MATHEMATICS

Chapter 10: STRAIGHT LINES

STRAIGHT LINES

Some Important Results

1. The distance between two points A (x_1, y_1) and B (x_2, y_2) is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. The distance of a point P(x, y) from the origin is given by OP = $\sqrt{x^2, y^2}$.

3. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the coordinates of the vertices of the triangle ABC. Then, the area of the triangle ABC is given by

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

4. If the three points and, then $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear, then

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

5. Let $A(x_1, y_1)$, and $B(x_2, y_2)$ be two points. Then, the coordinates of the point P(x, y) which divides the line segment joining A and B internally in the ratio m:n are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m\ n}\right)$.

6. Let $A(x_1, y_1)$, and $B(x_2, y_2)$ be two points. Then the coordinates of the point P(x, y) that divides the line segment joining A and B externally in the ratio m:n are $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-\ ny_1}{m\ n}\right)$.

7. Let $A(x_1, y_1)$, and $B(x_2, y_2)$ be two points. Then, the coordinates of the mid-point P(x, y) of the segment A and B are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

8. line If the three points $A(x_1, y_1)$, $B(x_2,y_2)$ and $C(x_3, y_3)$ are the vertices of the triangle ABC, then the coordinates of the centroid of the triangle are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

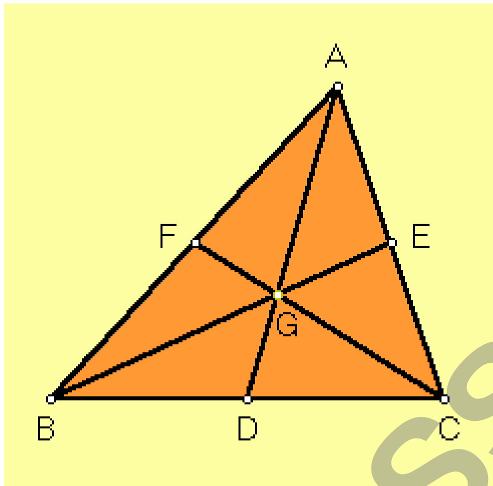
Key concepts

1. For two points on a line parallel to X–axis, the distance between them is just the modulus of thedifference between their x coordinates.

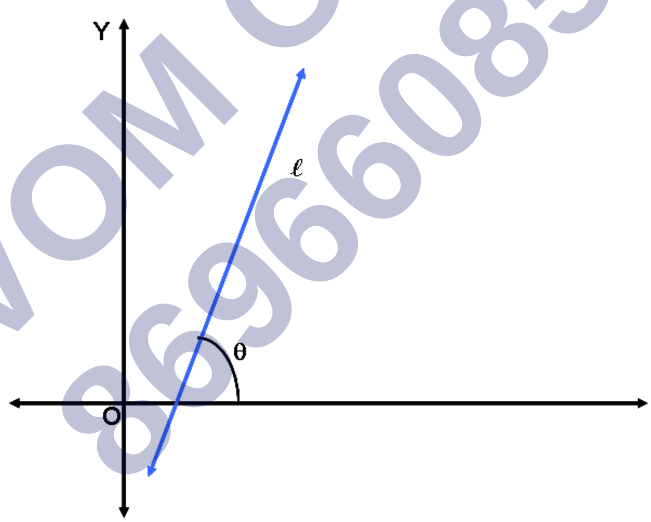
2. For two points, on a line parallel to Y-axis the distance between them is just the modulus of thedifference between their y coordinates.

3. Three points are collinear, i.e., they lie on the same line if the triangle formed by them has zero area.

4. The centroid G divides the medians in the ratio 2:1. A triangle can be divided into 3 triangles of equalarea by its centroid ΔGAB, ΔGBC and ΔGAC are equal in area.



5. The angle (say) θ made by the line ℓ with positive direction of X-axis and measured anticlockwise iscalled the inclination of the line 0° ≤ θ ≤ 180°.



6. A line which is parallel to the X-axis or coinciding with the X-axis has inclination 0°.

7. A line that is parallel to the Y-axis or coinciding with the Y-axis has inclination 90°.

8. The slope of a straight line is a measure indicating its inclination with respect to the positive direction of the X-axis.

9. Consider a line not parallel to the Y-axis. If it makes an angle θ with the X-axis (measured in theanticlockwise direction), then m = tan θ is called the slope of the line.

10. Slope of a line parallel to the Y-axis is not defined.

11. Slope of a line parallel to the X-axis is zero.

12. If the slope is positive, then the angle of inclination θ is an acute angle.

13. If the slope is zero, then the line is X-axis or is parallel to X-axis.

14. If the slope is negative, then the angle of inclination is an obtuse angle.

15. Two lines are parallel, i.e., they never meet, if and only if one of the following conditions holds:

i. They are both vertical lines, i.e., they are parallel to the Y-axis.
ii. If their slopes are equal, i.e., $m_1 = m_2$.

