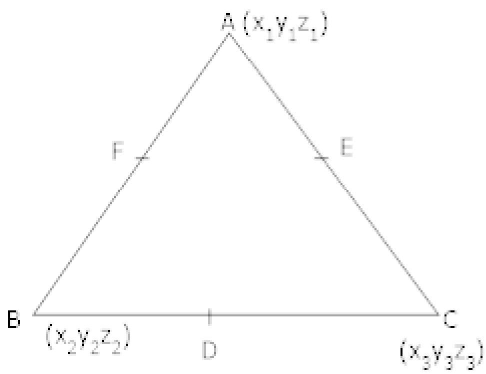
Similarly the point dividing CG2, AG3 and BG4 in the ratio 3:1 has the same coordinates.

Hence the point $G\left[\frac{x_1+x_2+x_3+x_4}{4},\frac{y_1+y_2+y_3+y_4}{4},\frac{z_1+z_2+z_3+z_4}{4}\right]$ is common to DG1, CG2, AG3 and BG4.

Hence they are concurrent.

2. Suppose vertices of ΔABC are A (x_1, y_1, z_1), B(x_2, y_2, z_2) and C (x_3, y_3, z_3) respectively

Given coordinates of mid point of side BC, CA, and AB respectively are D(1,5,-1), E(0,4,-2) and F(2,3,4)



$\therefore \frac{x_2+x_3}{2}=1 \quad \frac{y_2+y_3}{2}=5 \quad \frac{z_2+z_3}{2}=-1$

$x_2+x_3=2......(i)$

$\frac{x_1+x_3}{2}=0$

$y_2+y_3=10......(ii)$

$\frac{y_1+y_3}{2}=4$

$z_2+z_3=2......(iii)$

$\frac{z_1+z_3}{2}=-2$

$x_1+x_3=0.......(iv)$

$\frac{x_1+x_2}{2}=2$

$y_1+y_2=8.......(v)$

$\frac{y_1+y_2}{2}=3$

$z_1+z_3=-4.......(vi)$

$\frac{z_1+z_2}{2}=4$

$x_1+x_2=4.......(vii)$

$y_1 + y_2 = 6 \ldots\ldots (viii)$

$z_1 + z_2 = 8 \ldots\ldots (ix)$

Adding eq. (i), (iv), (viii)

$2(x_1 + x_2 + x_3) = 6$

$x_1 + x_2 + x_3 = 3 \ldots\ldots (x)$

Subtracting eq. (i), (iv), (vii) from (x) we get

$x_1 = 1,\ \ x_2 = 3,\ \ x_3 = -1$

Similarly, adding eq. (ii), (v) and (viii)

$y_1 + y_2 + y_3 = 12 \ldots\ldots (xi)$

Subtracting eq. (ii), (v) and (viii) from (ix)

$y_1 = 2,\ \ y_2 = 4,\ \ y_3 = 6$

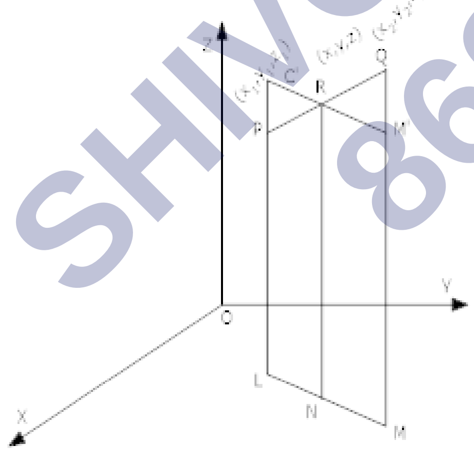
Similarly $z_1 + z_2 + z_3 = 3$

$z_1 = 1,\ \ z_2 = 7,\ \ z_3 = -5$

∴ Coordinates of vertices of ΔABC are A(1,3,-1), B(2,4,6) and C(1,7,-5)

