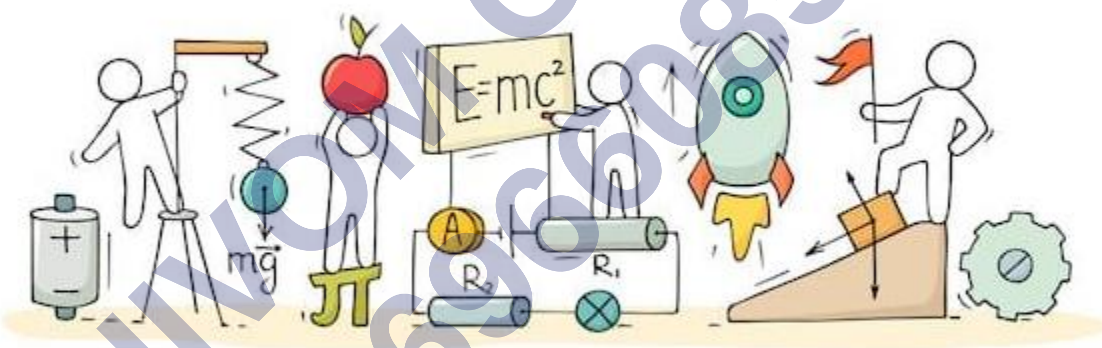
PHYSICS

CHAPTER 13: NUCLEI



NUCLEI

Nucleus:

The entire positive charge and nearly the entire mass of atom is concentrated in a very small space called the nucleus of an atom.

The nucleus consists of protons and neutrons. They are called nucleons.

Atomic Number:

The number of protons in the nucleus is called the atomic number. It is denoted by Z.

Mass number:

The total number of protons and neutrons present in a nucleus is called the mass number of the element. It is denoted by A.

Atomic Mass Unit:

The unit in which atomic and nuclear masses are measured is called atomic mass unit (u), defined as $\frac{1}{12}$th of the mass of an atom of $_6C^{12}$ isotope.

$$1\text{u} = \frac{1}{12} \times \frac{12}{6.02 \times 10^{23}}$$

$$1\text{u} = 1.66 \times 10^{-27}\text{kg}$$

1amu = 931 MeV

Nuclear Mass:

The total mass of the protons and neutrons present in a nucleus is called the nuclear mass.

Nuclide:

A nuclide is a specific nucleus of an atom characterized by its atomic number Z and mass number A. It is represented as, $_ZX^A$

Where X = chemical symbol of the element, Z = atomic number and A = mass number

Isotopes:

The atoms of an element which have the same atomic number, but different mass number are called isotopes.

Isotopes have similar chemical properties but different physical properties.

Isobars:

The atoms having the same mass number, but different atomic number are called isobars.

Isotones:

The nuclides having the same number of neutrons are called isotones.

Isomers:

These are nuclei with same atomic number and same mass number but in different energy states.

Electron Volt:

It is defined as the energy acquired by an electron when it is accelerated through a potential difference of 1 volt and is denoted by eV.

Discovery of Neutron:

Neutron was discovered experimentally by Chadwick in the year 1932 and was awarded Nobel Prize in Physics in 1935 for their discovery. A neutron is a neutral particle carrying no charge and having mass roughly equal to the mass of a proton.

Now the mass of a neutron is known to a high degree of accuracy and is equal to m_n = 1.67×10^{-27} kg. A free neutron is unstable and has a mean life of 1000 second. Whereas a free proton is stable. Neutron is however stable inside the nucleus.

Size of the Nucleus:

- It is found that a nucleus of mass number A has a radius.

$$R = R_0 A^{\frac{1}{3}}$$

$$\text{where}, R_0 = 1.2 \times 10^{-15} \text{m}$$

- This implies that the volume of the nucleus, which is proportional to R^3 is proportional A.

Mass-Energy and Nuclear Binding Energy:

Mass-Energy:

Einstein gave the famous mass–energy equivalence $E = mc^2$, here the energy equivalent of mass m is related by the above equation and c is the velocity of light in vacuum and is approximately equal to 3×10^8 m/s. Einstein's mass-energy relation has been experimentally verified in the study of nuclear reactions amongst nucleons, nuclei, electrons and other more recently discovered particles.

Nuclear Binding Energy:

Nucleus is made up of neutrons and protons. Therefore, mass of the nucleus (M) should be equal to the total mass of its protons and neutrons. However, it is found to be always less than this. This difference in mass (ΔM) is called the mass defect and is given by.

$$\Delta M = [Zm_p + (A - Z)m_n] - m$$

It is mass defect which appears in the form of binding energy, responsible for binding the nucleons together in the nucleus.

Nuclear Force:

The force acting inside the nucleus or acting between nucleons is called nuclear force.

- Nuclear forces are the strongest forces in nature.
- It is a very short-range attractive force.
- It is non-central. non-conservative force.
- It is neither gravitational nor electrostatic force.
- It is independent of charge.
- It is 100 times that of electrostatic force and 1038 times that of gravitational force.

According to the Yukawa, the nuclear force acts between the nucleon due to continuous exchange of meson particles.

Radioactivity:

It is the phenomenon of spontaneous disintegration of the nucleus of an atom with emission of one or more radiations like α-particle, β-particle or γ –rays.

Radioactive Decay:

It is a nuclear transformation process in which the radioactive rays are emitted from the nucleus of the atom. This process cannot be accelerated and slow down by any physical or chemical process.

Radioactivity Displacement Law:

It states that:

- When a radioactive nucleus emits an α -particle, atomic number decreases by 2 and mass number decreases by 4.
- When a radioactive nucleus emits β -particle, its atomic number increases by 1 but mass number remains same.
- The emission of a γ -particle does not change the mass number or the atomic number of the radioactive nucleus. The γ -particle emission by a radioactive nucleus lowers its energy state.

Alpha Decay:

In this process, parent nucleus disintegrates to give a daughter nucleus and helium nucleus or an alpha-particle. Mass number of the daughter nucleus decreases by four units and atomic number decreases by two units. A typical example of this decay mode is.

$$U_{92}^{238} \rightarrow He_2^4 + Th_{90}^{234}$$

Thus, daughter nucleus is shifted in periodic table by 2 unit in backward direction.

Beta Decay:

It is the process of emission of an electron from a radioactive nucleus. It may be represented as,

$${}_Z^AX \rightarrow {}_{Z+1}^{\;\;A}Y + {}_{-1}^{\;0}e + \overline{V}$$

Gamma Decay:

Alpha and beta decays of a radioactive nucleus leave the daughter nucleus in an excited state. If the excitation energy available with the daughter nucleus is not sufficient for further particle emission, it loses its energy by emitting electromagnetic radiations, also known as Gamma-rays. Mass and charge of the daughter nucleus remains the same as before the emission of Gamma-rays.

$$Ba_{56}^{137} \rightarrow Ba_{56}^{137} + \gamma$$

Alpha and beta decays of a radioactive nucleus leave the daughter nucleus in an excited state. If the excitation energy available with the daughter nucleus is not sufficient for further particle emission, it loses its energy by emitting electromagnetic radiations, also known as Gamma-rays. Mass and charge of the daughter nucleus remains the same as before the emission of Gamma-rays.