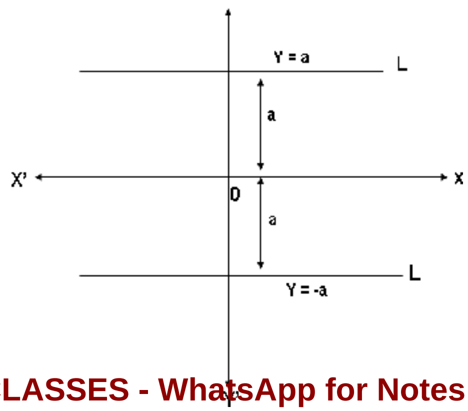
16. Two lines (not parallel to the Y-axis) are perpendicular if and only if their slopes m_1 and m_2 satisfies the condition that $m_1m_2 = -1$. If one of the lines is parallel to the Y-axis, i.e., a vertical line with its slope undefined, then any line parallel to the X-axis, i.e., a horizontal line with slope 0 is perpendicular to it.

Conversely, suppose a pair of lines, where one is horizontal and the other is vertical, then the given lines are perpendicular.

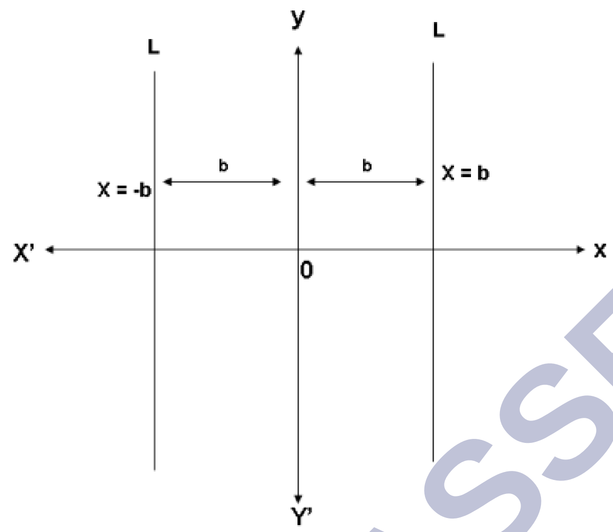
17. If X, Y and Z are three points in the XY plane, then they are collinear if and only if slope of XY is the same as the slope of YZ.

18. If θ is the inclination of a line L, then tanθ is called the slope or gradient of the line L.

19. If a horizontal line L is at a distance a units from the x-axis, then the ordinate of every point lying onthe line is a. Thus, the equation of such a line L is y = a where a is any real number.



20. Equation of a vertical line at a distance b from the Y-axis is x = b. Depending upon whether the line is on the left or right of the Y-axis, the constant b is positive or negative.



21. Various forms of equation of the line

a. Slope intercept form
b. Point slope form
c. Two-point form
d. Intercept form
e. Normal form

22. The general equation of a straight line is Ax + By + C = 0, where A, B and C are constants and A and B are not zero simultaneously.

Case 1: $A \neq 0, B = 0$
In this case, the equation reduces to

$Ax + C = 0$ or $x = -\frac{C}{A}$

which is a straight line parallel to the Y-axis.

Case 2: $A = 0, B \neq 0$ then as in case 1, the straight line is parallel to the X-axis.
By + C = 0

$$y = -\frac{C}{B}$$

Case 3: $A \neq 0, B \neq 0$

In this case, the equation may be written as

$$y = -\frac{A}{B} \times -\frac{C}{B}$$

which is the slope intercept form of a straight line with slope

$-\frac{A}{B}$ and y- intercept $-\frac{C}{B}$

Case 4: If C = 0, then Ax + By + C = 0 becomes

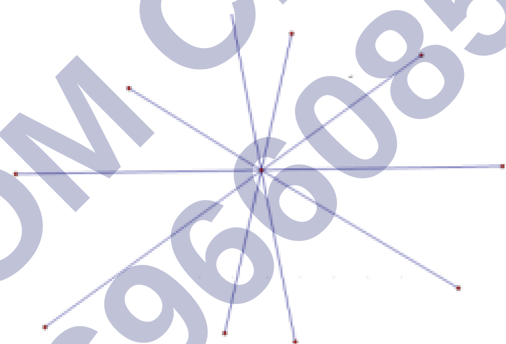
Ax + By = 0.

This is a line passing through the origin and therefore line has zero intercepts on the axes.

23. Equation of the line parallel to the line Ax + By + C = 0 is Ax + By + K = 0, where K is any arbitrary constant.

24. Equation of line perpendicular to the line Ax + By + C = 0 is Bx − Ay + K = 0.

25. Two or more lines are concurrent if they meet at a single point.



26. General equation of line Ax + By + C=0 can be reduced to other forms of line as well

i. Slope intercept form:
If B $\neq$ 0, then Ax + By + C = 0 can be written as
By = −Ax – C
$\Rightarrow y = -\frac{A}{B} \times -\frac{C}{B}$

ii. Intercept form: If C $\neq$ 0, then Ax + By + C = 0

$\Rightarrow Ax + By = -C$

$\Rightarrow (\frac{A}{-C})x + (\frac{B}{-C})y = 1$

$\Rightarrow \frac{x}{\left(-\frac{C}{A}\right)} + \frac{y}{\left(-\frac{C}{B}\right)} = 1$

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$

iii. Normal form:

$Ax + By + C = 0$ or $Ax + By = -C$.

$\Rightarrow \frac{A}{\cos\omega} = \frac{B}{\sin} = -\frac{C}{p}$

$\Rightarrow x\cos\omega + y\sin\omega = p$

iv. Distance form:

Let θ be the angle with the positive direction of X-axis is

$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin} = r$, where r is the distance of the point (x, y) on the line from the point (x_1, y).

