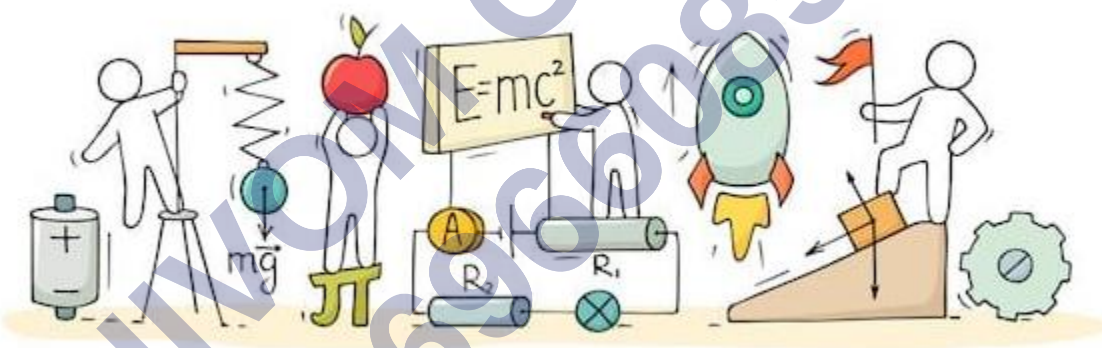
PHYSICS

CHAPTER 7: ALTERNATING CURRENT



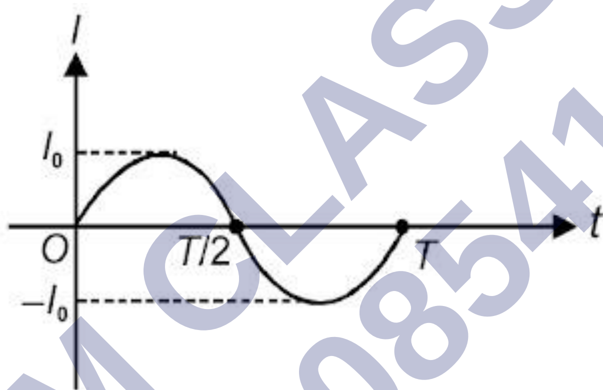
ALTERNATING CURRENT

Alternating Current:

The magnitude of alternating current changes continuously with time and its direction is reversed periodically. It is represented by

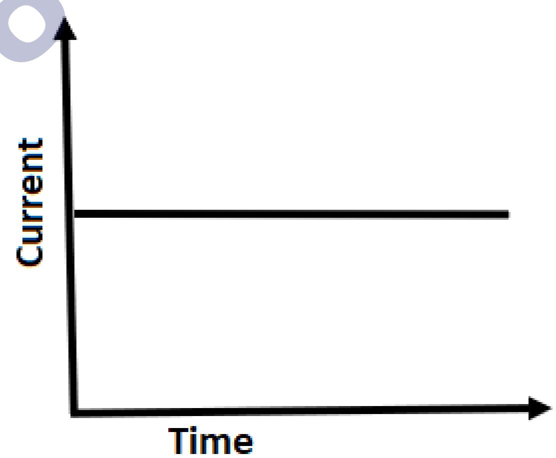
$$I = I_0 \sin \omega t \text{ or } I = I_0 \cos \omega t$$

$$\omega = \frac{2\pi}{T} = 2\pi v$$



Direct current (DC):

Direct current (DC) is electrical current which flows consistently in one direction. The current that flows in a flashlight or another appliance running on batteries is direct current.



Mean value for half cycle of AC:

Mean value of AC is the total charge that flows through a circuit element in a given time interval divided by the time interval. emf.

$$I_{mean} = \frac{\int_0^T Idt}{T}$$

For half cycle

$$I_{mean} = \frac{\int_0^{\frac{T}{2}} Idt}{\frac{T}{2}}$$

$$I_{mean} = \frac{2}{T}\int_0^{\frac{T}{2}} I_0 \sin \omega dt$$

$$I_{mean} = \frac{2I_0}{T}\left[\frac{-\cos \omega t}{\omega}\right]_0^{\frac{T}{2}}$$

$$I_{mean} = \frac{2I_0}{2\pi}[-\cos \pi - \cos 0] \ldots (\because \omega = \frac{2\pi}{T})$$

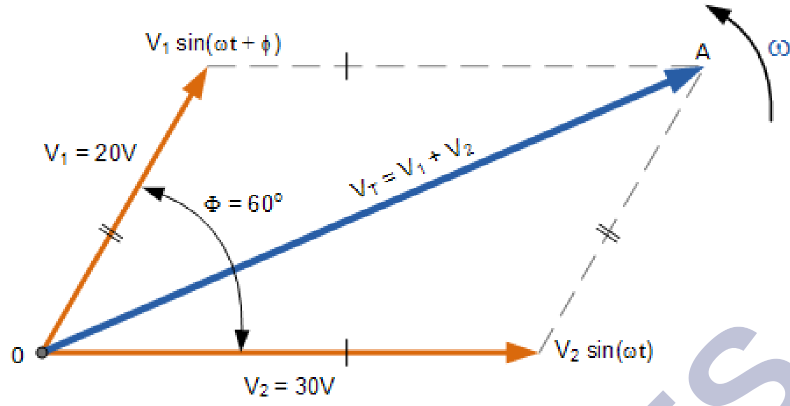
$$I_{mean} = \frac{2I_0}{\pi}$$

Note: For complete cycle, mean value = 0

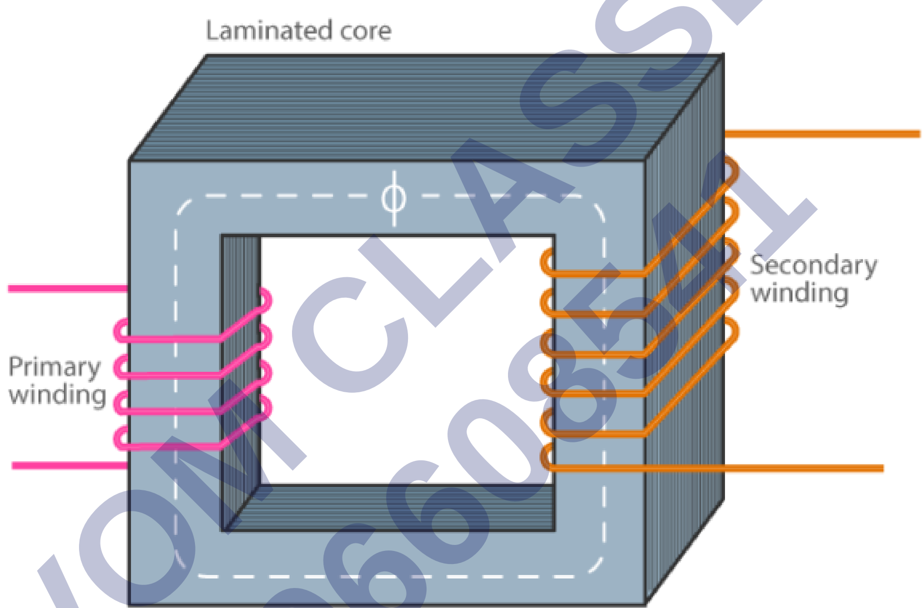
Phasor Diagram:

In the a.c. circuit containing R only, current and voltage are in the same phase. Therefore, in figure, both phasors $\overrightarrow{I_0}$ and $\overrightarrow{E_0}$ are in the same direction making an angle (ωt) with OX. This is so for all times. It means that the phase angle between alternating voltage and current through R is Zero.

$$I = I_0 \sin \omega t \text{ and } E = E_0 \sin \omega t$$



Parts of a Single-phase Transformer



The major parts of a single-phase transformer consist of;

Core

The core acts as a support to the winding in the transformer. It also provides a low reluctance path to the flow of magnetic flux. The winding is wound on the core as shown in the picture. It is made up of a laminated soft iron core in order to reduce the losses in a transformer. The factors such as operating voltage, current, power etc decide core composition. The core diameter is directly proportional to copper losses and inversely proportional to iron losses.

Windings

Windings are the set of copper wires wound over the transformer core. Copper wires are used due to:

- The high conductivity of copper minimizes the loss in a transformer because when the conductivity increases, resistance to current flow decreases.

- The high ductility of copper is the property of metals that allows it to be made into very thin wires.

There are mainly two types of windings. Primary windings and secondary windings.

- **Primary winding:** The set of turns of windings to which supply current is fed.
- **Secondary winding:** The set of turns of winding from which output is taken.

The primary and secondary windings are insulated from each other using insulation coating agents.

Insulation Agents

Insulation is necessary for transformers to separate windings from each other and to avoid short circuit. This facilitates mutual induction. Insulation agents have an influence on the durability and the stability of a transformer.

Following are used as an insulation medium in a transformer:

- Insulating oil
- Insulating tape
- Insulating paper
- Wood-based lamination

Capacitive Reactance (XC):

The opposing nature of capacitor to the flow of alternating current is called capacitive reactance.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fc}$$

Where, C = capacitance

Choke Coil:

A choke coil is an inductor having a small resistance. It is a device used in ac circuits to control current without wasting too much power. As it has low resistance, its power factor $\cos\phi$ is low.

Wattless Current:

The current in an AC circuit when average power consumption in AC circuit is zero, is referred as wattless current or idle current.

A.C. Generator or A.C. Dynamo:

An a.c. generator/ dynamo is a machine that produces alternating current energy from mechanical energy. It is one of the most important applications of the phenomenon of electromagnetic induction. The generator was designed originally by a Yugoslav scientist, Nikola Tesla. The word generator is a misnomer because nothing is generated by the machine. In fact, it is an alternator converting one form of energy into another.

Transformer:

A transformer which increases the a.c. voltage is called a step-up transformer. A transformer which decreases the a.c. voltages are called a step-down transformer.

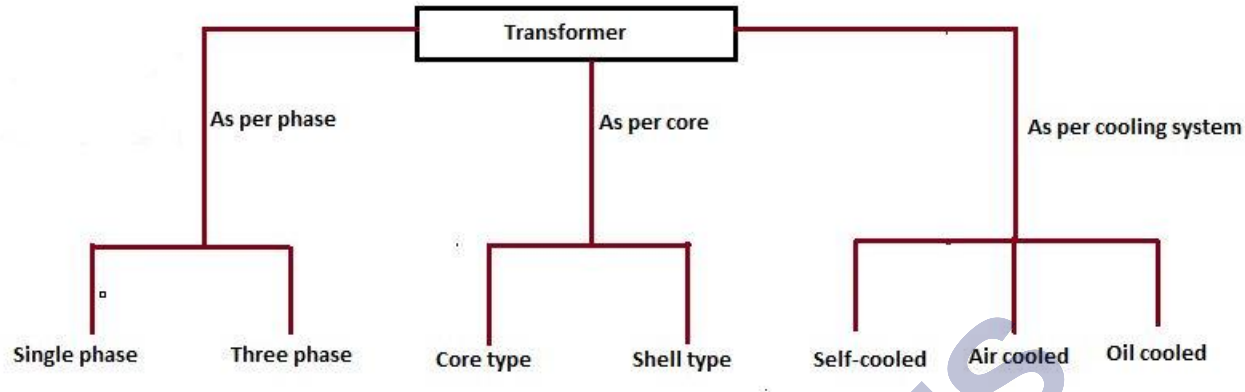
Step Up Transformer

If the secondary coil has more numbers of turns than the primary, the voltage is seen to step up, that is how the name is given for this type of transformer. If the secondary coil has less number of coils, it is referred to as a step-down type of transformer.

Transformer Types

Transformers are used in various fields like power generation grid, distribution sector, transmission and electric energy consumption. There are various types of transformers which are classified based on the following factors;

- Working voltage range.
- The medium used in the core.
- Winding arrangement.
- Installation location.



Based on Voltage Levels

Commonly used transformer type, depending upon voltage they are classified as:

Step-up Transformer: They are used between the power generator and the power grid. The secondary output voltage is higher than the input voltage.

Step down Transformer: These transformers are used to convert high voltage primary supply to low voltage secondary output.

Based on the Medium of Core Used

In a transformer, we will find different types of cores that are used.

Air core Transformer: The flux linkage between primary and secondary winding is through the air. The coil or windings wound on the non-magnetic strip.

Iron core Transformer: Windings are wound on multiple iron plates stacked together, which provides a perfect linkage path to generate flux.

Based on the Winding Arrangement

Autotransformer: It will have only one winding wound over a laminated core. The primary and secondary share the same coil. Auto also means "self" in language Greek.