Law of Radioactive Decay:

According to the law of radioactive disintegration the rate of spontaneous disintegration of a radioactive element is proportional to the number of nuclei present at that time.

Mathematically, it can be written as

$$\frac{dN}{dt} \propto N \ldots (1)$$

Where, N is the number of atoms present at time t. Removing Proportionality sign, we get

$$\frac{dN}{dt} = -\lambda N \ldots (2)$$

Where, λ is a constant of proportionality and is known as decay constant of the element. Negative sign indicates that as t increase N decreases.

$$\frac{dN}{N} = -\lambda dt \ldots (3)$$

Integrating both sides, we have

$$\int \frac{dn}{N} = -\lambda \int dt$$

$$\log_e(N) = -\lambda t + C \ldots (4)$$

where C is constant of integration and is evaluated by the fact that at t = 0, number of atoms of the radioactive element is N0. Using this condition, we get

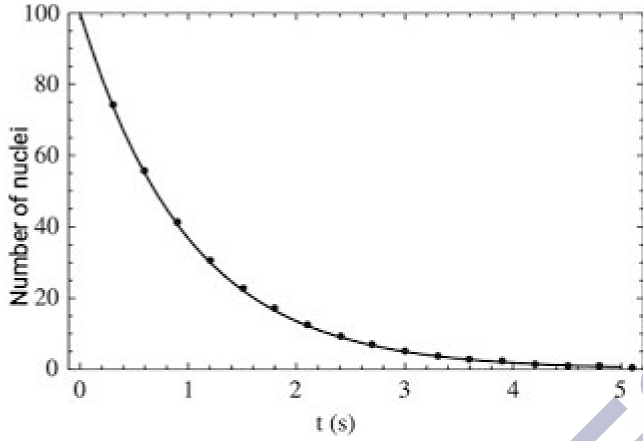
$$C = \log_e(N_0) \ \ldots (5)$$

Substituting this value of C in Eq. (5), we get

$$\log_e(N) = -\lambda + \log_e(N_0)$$

$$\log_e(N) - \log_e(N_0) = -\lambda t$$

Thus, $N = N_0 e^{-\lambda t} \ldots (6)$



Exponential decay curve

Decay or disintegration Constant:

It may be defined as the reciprocal or the time interval in which the number of active nuclei in a given radioactive sample reduces to 36.8% of its initial value.

Half-life:

The half-life of a radioactive substance is the time in which one-half of its nuclei will disintegrate. It is inversely proportional to the decay constant of the radioactive substance.

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

Mean Life:

The mean-life of a radioactive sample is defined as the ratio of the combined age of all the atoms and the total number of atoms in the given sample. It is given by,

$$\tau = \frac{T_{\frac{1}{2}}}{0.693} = 1.44T_{\frac{1}{2}}$$

Curie:

It is the SI unit of decay.

One curie is the decay rate of 3.7×10^{10} disintegrations per second.

Rutherford:

One Rutherford is the decay rate of 106 disintegrations per second.

Natural Radioactivity:

It is the phenomenon of the spontaneous emission of, α and γ radiations from the nuclei of naturally occurring isotopes.

Artificial or Induced Radioactivity:

It is the phenomenon of inducing radioactivity in certain stable nuclei by bombarding them by suitable high energy subatomic particles.

Nuclear Reaction:

It is a reaction which involves the change of stable nuclei of one element into the nucleus of another element.

Nuclear Energy:

Nuclear Fission:

The process of the splitting of a heavy nucleus into two or more lighter nuclei is called nuclear fission. When a slow-moving neutron strikes with a uranium nucleus (${}_{92}U^{235}$), it splits into ${}_{56}Ba^{144}$ and ${}_{36}Kr^{89}$ along with three neutrons and a lot of energy.

$$U_{92}^{235} + n_0^1 \rightarrow Ba_{56}^{144} + Kr_{36}^{89} + 3n_0^1$$

Nuclear fusion:

The process of combining of two lighter nuclei to form one heavy nucleus, is called nuclear fusion.

$$H_1^2 + H_1^2 \rightarrow He_2^3 + n_0^1$$

In this process, a large amount of energy is released. Hydrogen bomb is based on nuclear fusion. The source of Sun's energy is the nuclear fusion taking place at sun.

Critical size and Critical Mass:

- The size of the fissionable material for which reproduction factor is unity is called critical size and its mass is called critical mass of the material.
- The chain reaction in this case remains steady or sustained.

Moderator:

Any substance which is used to slow down fast-moving neutrons to thermal energies is called a moderator.
The commonly used moderators are water, heavy water (D_2O) and graphite.