27. The coordinates of any point on the line at a distance r from the point (x_1, y) are

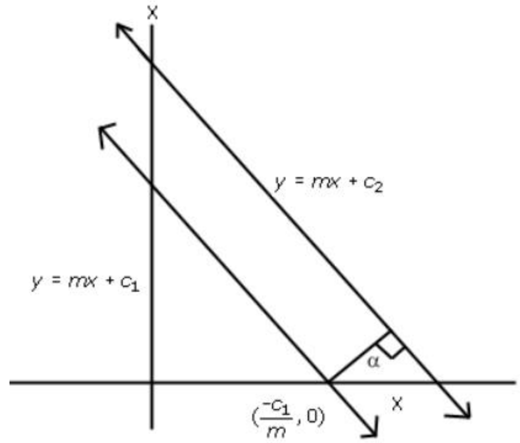
$(x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$

28. The slope of the line ax + by + c = 0 is $-\frac{a}{b} = -\frac{\text{coefficient of x}}{\text{coefficent of y}}$

29. The distance of a point from a line is the length of perpendicular drawn from the point on the line.

30. Distance between two parallel lines is equal to the length of the perpendicular from a point to line (2). Therefore, the distance between parallel lines y = mx + c and y = mx + d is given by

$$\text{Distance} = \frac{\left|(-m)(\frac{-c}{m})+(-d)\right|}{\sqrt{1+m^2}} = \frac{|c-d|}{\sqrt{1+m^2}}$$



31. Let $L_1 = a_1x + b_1y + c_1 = 0$

$L_2 = a_2x + b_2y + c_2 = 0$ and

$L_3 = a_3x + b_3y + c_3 = 0$ be three lines then they are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

32. If two lines having the same slope pass through a common point, then the two lines will coincide.

33. If θ is the inclination of a line ℓ, then tan θ is called the slope or gradient of the line ℓ.

34. Two lines are parallel if and only if their slopes are equal.

35. Two lines are perpendicular if and only if the product of their slopes is −1.

36. The equation of the line having a normal distance from the origin p and angle between the normaland the positive X-axis ω is given by x cos ω + y sin ω = p.

Key formulae

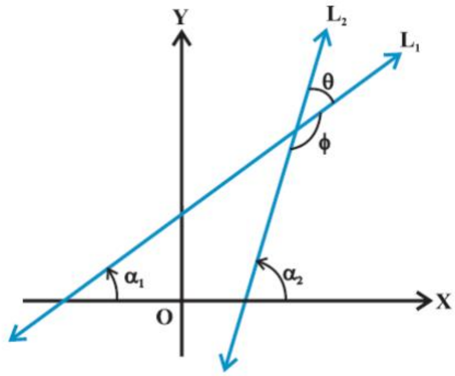
1. (a) If a line makes an angle á with the positive direction of X-axis, then the slope of the line is given by tan θ ≠ 90˚.

   (b) Slope or gradient of a line joining (x_1, y_1), (x_2, y_2) is $m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{sun}}$.

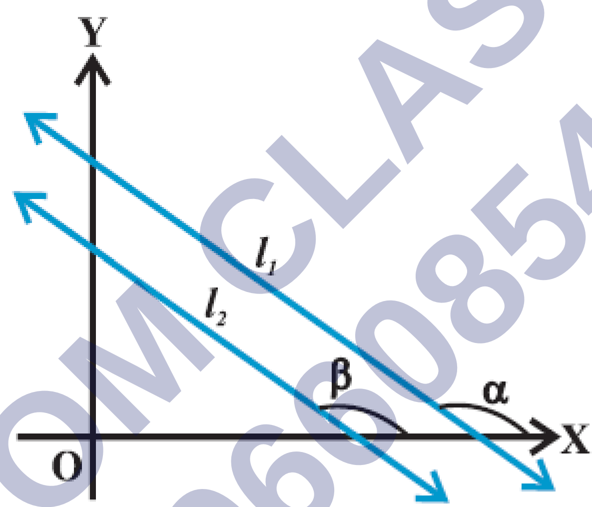
   (c) Slope of the horizontal line is zero and slope of vertical line is undefined.

2. Angle θ between two lines L_1 and L_2

$\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1 m}\right|$, as $1 + m_1 m_2 \neq 0$

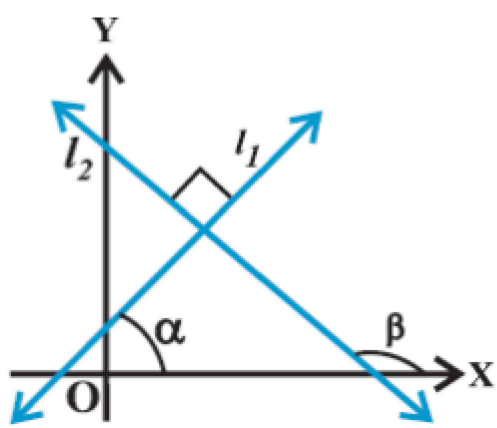


3. For parallel lines



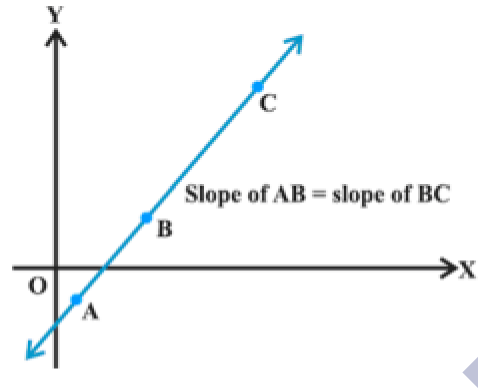
Slope of L_1 (m_1) = slope of L_2 (m_2) or $\tan\alpha = \tan\beta$.