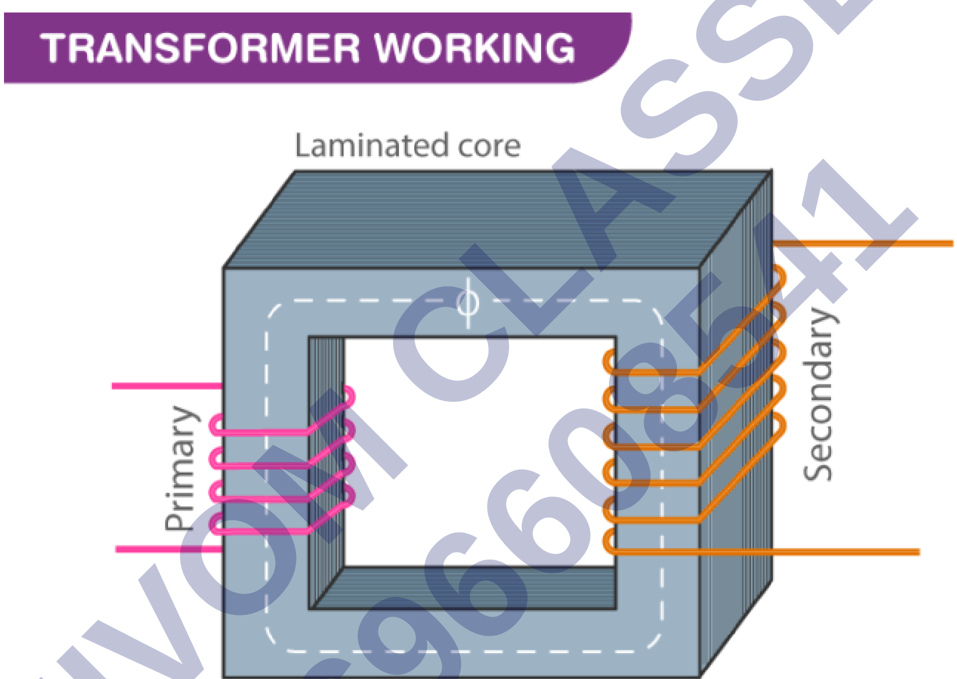
Based on Install Location

Power Transformer: It is used at power generation stations as they are suitable for high voltage application

Distribution Transformer: Mostly used at distribution lanes for domestic purposes. They are designed for carrying low voltages. It is very easy to install and characterized by low magnetic losses.

Measurement Transformers: These are further classified. They are mainly used for measuring voltage, current, power.

Protection Transformers: They are used for component protection purposes. In circuits, some components must be protected from voltage fluctuation etc. Protection transformers ensure component protection.

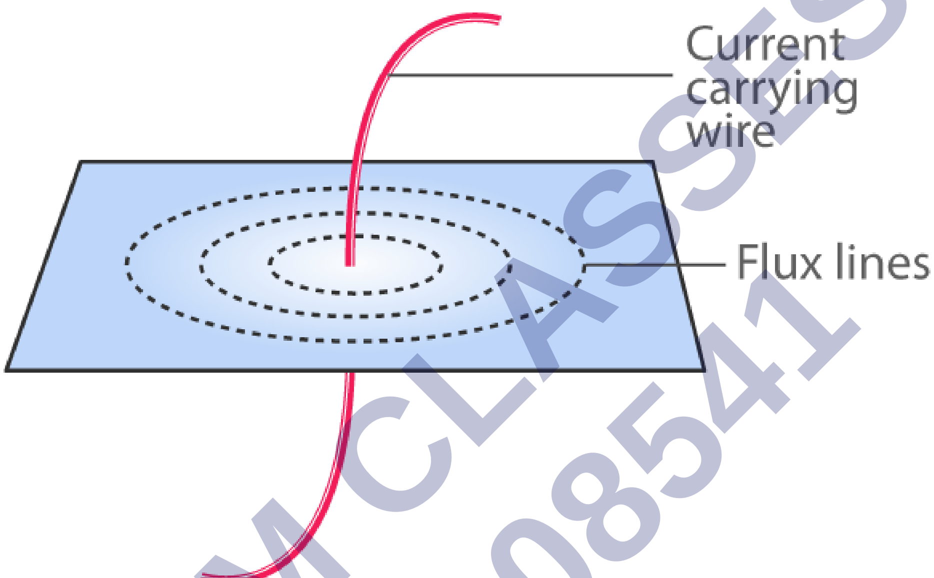


The transformer works on the principle of Faraday's law of electromagnetic induction and mutual induction.

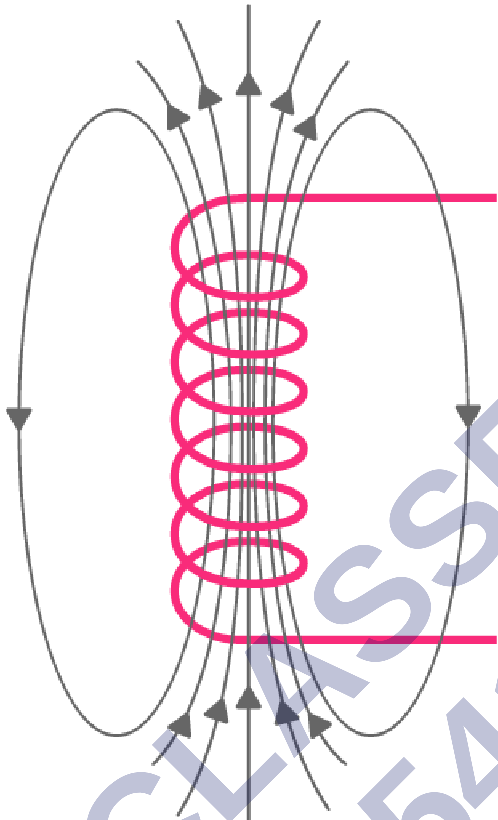
There are usually two coils primary coil and secondary coil on the transformer core. The core laminations are joined in the form of strips. The two coils have high mutual inductance. When an alternating current pass through the primary coil it creates a varying magnetic flux. As per faraday's law of electromagnetic induction, this change in magnetic flux induces an emf (electromotive force) in the secondary coil which is linked to the core having a primary coil. This is mutual induction.

Overall, a transformer carries the below operations:

- Transfer of electrical energy from circuit to another
- Transfer of electrical power through electromagnetic induction
- Electric power transfer without any change in frequency
- Two circuits are linked with mutual induction



The figure shows the formation of magnetic flux lines around a current-carrying wire. The normal of the plane containing the flux lines are parallel to normal of a cross-section of a wire.



The figure shows the formation of varying magnetic flux lines around a wire-wound. The interesting part is that reverse is also true, when a magnetic flux line fluctuates around a piece of wire, a current will be induced in it. This was what Michael faraday found in 1831 which is the fundamental working principle of electric generators as well as transformers.

Charging and Discharging of a Capacitor:

The instantaneous charge on a capacitor on charging at any instant of time t is given by

$$q = q_0\left(1 - e - \frac{t}{Rc}\right)$$

where RC = τ, is called time constant of a R – C circuit.

The instantaneous charge on a capacitor in discharging at any instant of time t is given by

$$q = q_0 e - \frac{t}{Rc}$$

Time constant of a R – C circuit is the time in which charge in the capacitor grows to 63.8% or decay to 36.8% of the maximum charge on capacitor.