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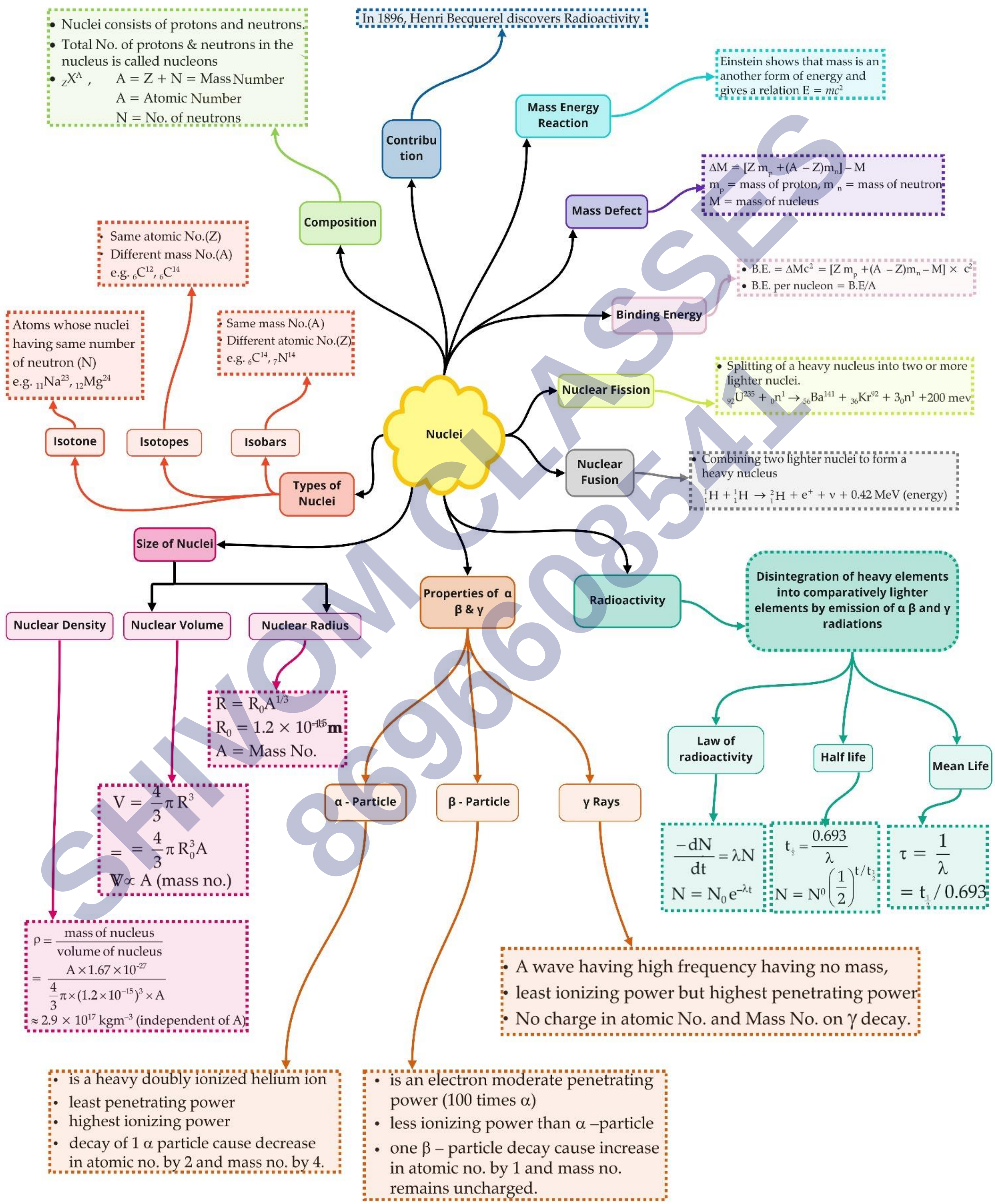


Important Questions

Multiple Choice questions-

Question 1. When a nucleus in an atom undergoes a radioactive decay, the electronic energy levels of the atom:
(a) do not change for any type of radioactivity.
(b) change for α and ß radioactivity but not for γ-radioactivity.
(c) change for α-radioactivity but not for others.
(d) change for ß-radioactivity but not for others.

Question 2. A radioactive isotope has a half-life of T years. The time it takes its activity to reduce to 3.125% is
(a) 5 T
(b) 6.654 T
(c) 5.645 T
(d) 6.654 T

Question 3. For a radioactive material, half-life is 10 minutes. If initially there are 600 number of nuclei, the time taken (in minutes) for the disintegration of 450 nuclei is:
(a) 20
(b) 10
(c) 30
(d) 15

Question 4. A nuclear explosive is designed to deliver 1 MW power in the form of heat energy. If the explosion is designed with nuclear fuel consisting of U^{235} to run a reactor at this power level for one year, then the amount of fuel needed is (given energy per fission is 200 MeV)
(a) 1 kg
(b) 0.01 kg
(c) 3.84 kg
(d) 0.384 kg

Question 5. When the radioactive isotope $_{88}Ra^{226}$ decays in a series by emission of three alpha (α) and a beta (ß) particle, the isotope X which remains undecayed is
(a) $_{83}X^{214}$
(b) $_{84}X^{218}$
(c) $_{84}X^{220}$
(d) $_{87}X^{223}$

Question 6. Fusion reaction takes place, at high temperature because:

(a) nuclei break up at high temperature
(b) atoms get ionised at high temperature
(c) kinetic energy is high enough to overcome the coulomb repulsion between nuclei
(d) molecules break up at high temperature

Question 7. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be:
(a) 1 : 16
(b) 4 : 1
(c) 1 : 4
(d) 5 : 4

Question 8. Radioactive material 'A' has decay constant '8 λ' and material 'B' has decay constant 'λ'. Initially they have same number of nuclei. After what time, the ratio of number of nuclei of material 'B' to that 'A' will be 1/e?
(a) $\frac{1}{7\lambda}$
(b) $\frac{1}{8\lambda}$
(c) $\frac{1}{9\lambda}$
(d) $\frac{1}{\lambda}$

Question 9. A radioactive nucleus A with a half-life T decays into a nucleus B. At t = 0, there is no nucleus B. At some time, t the ratio of the number of B to that of A is 0.3. Then, t is given by:
(a) t = T log(1.3)
(b) t = $\frac{T}{log(1.3)}$
(c) t = $\frac{Tlog(2)}{2log(1.3)}$
(d) t = $\frac{Tlog(1.3)}{log(2)}$

Very Short Answer Questions-

1. Complete the following nuclear reactions:

a) ${}_4Be^9 + {}_1H^1 \rightarrow {}_3Li^6 + \ldots\ldots\ldots\ldots$

b) ${}_5B^{10} + {}_2He^4 \rightarrow {}_7N^{13} + \ldots\ldots\ldots\ldots$

2. What is the Q-value of a nuclear reaction?

3. The wavelengths of some of the spectral lines obtained in hydrogen spectrum are 9546A°,

6463A° and 1216A°. Which one of these wavelengths belongs to the Lyman series?

4. Write the empirical relation for paschen series lines of hydrogen atoms.

5. What will be the ratio of the radii of two nuclei of mass numbers A_1 and A_2?

6. Two nuclei have mass numbers in the ratio 1: 2. What is the ratio of their nuclear densities?

7. A nucleus of mass number A has a mass defect Δm. Give the formula, for the binding energy per nucleon of this nucleus.

8. Write the relation between half-life and decay constant of a radioactive sample.

9. Write the nuclear decay process for β-decay of 15^{32}P.

10. State the relation between the mean life (τ) of a radioactive element and its decay constant λ.

Short Answer Questions-

Question 1. Draw the curve showing the binding energy/nucleon with a mass number of different nuclei. Briefly state, how nuclear fusion and nuclear fission can be explained on the basis of this graph.

Question 2. Define decay constant for a radioactive sample. Which of the following radiations α, β, and γ rays
(i) are similar to X-rays,
(ii) are easily absorbed by matter, and
(iii) are similar in nature to cathode rays?