4. For perpendicular lines

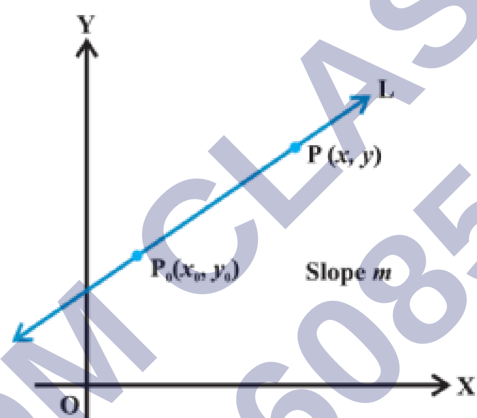


Slope of L_1 (m_1) × slope of L_2 (m_2) $= -1$, i.e., $m_1 m_2 = -1$.

5. Three points are collinear if and only if the slope of AB = slope of BC.

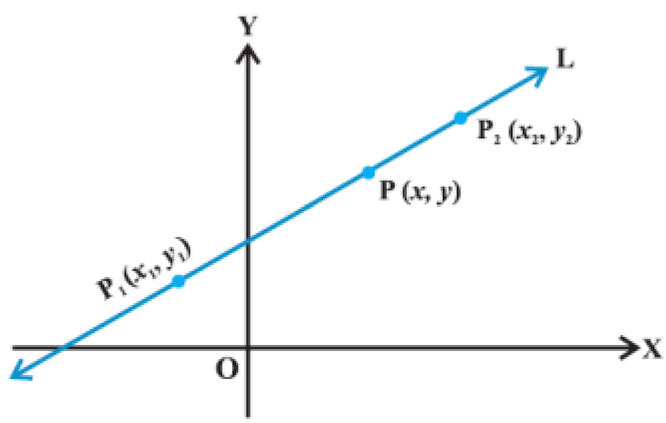


6. Point-slope form: $m = \frac{y - y_0}{x - x_0}$, i.e., $y - y = m(x - x_0)$

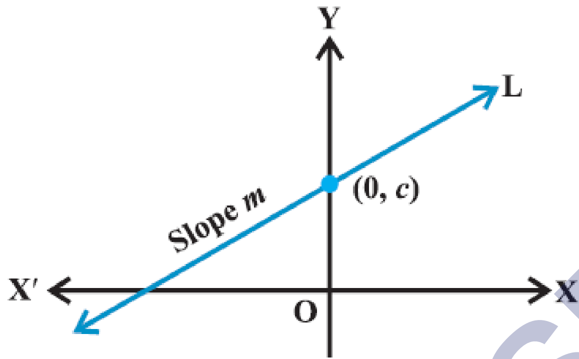


7. Two-point form: The equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



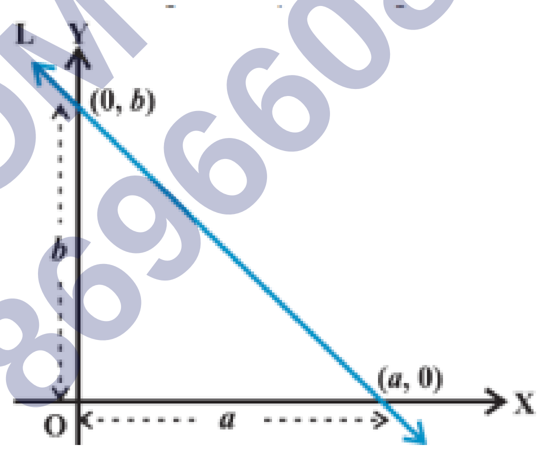
8. Slope-intercept form: Equation of line L with the point (x, y) and slope m and y-intercept c is y = mx + c.



(b) Suppose line L with slope m makes x-intercept d. The equation of L is y = m(x − d).

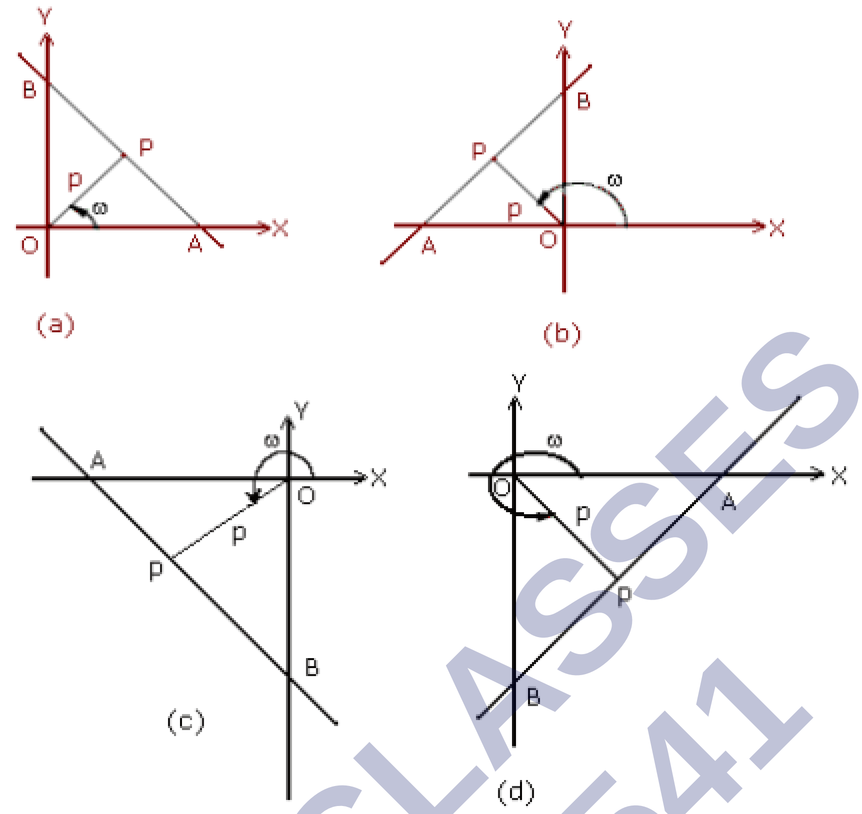
9. Intercept form: Equation of the line making intercepts a and b on the X and Y-axis, respectively.