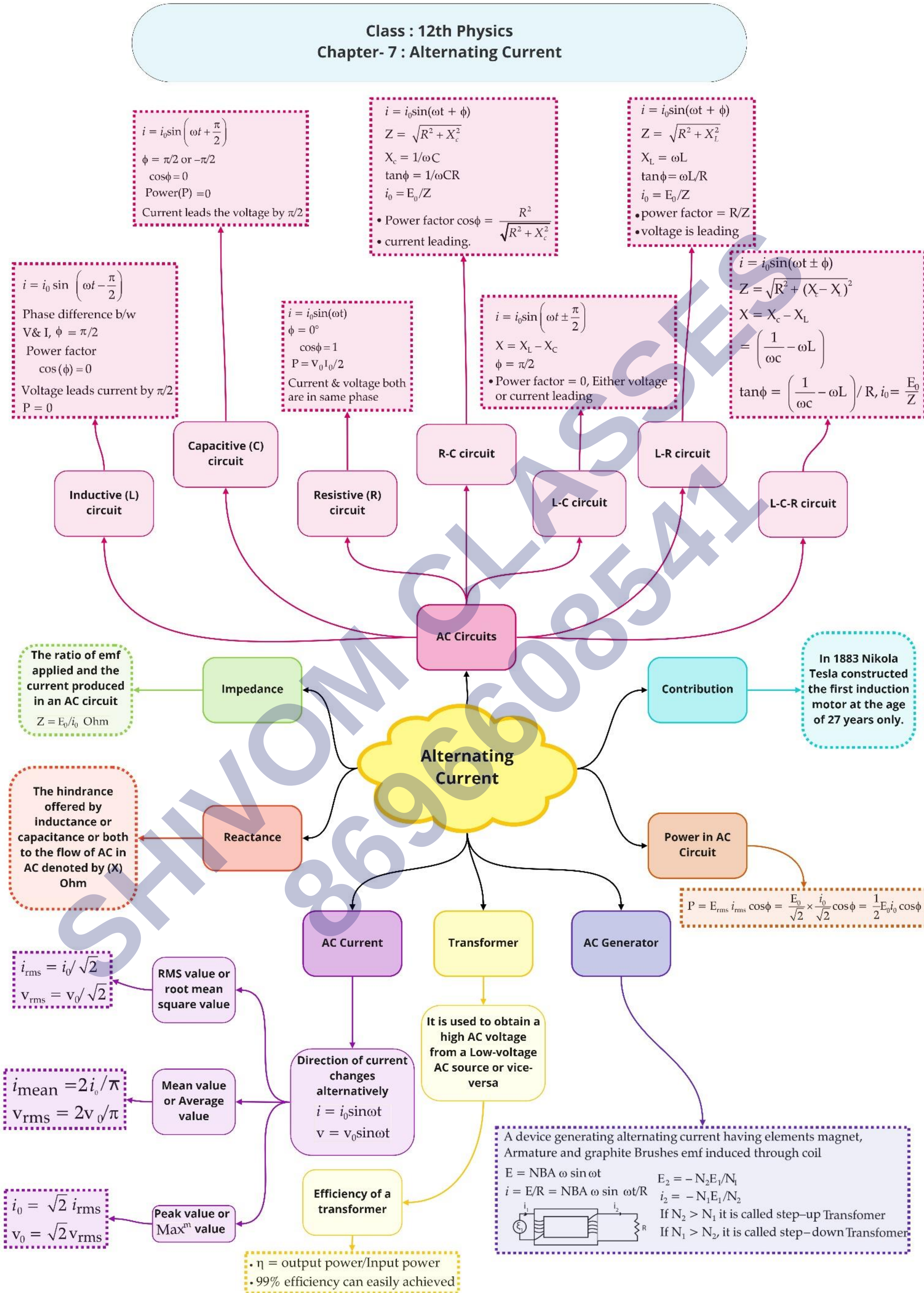
Transient Current: An electric current which very for a small finite time, while growing from zero to maximum or decaying from maximum to zero, is called a transient current.

Differences between Alternating Current and Direct Current:

Alternating Current	Direct Current
AC is safe to transfer longer distance even between two cities and maintain the electric power.	DC cannot travel for a very long distance. It loses electric power.
The rotating magnets cause the change in direction of electric flow.	The steady magnetism makes DC flow in a single direction.
The frequency of AC is dependent upon the country. But generally, the frequency is 50 Hz or 60 Hz.	DC has no frequency of zero frequency.
In AC the flow of current changes its direction backwards periodically.	It flows in a single direction steadily.
Electrons in AC keep changing its directions – backward and forward	Electrons only move in one direction – that is forward.

Use of Transformers in Transmission:

- **In electric power** transmission, transformers allow transmission of electric power at high voltages, which reduces the loss due to heating of the wires.
- In many **electronic devices,** a transformer is used to convert voltage from the distribution wiring to convenient values for the circuit requirements.
- **Signal and audio** transformers are used to couple stages of amplifiers and to match devices such as microphones and record players to the input of amplifiers.
- **Audio transformers** allowed telephone circuits to carry on a two-way conversation over a single pair of wires.
- **Resonant transformers** are used for coupling between stages of radio receivers, or in high-voltage Tesla coils.


Class : 12th Physics
Chapter- 7 : Alternating Current
Alternating Current
AC Circuits
Inductive (L) circuit
$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$
Phase difference b/w V& I, $\phi = \pi/2$
Power factor $\cos(\phi) = 0$
Voltage leads current by $\pi/2$
$P = 0$
Capacitive (C) circuit
$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$
$\phi = \pi/2$ or $-\pi/2$
$\cos\phi = 0$
Power(P) = 0
Current leads the voltage by $\pi/2$
Resistive (R) circuit
$i = i_0 \sin(\omega t)$
$\phi = 0°$
$\cos\phi = 1$
$P = V_0 I_0/2$
Current & voltage both are in same phase
R-C circuit
$i = i_0 \sin(\omega t + \phi)$
$Z = \sqrt{R^2 + X_c^2}$
$X_c = 1/\omega C$
$\tan\phi = 1/\omega CR$
$i_0 = E_0/Z$
• Power factor $\cos\phi = \frac{R^2}{\sqrt{R^2 + X_c^2}}$
• current leading.
L-C circuit
$i = i_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$
$X = X_L - X_C$
$\phi = \pi/2$
• Power factor = 0, Either voltage or current leading
L-R circuit
$i = i_0 \sin(\omega t + \phi)$
$Z = \sqrt{R^2 + X_L^2}$
$X_L = \omega L$
$\tan\phi = \omega L/R$
$i_0 = E_0/Z$
• power factor = R/Z
• voltage is leading
L-C-R circuit
$i = i_0 \sin(\omega t \pm \phi)$
$Z = \sqrt{R^2 + (X_c - X_L)^2}$
$X = X_c - X_L$
$= \left(\frac{1}{\omega c} - \omega L\right)$
$\tan\phi = \left(\frac{1}{\omega c} - \omega L\right)/R$, $i_0 = \frac{E_0}{Z}$
Impedance
The ratio of emf applied and the current produced in an AC circuit
$Z = E_0/i_0$ Ohm
Reactance
The hindrance offered by inductance or capacitance or both to the flow of AC in AC denoted by (X) Ohm
Contribution
In 1883 Nikola Tesla constructed the first induction motor at the age of 27 years only.
Power in AC Circuit
$P = E_{rms} i_{rms} \cos\phi = \frac{E_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \cos\phi = \frac{1}{2} E_0 i_0 \cos\phi$
AC Current
Direction of current changes alternatively
$i = i_0 \sin\omega t$
$v = v_0 \sin\omega t$
RMS value or root mean square value
$i_{rms} = i_0/\sqrt{2}$
$v_{rms} = v_0/\sqrt{2}$
Mean value or Average value
$i_{mean} = 2i_0/\pi$
$v_{rms} = 2v_0/\pi$
Peak value or Max^m value
$i_0 = \sqrt{2}\, i_{rms}$
$v_0 = \sqrt{2}\, v_{rms}$
Transformer
It is used to obtain a high AC voltage from a Low-voltage AC source or vice-versa
Efficiency of a transformer
• η = output power/Input power
• 99% efficiency can easily achieved
AC Generator
A device generating alternating current having elements magnet, Armature and graphite Brushes emf induced through coil
$E = NBA\,\omega \sin\omega t$
$i = E/R = NBA\,\omega \sin \omega t/R$
$E_2 = -N_2E_1/N_1$
$i_2 = -N_1E_1/N_2$
If $N_2 > N_1$ it is called step–up Transfomer
If $N_1 > N_2$, it is called step–down Transfomer