Question 3. State the law of radioactive decay.
Plot a graph showing the number of undecayed nuclei as a function of time (t) for a given radioactive sample having a half-life $T_{1/2}$.
Depict In the plot the number of undecayed nuclei at (i) t = $3T_{1/2}$ and (ii) t = 5 $T_{1/2}$ (CBSE Delhi 2011)

Question 4. Draw a plot of the potential energy of a pair of nucleons as a function of their separations. Mark the regions where the nuclear force is (i) attractive and (ii) repulsive. Write any two characteristic features of nuclear forces. (CBSE AI 2012)

Question 5.

(a) Write the relation for binding energy (BE) (in MeV) of a nucleus of mass ${}_Z^AM$ atomic number (Z) and mass number (A) in terms of the masses of its constituents – neutrons and protons.

(b) Draw a plot of BE/A versus mass number A for 2 ≤ A ≤ 170. Use this graph to explain the release of energy in the process of nuclear fusion of two light nuclei. (CBSE Delhi 2014C)

Question 6. If both the number of neutrons and the number of protons are conserved in each nuclear reaction, in what way is mass converted into energy (or vice versa) in a nuclear reaction? Explain. (CBSE AI2016C)

Question 7. State two properties of nuclear forces. Write the relation between half-life and decay constant of a radioactive nucleus. (CBSE AI 2017C)

Question 8.
(a) Draw a graph showing the variation of binding energy per nucleon (BE/A) vs mass number A for the nuclei in 20 ≤ A ≤ 170.

(b) A nucleus of mass number 240 and having binding energy/nucleon 7.6 MeV splits into two fragments Y, 1 of mass numbers 110 and 130 respectively. If the binding energy/ nucleon of Y, 1 is equal to 8.5 MeV each, calculate the energy released in the nuclear reaction. (CBSE Al 2017C)

Question 9. Explain with the help of an example, whether the neutron-proton ratio in a nucleus increases or decreases due to beta decay.

Question 10. How is the size of a nucleus experimentally determined? Write the relation between the radius and mass number of the nucleus. Show that the density of the nucleus is independent of its mass number. (CBSE Delhi 2011C)

Long Answer's Questions-

1. The wavelength of the first member of the Balmer series in the hydrogen spectrum is $6563A^0$. Calculate the wavelength of the first member of Lyman series in the same spectrum.

2. A neutron is absorbed by a ${}_3^6$Li nucleus with subsequent emission of α−particle. Write the corresponding nuclear reaction. Calculate the energy released in this reaction. Given mass of ${}_3^6$Li=6.015126a.m.u. , Mass of ${}_2^4$He=4.00026044 a.m.u., Mass of neutron ${}_0^1$n=1.0086654 a.m.u. Mass of tritium ${}_1^1$H=3.016049 a.m.u.

3. Define decay constant of a radioactive sample. Which of the following radiation α−rays, β−rays and γ−rays.

a) Are they similar to X – rays?

b) Are they easily absorbed by matter?

4. State radioactive decay law and hence derive the relation $N=N_0e^{-\lambda t}$ where symbols have their usual meanings.

5. Define half life and decay constant of a radioactive element. Write their S.I. unit. Define expression for half life.

6. Draw a curve between mass number and binding energy per nucleon. Give two salient features of the curve. Hence define binding energy.

7.

a) Two stable isotopes of lithium $^{6}_{3}$Li and $^{7}_{3}$Li have respective abundances of 7.5 and 92.5 . These isotopes have masses 6.01512u and 7.01600u respectively. Find the atomic mass of lithium.

b) Boron has two stable isotopes, $^{10}_{5}$B and $^{11}_{5}$B. Their respective masses are 10.01294u and 11.00931u , and the atomic mass of boron is 10.811u . Find the abundances of $^{10}_{5}$B and $^{11}_{5}$B.

8. Obtain the binding energy of the nuclei $^{56}_{26}$Fe and $^{209}_{83}$Bi in units of MeV from the following data: m($^{56}_{26}$Fe)=55.934939u, m($^{209}_{83}$Bi)=208.980388u.

Assertion and Reason Questions-

1. For question, statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a) (b) (c) and (d) as given below.

a) Both A and R are true, and R is the correct explanation of A.
b) Both A and R are true, but R is NOT the correct explanation of A.
c) A is true, but R is false.
d) A is false and R is also false.

Assertion (A): Thermonuclear fusion reactions may become the source of unlimited power for the mankind.

Reason (R): A single fusion event involving isotopes of hydrogen produces more energy than energy from nuclear fission of a single uranium.

2. For question, statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a) (b) (c) and (d) as given below.

a. Both A and R are true, and R is the correct explanation of A.
b. Both A and R are true, but R is NOT the correct explanation of A.
c. A is true, but R is false.
d. A is false and R is also false.

Assertion (A): A fission reaction can be more easily controlled than a fission reaction.

Reason (R): The percentage of mass converted to energy in a fission reaction is 0.1% whereas in a fission reaction it is 0.4%

Case Study Questions-

1. When subatomic particles undergo reactions, energy is conserved, but mass is not necessarily conserved. However, a particle's mass "contributes" to its total energy, in accordance with Einstein's famous equation, $E = mc^2$ In this equation, E denotes the energy carried by a particle because of its mass. The particle can also have additional energy due to its motion and its interactions with other particles. Consider a neutron at rest and well separated from other particles. It decays into a proton, an electron and an undetected third particle as given here: Neutron → proton + electron + ???

The given table summarizes some data from a single neutron decay. Electron volt is a unit of energy. Column 2 shows the rest mass of the particle times the speed of light squared.

Particle	**Mass × c^2 (MeV)**	**Kinetic energy (MeV)**
Neutron	940.97	0.00
Proton	939.67	0.01
Electron	0.51	0.39

(i) From the given table, which properties of the undetected third particle can be calculate?

a) Total energy, but not kinetic energy.
b) Kinetic energy, but not total energy.
c) Both total energy and kinetic energy.
d) Neither total energy nor kinetic energy.

(ii) Assuming the table contains no major errors, what can we conclude about the (mass × c^2) of the undetected third particle?

a) It is 0. 79 MeV
b) It is 0.39 MeV
c) It is less than or equal to 0.79 MeV; but we cannot be more precise.
d) It is less than or equal to 0.40 MeV; but we cannot be more precise.

(iii) Could this reaction occur?

Proton → neutron + other particles

a) Yes, if the other particles have much more kinetic energy than mass energy.
b) Yes, but only if the proton has potential energy (due to interactions with other particles).
c) No, because a neutron is more massive than a proton.
d) No, because a proton is positively charged while a neutron is electrically neutral.