$$\frac{x}{a}+\frac{y}{b}=1$$



10.Normal form: The equation of the line having normal distance p from the origin and angle to whichthe normal makes with the positive direction of x-axis is given by

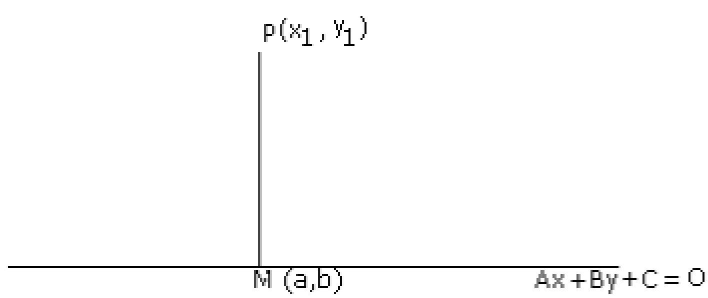
$$x \cos \omega + y \sin \omega = p$$



11. General orm of linear equation: Equation of the form Ax + By + C = 0, where A and B are not zerosimultaneously.

12. The perpendicular distance (d) of a line Ax + By + C = 0 from a point $P(x_1, y_1)$ not on it is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 +}}$$



13. Distance between two parallel lines Ax + by + C_1 = 0 and Ax + By + C_2 = 0 is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 +}}$$

14. The equation of the lines passing through (x_1, y) and making an angle θ with the line

y = mx + c by is given by

$$y - y_1 = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}(x - x_1)$$

Class : 11th Mathematics
Chapter- 10 : Straight lines

Straight lines

Distance Formula

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Slope Formula

- If θ is the angle at which a straight line is inclined to be+ve direction of x-axis and $0° \le \theta < 180°$, $\theta \neq 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m doesn't exist, but the line is parallel to y-axis. If $\theta = 0°$, then $m=0$ and the line is parallel to x-axis.
- If $A(x_1, y_1)$ and $B(x_2, y_2)$, $x_1 \neq x_2$ are points on straight line, then the slope m of the line is given by $m = (y_1 - y_2)/(x_1 - x_2)$

Perpendicular Lines

1. When two lines of the slope m_1 & m_2 are at right angles, the Product of their slope is -1, i.e. , $m_1m_2 = -1$. Thus, any line perpendicular to $y=mx+c$ is of the form , $y = \frac{-1}{m}x+d$ where d is any parameter.
2. Two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are perpendicular if $aa' + bb' = 0$. Thus, any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where k is any parameter.

Area of Triangle

Area of triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\frac{1}{2}\left|x_1(y_1-y_2)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

Reflection & foot of perpendicular a point about a line

1. The image of a point (x_1, y_1) about a line $ax + by + c = 0$ is:

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$$

2. Similarly, foot of perpendicular from a point on the line is:

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{-(ax_1+by_1+c)}{\sqrt{a^2+b^2}}$$

Parallel lines

1. When two lines are parallel their slopes are equal. Thus, any line parallel to $y = mx+c$ is of the type $y = mx + d$, where d is any parameter
2. Two lines $ax + by + c = 0$ and $a'x + b'x + c' = 0$ are parallel if $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$
3. The distance between two parallel lines with equations $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$

Length of the perpendicular from a point on a line

The length of the perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is: $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

Section Formula

The $P(x, y)$ divided the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio m:n, then $x = \frac{mx_2+nx_1}{m+n}$; $y_1 = \frac{my_2+ny_1}{m+n}$

Note:
- If m/n is +ve, the division is internal, but if m/n is –ve, the division is external.
- If m = n, then P is the mid-point of the line segment joining A & B.