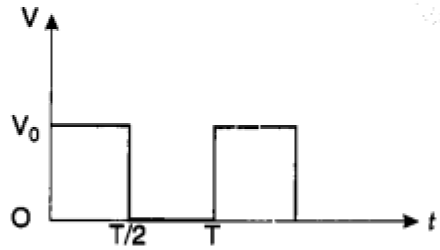
Important Questions

Multiple Choice questions-

1. Alternating voltage (V) is represented by the equation

(a) $V(t) = V_m e^{\omega t}$
(b) $V(t) = V_m \sin \omega t$
(c) $V(t) = V_m \cot \omega t$
(d) $V(t) = V_m \tan \omega t$

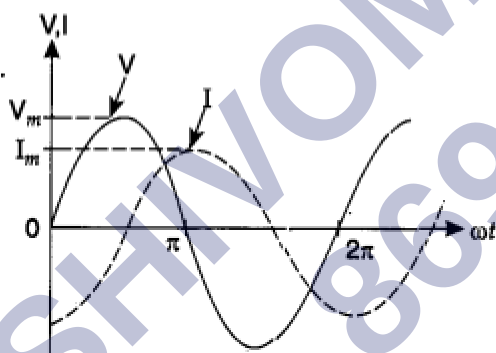
2. The rms value of potential difference V shown in the figure is



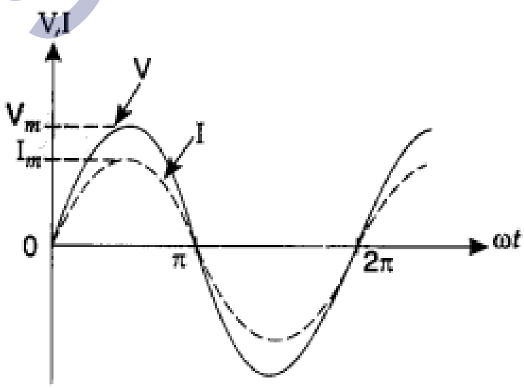
(a) $\frac{V_0}{\sqrt{3}}$ (b) V_0 (c) $\frac{V_0}{\sqrt{2}}$ (d) $\frac{V_0}{2}$

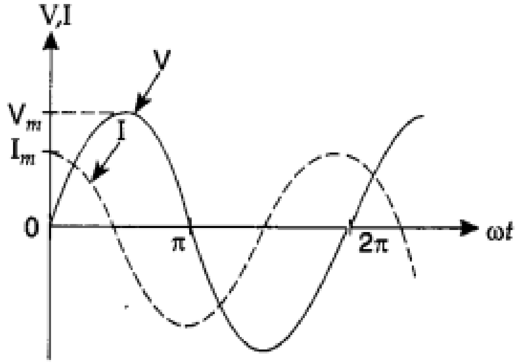
3. The phase relationship between current and voltage in a pure resistive circuit is best represented by

(a)



(b)



(c)


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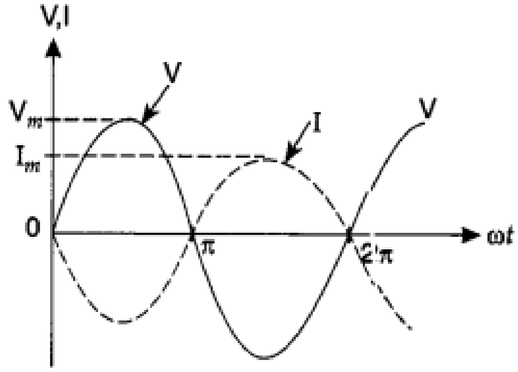


4. In the case of an inductor

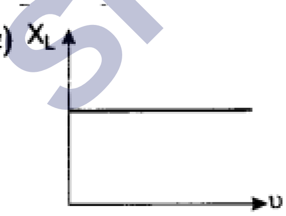
(a) voltage lags the current by $\frac{\pi}{2}$

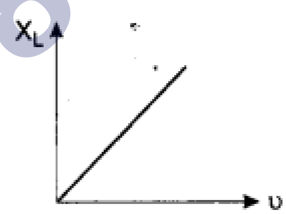
(b) voltage leads the current by $\frac{\pi}{2}$

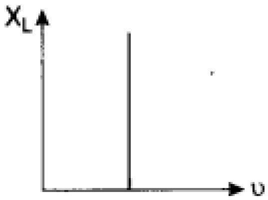
(c) voltage leads the current by $\frac{\pi}{3}$

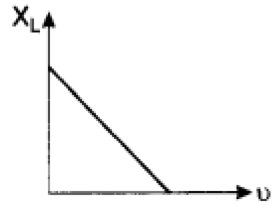
(d) voltage leads the current by $\frac{\pi}{4}$

5. Which of the following graphs represents the correct variation of inductive reactance X_L with frequency u?

(a)


(b)


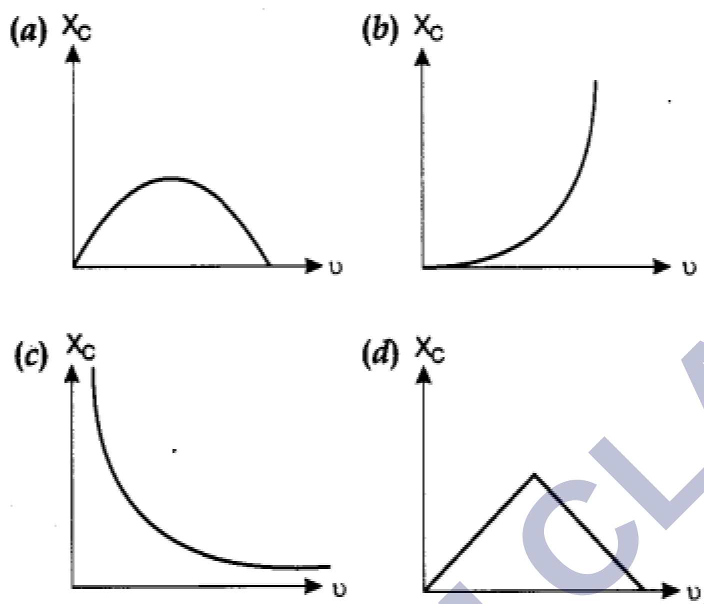
(c)


(d)


6. In a pure capacitive circuit if the frequency of ac source is doubled, then its capacitive reactance will be

(a) remains same
(b) doubled
(c) halved
(d) zero

7. Which of the following graphs represents the correct variation of capacitive reactance Xc with frequency v u?



8. In an alternating current circuit consisting of elements in series, the current increases on increasing the frequency of supply. Which of the following elements are likely to constitute the circuit?