(iv) How much mass has to be converted into energy to produce electric power of 500MW for one hour?

a) 2×10^{-5}kg
b) 1×10^{-5}kg
c) 3×10^{-5}kg
d) 4×10^{-5}kg

(v) The equivalent energy of 1g of substance is.

a) 9×10^{13}J
b) 6×10^{12}J
c) 3×10^{13}J
d) 6×10^{13}J

2. Neutrons and protons are identical particle in the sense that their masses are nearly the same and the force, called nuclear force, does into distinguish them. Nuclear force is the strongest force. Stability of nucleus is determined by the neutron proton ratio or mass defect or packing fraction. Shape of nucleus is calculated by quadrupole moment and spin of nucleus depends on even or odd mass number. Volume of nucleus depends on the mass number. Whole mass of the atom (nearly 99%) is centered at the nucleus.

(i) The correct statements about the nuclear force is/ are.

a) Change independent.
b) Short range force.
c) Non-conservative force.
d) All of these.

(ii) The range of nuclear force is the order of.

a) 2×10^{-10}m
b) 1.5×10^{-20}m
c) $1.2.\times 10^{-4}$m
d) 1.4×10^{-15}m

(iii) A force between two protons is same as the force between proton and neutron. The nature of the force is.

a) Electrical force.
b) Weak nuclear force.
c) Gravitational force.
d) Strong nuclear force.

(iv) Two protons are kept at a separation of 40 A. F_n is the nuclear force and F_e is the electrostatic force between them. Then.

a) $F_n << F_e$
b) $F_n = F_e$
c) $F_n >> F_e$
d) $F_n = F_e$

(v) AD the nucleons in an atom are held by.

a) Nuclear forces
b) Van der Waal's forces
c) Tensor forces
d) Coulomb forces

Multiple Choice Question's Answers-

1. **Answer: (b) change for α and ß radioactivity but not for γ-radioactivity.**
2. **Answer: (a) 5 T**
3. **Answer: (a) 20**
4. **Answer: (d) 0.384 kg**
5. **Answer: (a) $_{83}X^{214}$**

6. Answer: (c) kinetic energy is high enough to overcome the coulomb repulsion between nuclei
7. Answer: (d) 5 : 4
8. Answer: (a) $\frac{1}{7\lambda}$
9. Answer: (d) t = $\frac{Tlog(1.3)}{\log(2)}$

Very Short Answers-

1. Ans: (a)

$$_4Be^9 + {_1H^1} \rightarrow {_3Li^6} + {_2He^4}$$

Ans: (b)

$$_5B^{10} + {_2He^4} \rightarrow {_7N^{13}} + {_0n^1}$$

2. Ans: Q–value = (Mass of reactants – Mass of products)
3. Ans: 1216A° belong to the Lyman series.
4. Ans:

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \text{ where } n = 4, 5, 6, 7\ldots\ldots$$

5. Ans:

The ratio is $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$

6. Ans: The densities of both nuclei are equal as they do not depend upon mass number.
7. Ans:

The formula is E = $\frac{\Delta m \times c^2}{A}$

8. Ans:

The relation is $T_{1/2} = \frac{0.693}{\lambda}$

9. Ans:

The process is

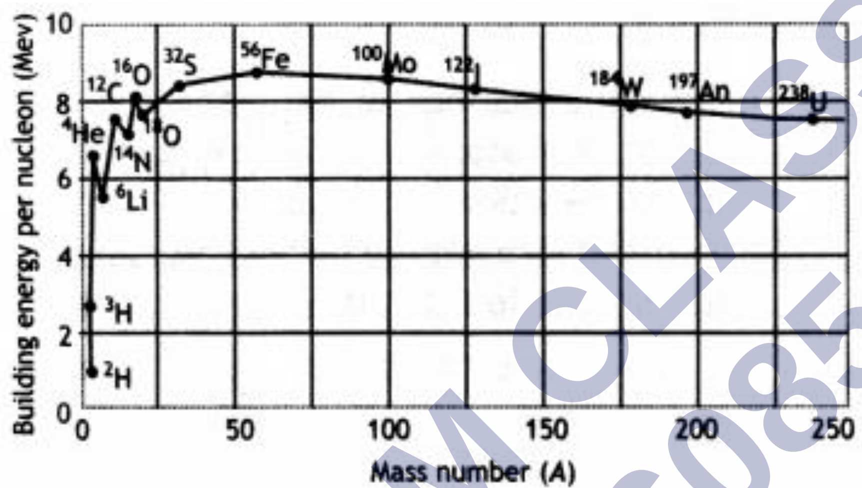
$$^{32}_{15}P \rightarrow ^{32}_{16}S + ^{0}_{-1}e + \bar{\nu} + Q$$

10. Ans:

The two are related as τ = 1 / λ.

Short Answers -

1. Answer: The diagram is as shown.



The binding energy per nucleon as a function of mass number

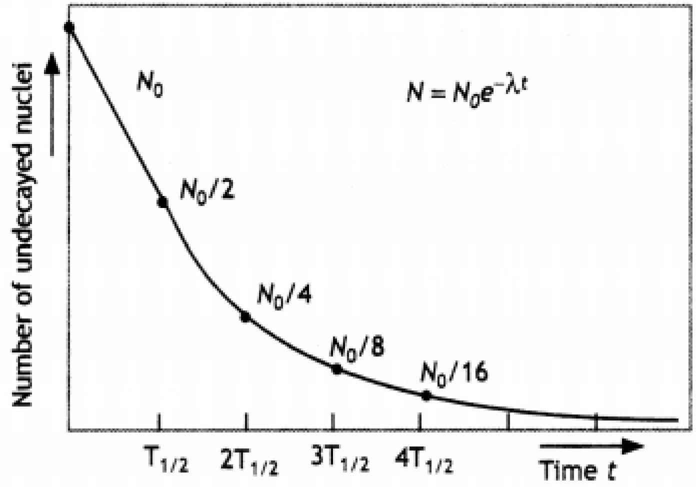
Light nuclei have a small value of binding energy per nucleon, therefore to become more stable they fuse to increase their binding energy per nucleon.

A very heavy nucleus, say A 240, has Lower binding energy per nucleon compared to that of a nucleus with A = 120. Thus if a nucleus A = 240 breaks into two A = 120 nuclei, nucleons get more tightly bound. This implies energy would be released in the process.