Equation of Straight line in various forms

1. **POINT-SLOPE FORM**: $y - y_1 = m(x - x_1)$ is the equation of a straight line whose slope is 'm' and passes through the point (x_1, y_1).
2. **SLOPE INTERCEPT FORM**: $y = mx + c$ is the equation of a straight line whose slope is 'm' and makes an intercept c on the y-axis.
3. **TWO POINT FORM**: $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$ is the equation of a straight line which passes through (x_1, y_1) & (x_2, y_2).
4. **INTERCEPT FORM**: $\frac{x}{a}+\frac{y}{b}=1$ is the equation of a straight line which makes intercepts a & b on x and y axis respectively.
5. **NORMAL / PERPENDICULAR FORM**: $x\cos\alpha + y\sin\alpha = p$ (where $p > 0, \le 0\ \alpha < 2\pi$) is the equation of a straight line where the length of the perpendicular from origin O on the line is p and this perpendicular makes an angle α with +ve x-axis.
6. **GENERAL FORM**: $ax + by + c = 0$ is the equation of a straight line in general form. In this case, slope of line $= -\frac{a}{b}$

Important Questions

Multiple Choice questions-

Question 1. In a ΔABC, if A is the point (1, 2) and equations of the median through B and C are respectively x + y = 5 and x = 4, then B is

(a) (1, 4)

(b) (7, -2)

(c) none of these

(d) (4, 1)

Question 2. The equation of straight line passing through the point (1, 2) and perpendicular to the line x + y + 1 = 0

(a) y – x + 1 = 0

(b) y – x – 1 = 0

(c) y – x + 2 = 0

(d) y – x – 2 = 0

Question 3. The points (-a, -b), (0, 0), (a, b) and (a^2, ab) are

(a) vertices of a square

(b) vertices of a parallelogram

(c) collinear

(d) vertices of a rectangle

Question 4. The equation of the line through the points (1, 5) and (2, 3) is

(a) 2x – y – 7 = 0

(b) 2x + y + 7 = 0

(c) 2x + y – 7 = 0

(d) x + 2y – 7 = 0

Question 5. The slope of a line which passes through points (3, 2) and (-1, 5) is

(a) 3/4

(b) -3/4

(c) 4/3

(d) -4/3

Question 6. The ratio of the 7th to the $(n-1)^{th}$ mean between 1 and 31, when n arithmetic means are inserted between them, is 5 : 9. The value of n is

(a) 15

(b) 12

(c) 13

(d) 14

Question 7. The ortho centre of the triangle formed by lines xy = 0 and x + y = 1 is :

(a) (0, 0)

(b) none of these

(c) (1/2, 1/2)

(d) (1/3, 1/3)

Question 8. Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel if

(a) $a_1 /a_2 = b_1 /b_2 \neq c_1 /c_2$

(b) $a_1 /a_2 \neq b_1 /b_2 = c_1 /c_2$

(c) $a_1 /a_2 \neq b_1 /b_2 \neq c_1 /c_2$

(d) $a_1 /a_2 = b_1 /b_2 = c_1 /c_2$

Question 9. If the line x/a + y/b = 1 passes through the points (2, -3) and (4, -5), then (a, b) is

(a) a = 1 and b = 1

(b) a = 1 and b = −1

(c) a = −1 and b = 1

(d) a = −1 and b = -1

Question 10. The angle between the lines x – 2y = y and y – 2x = 5 is

(a) $\tan^{-1}$ (1/4)

(b) $\tan^{-1}$ (3/5)

(c) $\tan^{-1}$ (5/4)

(d) $\tan^{-1}$ (2/3)

Very Short Questions:

1. Find the slope of the lines passing through the point (3,-2) and (-1,4)

2. **Three points $P(h,k)$, $Q(x_1,y_1)$ and $R(x_2,y_2)$ lie on a line. Show**

that $(h-x_1)(y_2-y_1)=(k-y_1)(x_2-x_1)$

3. Write the equation of the line through the points (1, -1) and (3, 5)
4. Find the measure of the angle between the lines x + y + 7 = 0 and x – y + 1 = 0.
5. Find the equation of the line that has y-intercept 4 and is $\perp$ to the line y = 3x – 2.
6. Find the equation of the line, which makes intercepts -3 and 2 on the x and y-axis respectively.
7. Equation of a line is 3x – 4y + 10 = 0 find its slope.
8. Find the distance between the parallel lines 3x – 4y + 7 = 0 and 3x – 4y + 5 = 0.
9. Find the equation of a straight line parallel to y-axis and passing through the point (4,-2)
10. If 3x – by + 2 = 0 and 9x + 3y + a = 0 represent the same straight line, find the values of a and b.

Short Questions:

1. If p is the length of the from the $\perp$ origin on the line whose intercepts on the axes are a and b. show that

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

2. Find the value of p so that the three lines 3x + y – 2 = 0, px + 2y – 3 = 0 and may intersect at one point.
3. Find the equation to the straight line which passes through the point (3,4) and has intercept on the axes equal in magnitude but opposite in sign.
4. By using area of Δ . Show that the points (a, b + c), and (c, a + b) are collinear.
5. Find the slope of a line, which passes through the origin, and the midpoint of the line segment joining the point p(0, 4) and Q (8, 0)

Long Questions:

1. Find the values of for the line (k-3)x – (4 - k^2) y + k^2 – 7k + 6 = 0

   (a). Parallel to the x-axis

   (b). Parallel to y-axis

   (c). Passing through the origin.

2. If p and q are the lengths of $\perp$ from the origin to the lines.

   $x\cos\theta - y\sin\theta = k\cos 2\theta$, **and** $x\sec\theta + y\,\mathrm{cosec}\,\theta = k$ **respectively, prove that** $p^2+4q^2=k^2$

3. Prove that the product of the $\perp$ drawn from the points $(\sqrt{a^2-b^2}, 0)$ and

$(-\sqrt{a^2-b^2},0)$ to the line.