(a) Only resistor
(b) Resistor and inductor
(c) Resistor and capacitor
(d) Only inductor

9. In which of the following circuits the maximum power dissipation is observed?

(a) Pure capacitive circuit
(b) Pure inductive circuit
(c) Pure resistive circuit
(d) None of these

10. In series LCR circuit, the phase angle between supply voltage and current is

(a) $\tan\phi = \frac{X_L - X_C}{R}$ (b) $\tan\phi = \frac{R}{X_L - X_C}$

(c) $\tan\phi = \frac{R}{X_L + X_C}$ (d) $\tan\phi = \frac{X_L + X_C}{R}$

Very Short:

1. The instantaneous current flowing from an ac source is I = 5 sin 314 t. What is the rms value of current?
2. The instantaneous emf of an ac source is given by E = 300 sin 314 t. What is the rms value of emf?
3. Give the phase difference between the applied ac voltage and the current in an LCR circuit at resonance.
4. What is the phase difference between the voltage across the inductor and the capacitor in an LCR circuit?
5. What is the power factor of an LCR series circuit at resonance?
6. In India, the domestic power supply is at 220 V, 50 Hz, while in the USA it is 110 V, 50 Hz. Give one advantage and one disadvantage of 220 V supply over 110 V supply.
7. Define the term 'wattles current'. (CBSE Delhi 2011)
8. In a series LCR circuit, $V_L = V_C \neq V_R$. What is the value of the power factor? (CBSE AI 2015)
9. Define capacitor reactance. Write its SI units. (CBSE Delhi 2015)
10. Define quality factor in series LCR circuit. What is its SI unit? (CBSE Delhi 2016)

Short Questions:

1. State the phase relationship between the current flowing and the voltage applied in an ac circuit for (i) a pure resistor (ii) a pure inductor.
2. A light bulb is in turn connected in a series (a) across an LR circuit, (b) across an RC circuit, with an ac source. Explain, giving the necessary mathematical formula, the effect on the brightness of the bulb in case (a) and (b), when the frequency of the ac source is increased. (CBSE 2019C)
3. An air-core solenoid is connected to an ac source and a bulb. If an iron core is inserted in the solenoid, how does the brightness of the bulb change? Give reasons for your answer.
4. A bulb and a capacitor are connected in series to an ac source of variable frequency. How will the brightness of the bulb change on increasing the frequency of the ac source? Give reason.
5. An ideal inductor is in turn put across 220 V, 50 Hz, and 220 V, 100 Hz supplies. Will the current flowing through it in the two cases be the same or different?
6. State the condition under which the phenomenon of resonance occurs in a series LCR circuit, plot a graph showing the variation of current with a frequency of ac source in a series LCR circuit.

7. Give two advantages and two disadvantages of ac over dc.
8. In a series, LCR circuit connected to an ac source of variable frequency and voltage v = vm sin ωt, draw a plot showing the variation of current (I) with angular frequency (ω) for two different values of resistance R1 and R2 ($R_1 > R_2$). Write the condition under which the phenomenon of resonance occurs. For which value of the resistance out of the two curves, a sharper resonance is produced? Define the Q-factor of the circuit and give its significance. (CBSE Delhi 2013C)

Long Questions:

1. Prove mathematically that the average power over a complete cycle of alternating current through an Ideal inductor is zero.
2. Draw the phasor diagram of a series LCR connected across an ac source V= V_o sin ωt. Hence, derive the expression for the impedance of the circuit. Obtain the conditions for the phase angle under which the current is
   (i) maximum and
   (ii) minimum. (CBSE AI 2019)

Assertion and Reason Question:

1. For two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is also false.

Assertion: A bulb connected in series with a solenoid is connected to A.C. source. If a soft iron core is introduced in the solenoid, the bulb will glow brighter.

Reason: On introducing soft iron core in the solenoid, the inductance decreases.

2. For two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

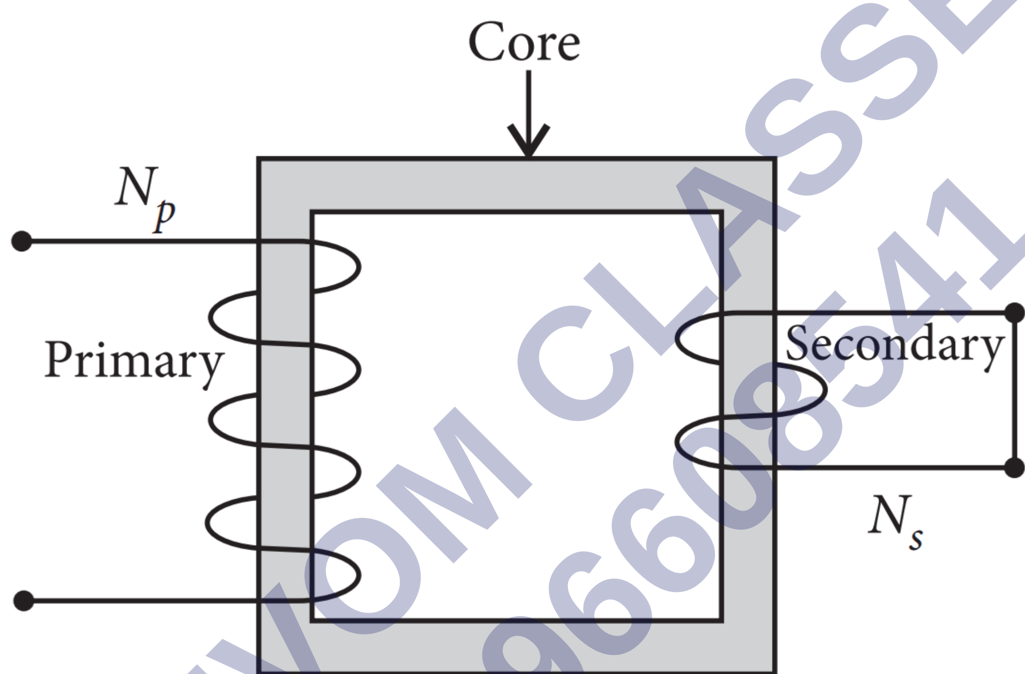
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is also false.