3. Let co-ordinate of Point R be (x, y, z) which divider line segment joining the point PQ in the ratio $m_1 : m_2$

Clearly $\Delta PRL' \sim \Delta QRM'$ $[By\ AA\ sinilsrity]$



$\therefore \dfrac{PL'}{MQ'} = \dfrac{PR}{RQ}$

$\Rightarrow \dfrac{LL' - LP}{MQ - MM'} = \dfrac{m_1}{m_2}$

$\Rightarrow \dfrac{NR - LP}{MQ - NR} = \dfrac{m_1}{m_2}$ $\left[\begin{array}{l}\because LL' = NR \\ \text{and } MM' = NR\end{array}\right]$

$$\Rightarrow \frac{z-z_1}{z_2-z} = \frac{m_1}{m_2}$$

$$\Rightarrow z = \frac{m_1.z_2 + m_2 z_1}{m_1 + m_2}$$

Similarly, $x = \dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$ and

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

4. Suppose the plane ax + by + cz + d = 0 divides the line joining the points (x_1, y_1, z_1) and (x_1, y_2, z_2) in the ratio λ : 1

$$\therefore\ x = \frac{\lambda x_2 + x_1}{\lambda + 1},\quad y = \frac{\lambda y_2 + y_1}{\lambda + 1},\quad z = \frac{\lambda z_2 + z_1}{\lambda + 1}$$

∵ Plane $ax + by + cz + d = 0$ Passing through (x, y, z)

$$\therefore\ Q\frac{(\lambda x_2 + x_1)}{\lambda + 1} + b\frac{(\lambda y_2 + y_1)}{\lambda + 1} + c\frac{(\lambda z_2 + z_1)}{\lambda + 1} + d = 0$$

$$a(\lambda x_2 + x_1) + b(\lambda y_2 + y_1) + c(\lambda z_2 + z_1) + d(\lambda + 1) = 0$$

$$\lambda(ax_2 + by_2 + cz_2 + d) + (ax_1 + by_1 + cz_1 + d) = 0$$

$$\lambda = -\frac{(ax_1 + by_1 + cz_1 + d)}{(ax_2 + by_2 + cz_2 + d)}$$

Hence Proved.

5. To prove O, A, B, C are vertices of regular tetrahedron.

We have to show that

|OA|=|OB|=|OC|=|AB|=|BC|=|CA|

$$|OA| = \sqrt{(0-2)^2 + 0^2 + 0^2} = 2 \text{ unit}$$

$$|OB| = \sqrt{(0-1)^2 + \left(0-\sqrt{3}\right)^2 + 0^2} = \sqrt{1+3} = \sqrt{4} = 2 \text{ unit}$$

$$|OC| = \sqrt{(0-1)^2 + \left(0-\frac{1}{\sqrt{3}}\right) + \left(0-\frac{2\sqrt{2}}{3}\right)^2}$$

$$= \sqrt{1+\frac{1}{3}+\frac{8}{3}}$$

$$= \sqrt{\frac{12}{3}} = \sqrt{4} = 2 \text{ unit}$$

$$|AB| = \sqrt{(2-1)^2 + \left(0-\sqrt{3}\right)^2 + (10-0)^2} = \sqrt{1+3+0}$$

$$= \sqrt{4} = 2 \text{ unit}$$

$$|BC| = \sqrt{(1-1)^2 + \left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)^2 + \left(0-\frac{2\sqrt{2}}{\sqrt{3}}\right)^2}$$

$$= \sqrt{0+\left(\frac{2}{\sqrt{3}}\right)^2+\frac{8}{3}}$$

$$= \sqrt{\frac{12}{3}} = 2 \text{ unit}$$

$$|CA| = \sqrt{(1-2)^2 + \left(\frac{1}{\sqrt{3}}-0\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{3}}-0\right)^2}$$

$$= \sqrt{1+\frac{1}{3}+\frac{8}{3}}$$

$$= \sqrt{\frac{12}{3}} = 2 \text{ unit}$$

∴ |AB| = |BC| = |CA| = |OA| = |OB| = |OC| = 2 unit

∴ O, A, B, C are vertices of a regular tetrahedron.