4. Find equation of the line mid way between the parallel lines 9x + 6y -7 = 0 and 3x + 2y + 6 = 0.

5. Assuming that straight lines work as the plane mirror for a point, find the image of the point (1,2) in the line x – 3y + 4 = 0.

Assertion Reason Questions:

1. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : The point (3, 0) is at 3 units distance from the Y -axis measured along the positive X -axis and has zero distance from the X -axis.

   Reason (R) : The point (3, 0) is at 3 units distance from the X -axis measured along the positive Y -axis and has zero distance from the Y -axis.

   (i) Both assertion and reason are true and reason is the correct explanation of assertion.

   (ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

   (iii) Assertion is true but reason is false.

   (iv) Assertion is false but reason is true.

2. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : Slope of X -axis is zero and slope of Y -axis is not defined.

   Reason (R) : Slope of X -axis is not defined and slope of Y -axis is zero.

   (i) Both assertion and reason are true and reason is the correct explanation of assertion.

   (ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

   (iii) Assertion is true but reason is false.

   (iv) Assertion is false but reason is true.

Answer Key:

MCQ

1. (b) (7, -2)
2. (b) $y - x - 1 = 0$
3. (c) collinear
4. (c) $2x + y - 7 = 0$
5. (b) -3/4
6. (d) 14
7. (a) (0, 0)
8. (a) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$
9. (d) $a = -1$ and $b = -1$
10. (c) $\tan^{-1}(5/4)$

Very Short Answer:

1. Slope of line through (3,-2) and (-1, 4)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-(-2)}{-1-3}$$

$$= \frac{6}{-4} = \frac{-3}{2}$$

2. Since P, Q, R are collinear

Slope of PQ = slope of QR

$$\frac{y_1 - k}{x_1 - h} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{\cancel{-}(k - y_1)}{\cancel{-}(h - x_1)} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$$

3.

Req. eq. $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$$y + 1 = \frac{5+1}{2}(x - 1)$$

$$-3x + y + 4 = 0$$

4.

$x+y+7=0$

$m_1=\frac{-1}{1}$

$x-y+1=0$

$m_2=\frac{-1}{-1}=1$

Slopes of the two lines are 1 and -1 as product of these two slopes is -1, the lines are at right angles.

5.

$y=3x-2$

Slope $(m)=\frac{-3}{-1}=3,$ slope of any line $\perp$ it is $-\frac{1}{3}$

$C=4$

Req. eq. is $y=mx+c$

$y=\frac{-1}{3}x+4$

6.

Req. eq. $\frac{x}{a}+\frac{y}{b}=1$

$a=-3, b=2$

$\therefore \frac{x}{-3}+\frac{y}{2}=1$

2x - 3y + 6 = 0

7.

$m=\frac{-\text{coff. of } x}{\text{coff. of } y}$

$=\frac{-3}{-4}=\frac{3}{4}$

8.

$A=3, B=-4, C_1=7$ and $C_2=5$

$d=\frac{|C_1-C_2|}{\sqrt{a^2+b^2}}$

$=\frac{|7-5|}{\sqrt{(3)^2+(-4)^2}}$

$= \frac{2}{5}$

9.

Equation of line parallel to y-axis is $x = a.....(i)$

Eq. (i) passing through (-4,2)

$a = -4$

So $x = -4$

$x + 4 = 0$

10.

ATQ

$\frac{3}{9} = \frac{-b}{3} = \frac{2}{a}$

$b = -1$

$\Rightarrow a = 6$

Short Answer:

1. Equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow \frac{x}{a} + \frac{y}{b} - 1 = 0$

The distance of this line from the origin is P

$$\therefore P = \frac{\left|\frac{0}{a} + \frac{0}{b} - 1\right|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}} \qquad \left[d = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}\right]$$

$$\frac{P}{1} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

$$\frac{1}{P} = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$$

Sq. both side

$$\frac{1}{P^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

2.

$3x + y - 2 = 0........(i)$

$px+2y-3=0.......(ii)$

$2x-y+3=0.......(iii)$

On solving eq. (i) and (iii)

$x=1$, And $y=-1$

Put x, y in eq. (ii)

$P(1)+2(-1)-3=0$

$p-2-3=0$

$p=5$

3.

Let intercept be a and –a the equation of the line is

$\frac{x}{a}+\frac{y}{-a}=1$

$\Rightarrow x-y=a.......(i)$

Since it passes through the point (3, 4)

$3-4=a$

$a=-1$

Put the value of a in eq. (i)

$x-y=-1$

$x-y+1=0$

4.

Area of $\Delta=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

$=\frac{1}{2}|a(c+a)-b(b+c)+b(a+b)-c(c+a)+c(b+c)-a(a+b)|$

$=\frac{1}{2}.0=0$

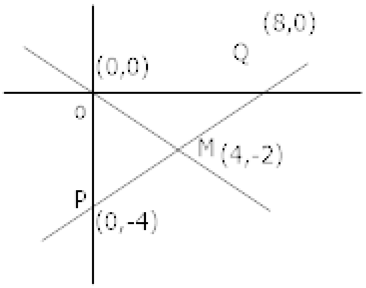
5.