Assertion: An alternating current shows magnetic effect.

Reason: Magnitude of alternating current varies with time.

Case Study Questions:

1. Step-down transformers are used to decrease or step-down voltages. These are used when voltages need to be lowered for use in homes and factories. A small town with a demand of 800kW of electric power at 220V is situated 15km away from an electric plant generating power at 440V. The resistance of the two wire line carrying power is 0.5Q per km. The town gets power from the line through a 4000 - 220V step-down transformer at a sub-station in the town.



(i) The value of total resistance of the wires is:

a) 25Ω
b) 30Ω
c) 35Ω
d) 15Ω

(ii) The line power loss in the form of heat is:

a) 550kW
b) 650kW
c) 600kW
d) 700kW

(iii) How much power must the plant supply, assuming there is negligible power loss due to leakage?

a) 600kW
b) 1600kW
c) 500W
d) 1400kW

(iv) The voltage drop in the power line is:

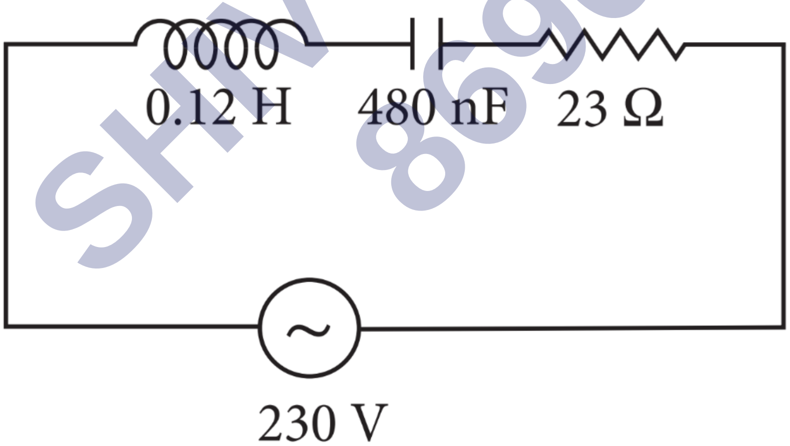
a) 1700V
b) 3000V
c) 2000V
d) 2800V

(v) The total value of voltage transmitted from the plant is:

a) 500V
b) 4000V
c) 3000V
d) 7000V

2. When the frequency of ac supply is such that the inductive reactance and capacitive reactance become equal, the impedance of the series LCR circuit is equal to the ohmic resistance in the circuit. Such a series LCR circuit is known as resonant series LCR circuit and the frequency of the ac supply is known as resonant frequency. Resonance phenomenon is exhibited by a circuit only if both Land Care present in the circuit. We cannot have resonance in a RL or RC circuit.

A series LCR circuit with L = 0.12H, C = 480nF, R=23Ω is connected to a 230V variable frequency supply.



(i) Find the value of source frequency for which current amplitude is maximum.

a) 222.32Hz
b) 550.52Hz
c) 663.48Hz
d) 770Hz

(ii) The value of maximum current is:

a) 14.14A
b) 22.52A
c) 50.25A
d) 47.41A

(iii) The value of maximum power is:

a) 2200W
b) 2299.3W
c) 5500W
d) 4700W

(iv) What is the Q-factor of the given circuit?

a) 25
b) 42.21
c) 35.42
d) 21.74

(v) At resonance which of the following physical quantity is maximum?

a) Impedance
b) Current
c) Both (a) and (b)
d) Neither (a) nor (b)

✓ Answer Key:

Multiple Choice Answers-

1. Answer: b
2. Answer: c
3. Answer: b
4. Answer: b
5. Answer: b
6. Answer: c
7. Answer: c
8. Answer: c
9. Answer: c
10. Answer: a

Very Short Answers:

1. Answer:

The rms value of current is $\frac{5}{\sqrt{2}}$.

2. Answer:

   The rms value of voltage is $\frac{300}{\sqrt{2}}$

3. Answer:

   The applied ac voltage and the current in an LCR circuit at resonance are in phase.

   Hence phase difference = 0.

4. Answer: The phase difference is 180°.
5. Answer: The power factor is one.
6. Answer:

   Advantage: less power loses

   Disadvantage: more fatal.

7. Answer: It is the current at which no power is consumed.
8. Answer: One.
9. Answer: It is the opposition offered to the flow of current by a capacitor. It is measured in ohm.
10. Answer: The quality factor is defined as the ratio of the voltage developed across the capacitor or inductor to the applied voltage. It does not have any unit.

Short Questions Answers:

1. Answer:

   (i) Electric current and voltage applied in a pure resistor are in same phase, i.e. Φ = 0°

   (ii) Applied voltage leads electric current flowing through pure-inductor in an ac circuit by phase angle of π/2.

2. Answer:

   a) The current in LR circuit is given by

   $$I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$$

   When the frequency of ac source ω increases, I decreases, and hence brightness decreases.

   (b) The current in RC circuit is given by

I = $\frac{V}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}$

When the frequency of ac source ω increases, I increases, and hence brightness increases.

3. Answer: Insertion of an iron core in the solenoid increases its inductance. This in turn increases the value of inductive reactance. This decreases the current and hence the brightness of the bulb.

4. Answer:

   When the frequency of the ac is increased, it will decrease the impedance of the circuit as Z = $\sqrt{R^2+(1/2\pi fC)^2}$. As a result, the current and hence the brightness of the bulb will increase.

5. Answer:

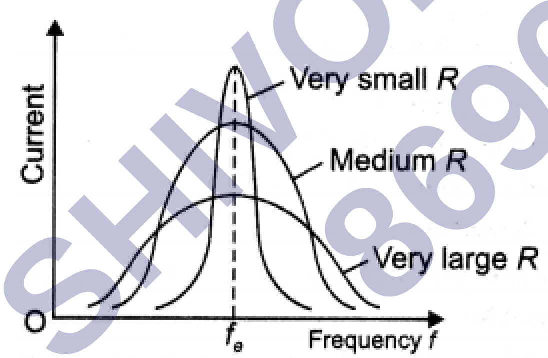
   The current through the inductor is given by I = $\frac{V}{X_L} = \frac{V}{2\pi fL}$. The current is inversely proportional to the frequency of applied ac.

6. Answer: The phenomenon occurs when the inductive reactance becomes equal to the capacitive reactance., i.e., X_L - X_C

   $\Rightarrow \omega L = \frac{1}{\omega C}$

   $\Rightarrow \omega = \frac{1}{\sqrt{LC}}$

   The graph is as shown below.



7. Answer:

   Advantages of ac:

   (a) The generation and transmission of ac are more economical than dc.

   (b) The alternating voltage may be easily stepped up or down as per need by using suitable transformers.