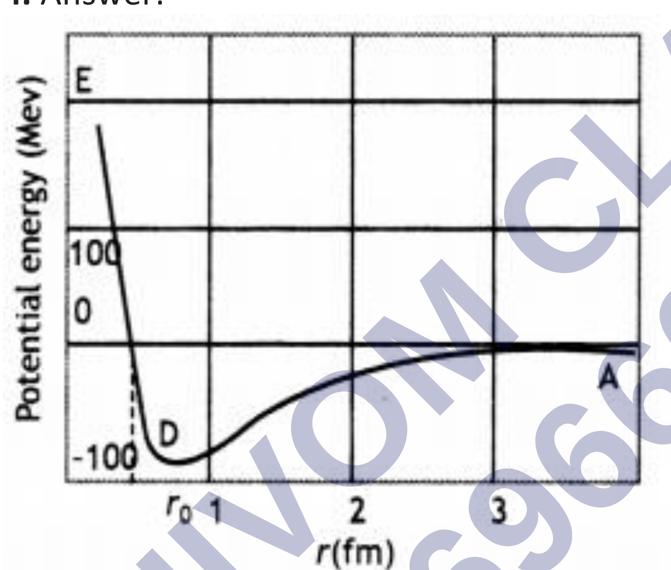
2. Answer: The decay constant is defined as the reciprocal of that time duration for which the number of nuclei of the radioactive sample decays to 1 / e or 37 % of its original value.
(i) Gamma
(ii) Alpha
(iii) Beta

3. Answer: The number of nuclei disintegrating per second is proportional to the number of nuclei present at the time of disintegration and is independent of alt physical conditions like temperature, pressure, humidity, chemical composition, etc.

The plot is as shown.



4. Answer:



For $r > r_0$ (attraction), For $r < r_0$ (repulsion)

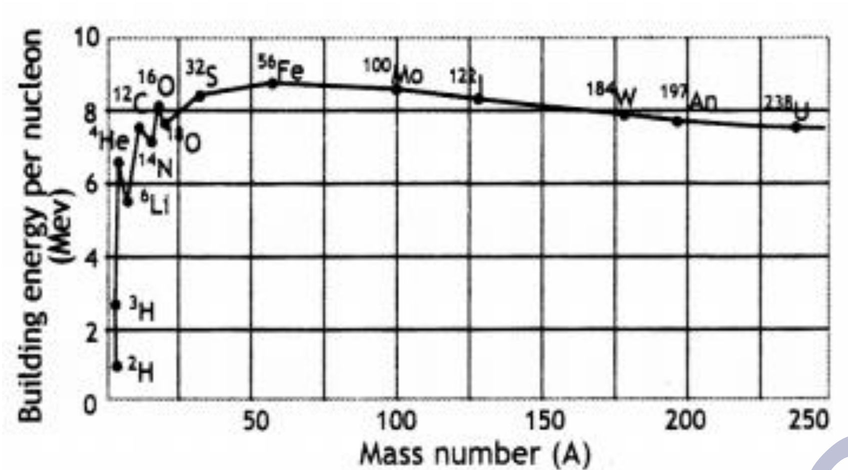
1. Strong attractive force (stronger than the repulsive electric force between the protons)
2. Are short-range forces.

5. Answer:

(a) The required expression is

$\Delta E = (Zm_p + (A - Z)\, m_n - M) \times 931$ MeV

(b)



The binding energy per nucleon as a function of mass number

Since the binding energy of the smaller nuclei like hydrogen is less, therefore they fuse together to form helium in order to increase their binding energy per nucleon and become stable. This means that the final system is more tightly bound than the initial system. Again energy would be released in such a process of fusion.

6. Answer: We know that the binding energy of a nucleus gives a negative contribution to the mass of the nucleus (mass defect). Now, since proton number and neutron number are conserved in a nuclear reaction the total rest mass of neutrons and protons is the same on either side of a reaction. But the total binding energy of nuclei on the left side need not be the same as that on the right-hand side.

The difference in these binding energies appears as the energy released or absorbed in a nuclear reaction. Since binding energy contributes to mass, we say that the difference in the total mass of nuclei on the two sides gets converted into energy or vice-versa.

7. Answer:

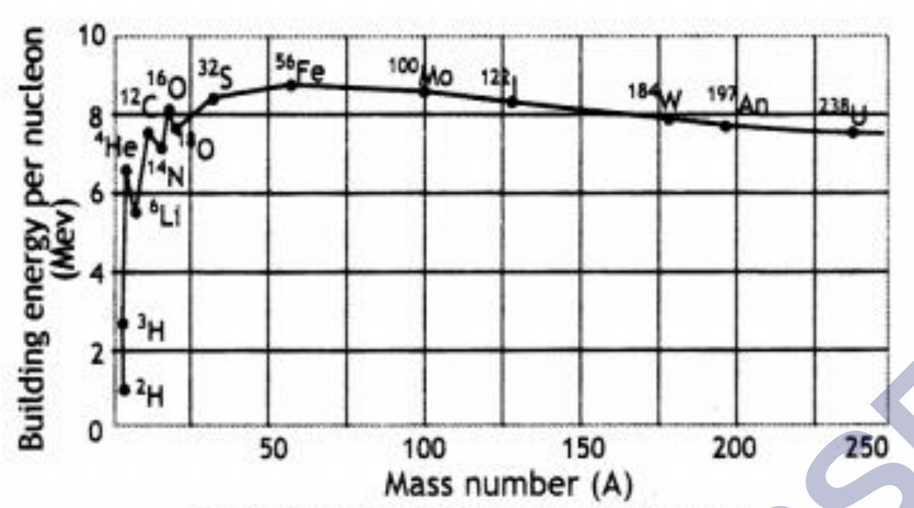
1.They are saturated forces.
2.They are charge - independent.

The required relation is

$$T = \frac{\ln 2}{\lambda} = \frac{2.303 \log 2}{\lambda} = \frac{0.693}{\lambda}$$

8. Answer:

(a)



The binding energy per nucleon as a function of mass number

Since the binding energy of the smaller nuclei like hydrogen is less, therefore they fuse together to form helium in order to increase their binding energy per nucleon and become stable. This means that the final system is more tightly bound than the initial system. Again energy would be released in such a process of fusion.

(B) Energy released per fission
= (110 + 130) × 8.5 – 240 × 7.6
= 240 × (8.5 – 7.6) MeV
= 240 × 0.9
= 216.0 MeV

9. Answer:
Consider the following decay

$$^{234}_{90}Th \xrightarrow{\beta^-} {}^{234}_{91}Pa$$

Number of neutrons before beta decay
= 234-90 = 144
Number of neutrons after beta decay
= 234-91 =143
Number of protons before beta decay
= 90
Number of protons after beta decay
= 91
Neutron-proton ratio before beta decay

$= \frac{144}{90} = 1.6$

Neutron-proton ratio after beta decay

$= \frac{143}{91} = 1.57$

Thus neutron-proton ratio decreases during beta decay.

10. Answer: The size of the nucleus can be determined by the Rutherford experiments on alpha particles scattering. The distance of the nearest approach is approximately the size of the nucleus. Here it is assumed that only coulomb repulsive force caused scattering. With alpha rays of 5.5 MeV, the size of the nucleus was found to be less than 4×10^{-14} m. By doing scattering experiments with fast electrons bombarding targets of different elements, the size of the nuclei of various elements determined accurately.