Let m be the midpoint of segment PQ then $M=\left(\frac{0+8}{2},\frac{-4+0}{2}\right)$

$=(4,-2)$

Slope of $OM=\frac{y_2-y_1}{x_2-x_1}$

$= \frac{-2-0}{4-0} = \frac{-1}{2}$



Long Answer:

1. (a) The line parallel to x-axis if coeff. Of x=0

$k-3=0$

$k=3$

(b) The line parallel to y-axis if coeff. Of y=0

$4-k^2=0$

$k=\pm 2$

(c)Given line passes through the origin if (0, 0) lies on given eq.

$(k-3).(0)-(4-k^2)(0)+k^2-7k+6=0$

$(k-6)(k-1)=0$

$k=6,1$

2.

$P=\frac{|0.\cos\theta-0\sin\theta-k\cos 2\theta|}{\sqrt{(\cos\theta)^2+(-\sin\theta)^2}}$ $\left[\begin{array}{l}\perp \text{ from origin} \\ \because (0,0)\end{array}\right.$

$P=K\cos 2\theta .....(i)$

$q=\frac{|0.\sec\theta+0\operatorname{cosec}\theta-k|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}$

$=\frac{K}{\sqrt{\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}}}$

$=\frac{k\cos\theta.\sin\theta}{\sqrt{\sin^2\theta+\cos^2\theta}}=\frac{1}{2}k.\sin\theta.\cos\theta$

$2q=k.\sin 2\theta ......(ii)$

Squaring (i) and (ii) and adding

$$P^2+(2q)^2=K^2\cos^2 2\theta+K^2\sin^2 2\theta$$

$$P^2+4q^2=K^2\left(\cos^2 2\theta+\sin^2 2\theta\right)$$

$$p^2+4q^2=k^2$$

3. Let

$$p_1=\frac{\left|\frac{\sqrt{a^2-b^2}}{a}.\cos\theta-1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\left[\because\perp \text{ from the points } \sqrt{a^2-b^2},0\right]$$

Similarly p_2 be the distance $\left(-\sqrt{a^2-b^2},0\right)$ from to given line

$$p_2=\frac{\left|-\frac{\sqrt{a^2-b^2}}{a}\cos\theta-1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}$$

$$p_1p_2=\frac{\left|\left(\frac{\sqrt{a^2-b^2}}{a}\cos\theta-1\right)\left(-\frac{\sqrt{a^2-b^2}}{a}\cos\theta-1\right)\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}$$

$$=\frac{\left|\left(\frac{a^2-b^2}{a^2}\right).\cos^2\theta-1\right|}{\frac{b^2\cos^2\theta+a^2\sin^2\theta}{a^2b^2}}$$

$$=\frac{\left|a^2\cos^2\theta-b^2\cos^2\theta-a^2\right|a^2b^2}{a^2(a^2\sin^2\theta+b^2\cos^2\theta)}$$

$$=\frac{\left|-(a^2\sin^2\theta+b^2\cos^2\theta)\right|b^2}{a^2\sin^2\theta+b^2\cos^2\theta}\qquad\left[\because a^2\cos^2\theta-a^2=a^2(\cos^2\theta-1)\right.$$

$$=\frac{(a^2\sin^2\theta+b^2\cos^2\theta)\,b^2}{a^2\sin^2\theta+b^2\cos^2\theta}$$

$$=b^2$$

4.

The equations are

$$9x+6y-7=0$$

$$3\left(3x+2y-\frac{7}{3}\right)=0$$

$3x+2y-\frac{7}{3}=0.......(i)$

$3x+2y+6=0.....(ii)$

Let the eq. of the line mid way between the parallel lines (i) and (ii) be

$3x+2y+k=0.......(iii)$

ATQ

Distance between (i) and (iii) = distance between (ii) and (iii)

$$\left|\frac{K+\frac{7}{3}}{\sqrt{9+4}}\right|=\left|\frac{K-6}{\sqrt{9+4}}\right| \quad \left[\because d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}\right]$$

$K+\frac{7}{3}=K-6$

$K=\frac{11}{6}$

Req. eq. is

$3x+2y+\frac{11}{6}=0$

5.

Let Q(h, k) is the image of the point p(1, 2) in the line.

$x-3y+4=0......(i)$

Coordinate of midpoint of $PQ=\left(\frac{h+1}{2},\frac{k+2}{2}\right)$

This point will satisfy the eq. ......(i)

$\left(\frac{h+1}{2}\right)-3\left(\frac{k+2}{2}\right)+4=0$

$h-3k=-3.......(i)$

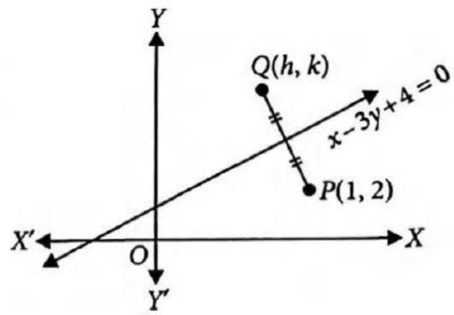
(Slope of line PQ) × (slope of line x – 3y + 4 = 0) = -1

$\left(\frac{k-2}{h-1}\right)\left(\frac{-1}{-3}\right)=-1$

$3h+k=5......(ii)$

On solving (i) and (ii)

$h=\frac{6}{5}$ and $k=\frac{7}{5}$



Assertion Reason Answer:

1. (iii) Assertion is true but reason is false.
2. (iii) Assertion is true but reason is false.