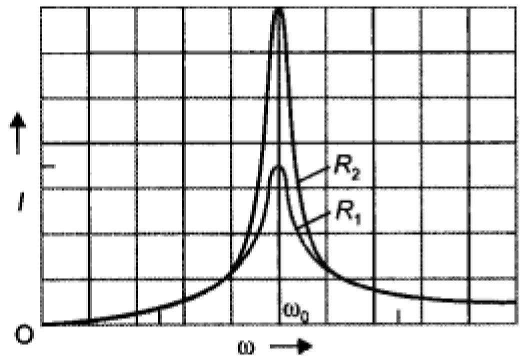
   Disadvantages of ac:

   (a) It is more fatal than dc.

   (b) It cannot be used for electrolysis.

8. Answer:

The plot is as shown.



Resonance occurs in an LCR circuit when

$X_L = X_C$.

The smaller the value of R sharper is the resonance. Therefore, the curve will be sharper for R_2. It determines the sharpness of the resonance. The larger the value of Q sharper is the resonance.

Long Questions Answers:

1. Answer:

Let the instantaneous value of voltage and current in the ac circuit containing a pure inductor are

$V = V_m \sin \omega t$ and

$I = I_m \sin (\omega t - \pi/2) = - I_m \cos \omega t$

where $\pi/2$ is the phase angle by which voltage Leads currently when ac flows through an inductor. Suppose the voltage and current remain constant for a small-time dt. Therefore, the electrical energy consumed in the small-time dt is

$dW = V I\, dt$

The total electrical energy consumed in one time period of ac is given by

$$W = \int_0^T VI\,dt = -\int_0^T V_m \sin\omega t .\ I_m \cos\omega t dt$$

$$= -V_m I_m \int_0^T \sin\omega t \cos\omega t dt$$

$$\text{or } W = -\frac{I_m V_m}{2}\int_0^T 2\sin\omega t \cos\omega t\,dt$$

$$\text{or } W = -\frac{I_m V_m}{2}\int_0^T \sin 2\omega t\,dt$$

$$\text{or } W = -\frac{I_m V_m}{2}\left[-\frac{\cos(2\omega t)}{2\omega}\right]_0^T = 0$$

Therefore, the total electrical energy consumed in an ac circuit by a pure inductor is W = 0

Now average power is defined as the ratio of the total electrical energy consumed over the entire cycle to the time period of the cycle, therefore

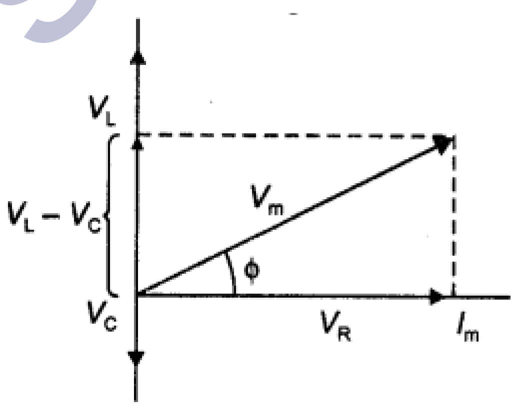
$P_{av} = \frac{W}{T} = 0$

Hence, the average power consumed in an ac circuit by a pure inductor is $P_{av} = 0$

Thus, a pure inductor does not consume any power when ac flows through it. Whatever energy is used in building up current is returned during the decay of current.

2. Answer:

The voltages across the various elements are drawn as shown in the figure below.



From the diagram, we observe that the vector sum of the voltage amplitudes V_R, V_L, and V_C equals a phasor whose length is the maximum applied voltage V_m, where the phasor Vm makes an angle ϕ with the current phasor Im. Since the voltage phasors, V_L and V_C are in opposite direction, therefore, a difference phasor (V_L - V_C) is drawn which is perpendicular to the phasor V_R. Adding vectorially we have

$$V_m = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= \sqrt{(I_m R)^2 + (I_m X_L - I_m X_C)^2}$$

$$\text{or } V_m = I_m \sqrt{R^2 + (X_L - X_C)^2}$$

where X_L = ω L and X_C = 1 / ω C, therefore, we can express the maximum current as

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The impedance Z of the circuit is defined as $Z = \sqrt{R^2 + (X_L - X_C)^2}$

For maximum Im, Z should be minimum (Z = R) or $X_C = X_L = 0$ and Φ = 0

For (I_m)min $\Phi \to 90°$ $(|X_C - X_L| >> R)$ $Z \to \infty$

Assertion and Reason Answers:

1. (d) A is false and R is also false.

Explanation:

On introducing soft iron core, the bulb will glow dimmer. This is because on introducing soft iron core in the solenoid, its inductance L increases, the inductive reactance, $X_L = W_L$ increases and hence the current through the bulb decreases.

2. (b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Like direct current, an alternating current also produces magnetic field. But the magnitude and direction of the field goes on changing continuously with lime.

Case Study Answers:

1. Answer :

(i) (d) 15Ω

Explanation:

Resistance of the two wire lines carrying power $= 0.5 \frac{\Omega}{\text{Km}}$

Total resistance $= (15 + 15)0.5 = 15\Omega$

(ii) (c) 600kW

Explanation:

Line power loss = I^2R

RMS current in the coil,

$$I = \frac{P}{V_1} = \frac{800 \times 10^3}{4000} = 200A$$

∴ Power loss = $(200)^2 \times 15 = 600kW$

(iii) (d) 1400kW

Explanation:

Assuming that the power loss is negligible due to the leakage of the current.

The total power supplied by the plant,

= 800kW + 600kW = 1400kW

(iv) (b) 3000V

Explanation:

Voltage drop in the power line = IR

= 200 × 15 = 3000V

(v) (d) 7000V

Explanation:

Total voltage transmitted from the plant,

= 3000V + 4000V = 7000V

2. Answer :

i. (c) 663.48Hz

Explanation:

Here, L = 0.12H, e = 480nF = 480 × 10^{-9}F

$R = 23\Omega$, V = 230V

$$V_0 = \sqrt{2} \times 230 = 325.22V$$

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

At resonance, $\omega L - \frac{1}{\omega C} = 0$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.12 \times 480 \times 10^9}} = 4166.67 \text{ rad s}^{-1}$$

$$\upsilon_R = \frac{4166.67}{2 \times 3.14} = 663.48H_z$$

ii. (a) 14.14A

Explanation:

Current, $I_0 = \frac{V_0}{R} = \frac{325.22}{23} = 14.14A$

iii. (b) 2299.3W

Explanation:

Maximum power, $P_{max} = \frac{1}{2}(I_0)^2 R$

$$= \frac{1}{2} \times (14.14)^2 \times 23 = 2299.3W$$

iv. (d) 21.74

Explanation:

Quality factor, $Q = \frac{X_R}{R} = \frac{\omega_r L}{R}$

$$= \frac{4166.67 \times 0.12}{23} = 21.74$$

v. (b) Current