The required relation is

$R = R_o A_1/3$, where $R_o = 1.2 \times 10^{-15}$ m

The density of a nucleus of mass number A and radius R is given by

$$\text{Nuclear density} = \frac{\text{Mass of nucleus}}{\text{Volume of the nucleus}}$$

$$= \frac{A \text{ amu}}{\frac{4}{3}\pi R^3} = \frac{A \times 1.660565 \times 10^{-27}}{\frac{4}{3}\pi R_0^3 A}$$

$$= 2.3 \times 10^{17} \text{ kg m}^{-3}$$

which is independent of the mass number A.

Long Answers-

1. Ans: It is known that,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{{n_i}^2}\right), n = 3, 4, 5, \ldots\ldots$$

For first member $n_i = 3$ (Balmer series)

$$\Rightarrow \frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \frac{1}{\lambda_1} = R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\Rightarrow \lambda_1 = \frac{36}{5R} \quad \ldots\ldots\ldots(1)$$

For first member of Lyman series

$$\Rightarrow \frac{1}{{\lambda_1}'} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\Rightarrow \frac{1}{{\lambda_1}'} = R\left(1 - \frac{1}{4}\right)$$

$$\Rightarrow {\lambda_1}' = \frac{4}{3R} \quad \ldots\ldots\ldots(2)$$

From (1) and (2)

$$\Rightarrow \frac{{\lambda_1}'}{\lambda_1} = \frac{4}{3R} \times \frac{5R}{36}$$

$$\Rightarrow {\lambda_1}' = \frac{5}{27}\lambda_1$$

$$\Rightarrow {\lambda_1}' = \frac{5}{27} \times 6563$$

$$\Rightarrow {\lambda_1}' = 1215.4\overset{\circ}{A}$$

Therefore, the wavelength of the first member of the Lyman series is $1215.4\overset{\circ}{A}$.

2. Ans:

Nuclear reaction is: ${}_0^1n + {}_3^6Li \rightarrow {}_2^4He + {}_1^3H$

Mass of reactants $= m\left({}_{0}^{1}n\right) + m\left({}_{3}^{6}Li\right) =$

⇒ Mass = 1.0086654 + 6.015126 = 7.0237914 a.m.u

Mass Defect, Δm=mass of reactant mass of product

⇒ Δm = 7.02371947.0186534

⇒ Δm = 0.005138 a.m.u.

It is known that, 1a.m.u.= 931MeV

Energy released, E=Δm × 931MeV

⇒ E = 0.005138 × 931

⇒ E = 4.783MeV

3. Ans:

(a) Radioactive decay constant(λ) is the reciprocal of time during which the number of atoms in the radioactive substance is reduced to 36.8 of the original number of atoms in it.

γ−rays are similar to X-rays.

(b) Penetration power of α−rays is less than that of β−rays and γ−rays. So γ−rays are easily absorbed by matter.

4. Ans: From the radioactive decay law, the rate of disintegration of a radioactive substance at an instant is directly proportional to the number of nuclei in the radioactive substance at that time i.e.

N=N0e−λt where symbols have their usual meanings

Consider a radioactive substance having N0 atoms initially at time(t=0). After time(t), let the number of atoms left undecayed be N.

If dN is the number of atoms decayed in time dt, then

From the law of radioactive decay: $\frac{-dN}{dt} \prec N$ or $\frac{-dN}{dt} = \lambda N$ ....(1)

Where,

λ is the decay constant and negative sign indicates that a radioactive sample goes on decreasing with time.

$$(1) \Rightarrow \frac{dN}{N} = -\lambda dt$$

Integrating both the sides

$$\log_e N = -\lambda t + K \text{ ....}(2)$$

Where K is constant of integration

For $t = 0$, $N = N_0$

$$\Rightarrow K = log_e N_0$$

Substituting K in equation (2)

$$\Rightarrow \log_e N = -\lambda t + log_e N_0$$

$$\Rightarrow \log_e N - log_e N_0 = -\lambda t \ \left[log_e m - log_e n = log_e\left(\frac{m}{n}\right)\right]$$

$$\Rightarrow \log_e\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t}$$

$$\Rightarrow N = N_0 e^{-\lambda t}$$

Hence derived.

5. Ans: The time during which half of the atoms of the radioactive substance disintegrate is called half life of a radioactive substance.

It is known that, $N = N_0 e^{-\lambda t}$

If $t = T_{1/2}$ $(Half\ \text{life})$, $N = \dfrac{N_0}{2}$

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\Rightarrow e^{\lambda T_{1/2}} = 2$$

$$\Rightarrow \lambda T_{1/2} = \log_e 2$$

$$\Rightarrow \lambda T_{1/2} = 2.303 \times \log_{10} 2$$

$$\Rightarrow \lambda T_{1/2} = 2.303 \times 0.3010$$

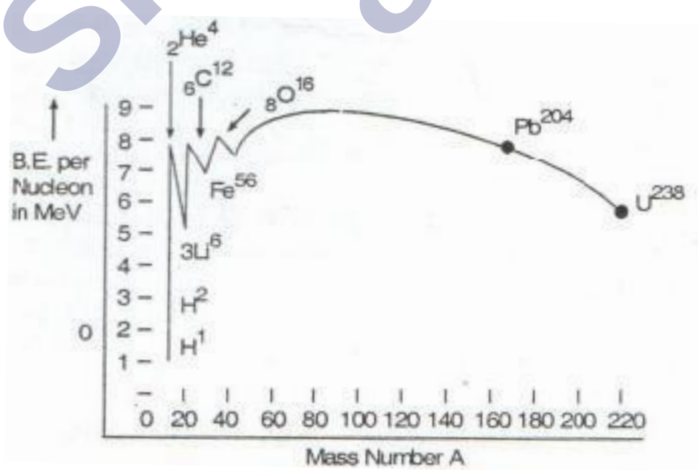
$$\Rightarrow T_{1/2} = \frac{0.6931}{\lambda}$$

S.I. unit - $second\ (s)$

Radioactive decay constant(λ) is the reciprocal of the time during which the number of atoms in the radioactive substance reduces to 36.8 of the original number of atoms in it.

S.I. unit – s^{-1} or min^{-1}

6. Ans: The total energy required to disintegrate the nucleus into its constituent particles is called binding energy of the nucleus.



Salient features of the curve

(i) The intermediate nuclei have a large value of binding energy per nucleon, so they are most stable. (For 30<A>63)

(ii) The binding energy per nucleon has low value for both the light and heavy nuclei. So, they are unstable nuclei.

7. Ans: (a) Given that,

Mass of lithium isotope ${}_3^6Li$, $m_1 = 6.01512\ u$

Mass of lithium isotope ${}_3^7Li$, $m_2 = 7.01600\ u$

Abundance of ${}_3^6Li$, $\eta_1 = 7.5$

Abundance of ${}_3^7Li$, $\eta_2 = 92.5$

The atomic mass of lithium atom, $m = \dfrac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$

$$\Rightarrow m = \frac{6.0512 \times 7.5 + 7.01600 \times 92.5}{7.5 + 92.5}$$

$$\Rightarrow m = 6.940934u$$

Therefore, the atomic mass of lithium is $6.940934u$.

(b) It is given that,

Mass of boron isotope ${}_{5}^{10}B$, $m_1 = \mathbf{10.01294}u$

Mass of boron isotope ${}_{5}^{11}B$, $m_2 = \mathbf{11.00931u}$

Abundance of ${}_{5}^{10}B$, $\eta_1 = x$

Abundance of ${}_{5}^{11}B$, $\eta_2 = (100 - x)$

Atomic mass of boron, $m = 10.811u$

The atomic mass of boron atom, $m = \dfrac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$

$$\Rightarrow 10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + 100 - x}$$

$$\Rightarrow 108.11 = 10.01294x + 1100.931 - 11.00931x$$

$$\Rightarrow x = \frac{19.821}{0.99637} = 19.89$$

$$\Rightarrow 100 - x = 80.11$$

Therefore, the abundance of ${}_{5}^{10}B$ is 19.89 and abundance of ${}_{5}^{11}B$ is 80.11.

8. Ans. Given that,

Atomic mass of ${}_{26}^{56}Fe$, $m_1 = 55.934939u$

${}_{26}^{56}Fe$ nucleus has 26 protons and $(56 - 26) = 30$ neutrons

Therefore, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where,

Mass of proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

$\Rightarrow \Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$

$\Rightarrow \Delta m = 26.20345 + 30.25995 - 55.934939$

$\Rightarrow \Delta m = 0.528461u$

It is known that, $1u = 931.5 \frac{MeV}{c^2}$

The binding energy of this nucleus is $E_{b_1} = \Delta mc^2$

c is the speed of light

$$\Rightarrow E_{b_1} = 0.528461 \times 931.5 \left(\frac{MeV}{c^2}\right) \times c^2$$

$$\Rightarrow E_{b_1} = 492.26 MeV$$

Average binding energy per nucleon$= \frac{492.26}{56} = 8.79 MeV$

Atomic mass of ${}^{209}_{83}Bi$, $m_2 = 208.980388u$

${}^{209}_{83}Bi$ nucleus has 83 protons and $(209 - 83) = 126$ neutrons.

Therefore, the mass defect of this nucleus, $\Delta m' = 83 \times m_H + 126 \times m_n - m_2$

Where,

Mass of proton,$m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

$$\Rightarrow \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$$

$$\Rightarrow \Delta m' = 83.649475 + 127.091790 - 208.980388$$

$$\Rightarrow \Delta m' = 1.760877u$$

It is known that, $1u = 931.5 \frac{MeV}{c^2}$

The binding energy of this nucleus is $E_{b_2} = \Delta m' c^2$

Where,

c is the speed of light

$$\Rightarrow E_{b_2} = 1.760877 \times 931.5 \left(\frac{MeV}{c^2}\right) \times c^2$$

$$\Rightarrow E_{b_2} = 1640.26 MeV$$

Clearly, average binding energy per nucleon$= \frac{1640}{209} = 7.848 MeV$

Assertion and Reason Answers-

1. (c) A is true, but R is false.

Explanation:

When fusion is achieved by raising the temperature of the system so that particles have enough kinetic energy to overcome the coulomb repulsive behaviour, it is called thermonuclear fusion. It is clean source of energy, but energy released in one fusion is much less than a single uranium fission.

2. Both A and R are true, but R is NOT the correct explanation of A.

Explanation:

Percentage of mass converted to energy in a fission reaction is 0.1% whereas in a fusion reaction it is 0.4%. Consequently, the amount of energy released is more in a fusion than in a fission reaction. II is not easy Jo control a fusion reaction.

Case Study Answers-

1. Answer :

(i) (a) Total energy, but not kinetic energy.

Explanation:

As just shown, energy conservation allows us to calculate the third particle's total energy. But we do not know what percentage of that total is mass energy.

(ii) (d) It is less than or equal to 0.40 MeV, but we cannot be more precise.

Explanation:

According to the passage, subatomic reactions do not conserve mass. So, we cannot find the third particle's mass by setting $m_{neutron}$ equal to-

$$m_{proton} + m_{electron} + E_{third\ particle}$$

The neutron has energy 940.97 MeV. The proton has energy 939.67 MeV + 0.01 MeV = 939.69 MeV. The electron has energy 0.51 MeV + 0.39 MeV = 0.90 MeV. Therefore, the third particle has energy.

$E_{third\ particle} = E_{neutron} - E_{proton} - E_{electron}$

= 940.97 - 939.67 - 0.90 = 0.40 MeV

We just found the third particle's total energy, the sum of its mass energy and kinetic energy. Without more information, we cannot figure out how much of that energy is mass energy.

(iii) (b) Yes, but only if the proton has potential energy (due to interactions with other particles).

(iv) (a) 2×10^{-5}kg

Explanation:

Here, P = 500MW = 5×10^8W,

t = 1h = 3600s

Energy produced, E = P × t = $5 \times 10^8 \times 3600 = 18 \times 10^{11}$J

$$\text{As } E = \Delta mc^2$$

$$\therefore \Delta m = \frac{E}{c^2} = \frac{18\times10^{11}}{(3\times10^8)^2}$$

$$= \frac{18\times10^{11}}{(3\times10^8)^{16}} = 2 \times 10^{-5}\text{kg}$$

(v) (a) 9×10^{13}J

Explanation:

Using, $E = mc^2$

Here, m = 1g = 1×10^{-3}kg, c = 3×10^8m s^{-1}

∴∴ E = $10^{-3} \times 9 \times 10^{16} = 9 \times 10^{13}$J

2. Answer :

(i) (d) All of these.

Explanation:

All options are basic properties of nuclear forces. So, all options are correct.

(ii) (d) 1.4×10^{-15}m

Explanation:

The nuclear force is of short range and the range of nuclear force is the order of 1.4×10^{-15}m. Now, volume $\infty R3 \infty A$

(iii) (d) Strong nuclear force.

(iv) (a) $F_n << F_e$

Explanation:

Nuclear force is much stronger than the electrostatic force inside the nucleus i.e., at distances of the order of fermi. At 40 A, nuclear force is ineffective and only electrostatic force of repulsion is present. This is very high at this distance because nuclear force is not acting now and the gravitational force is very feeble. $F_{nuclear} << F_{elcctrostatic}$ in this case.

(v) (a) Nuclear forces