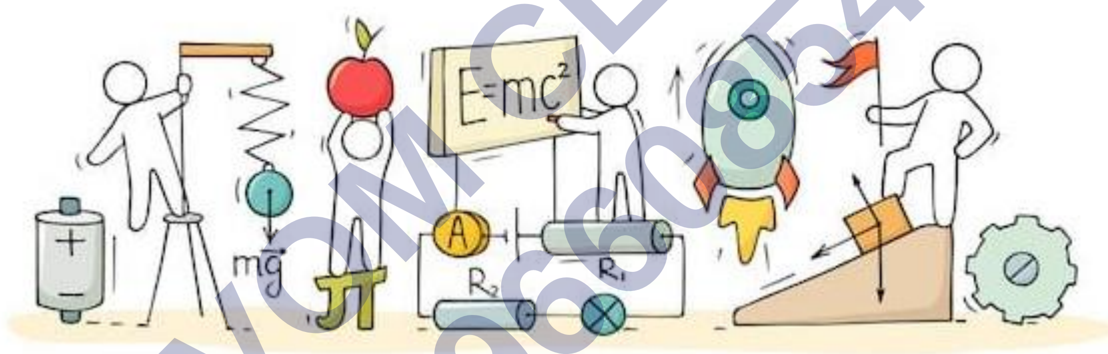
PHYSICS

CHAPTER 1: ELECTRIC CHARGES AND FIELDS



ELECTRIC CHARGES AND FIELDS

Introduction:

Study of static charges is called electrostatics and this complete electrostatic will be discussed in two chapters. In this chapter we begin with a discussion of electric charge, some properties of charged bodies, and fundamental electric force between two charged bodies.

What is Electric Charge?

Electric Charge is a fundamental property of a matter which is responsible for electric forces between the bodies. Two electrons placed at small separation are found to repeal each other, this repulsive force (Electric force) is only because of electric charge on electrons.

When a glass rod is rubbed with silk, the rod acquires one kind of charge, and the silk acquires the second kind of charge. This is true for any pair of objects that are rubbed to be electrified. Now if the electrified glass rod is brought in contact with silk, with which it was rubbed, they no longer attract each other.

Types of Electric Charge:

There are two types of charge exist in our nature.

- Positive Charge
- Negative Charge

If any object loses their electrons then they get positive charge. It is denoted by (+q) sign. If any object gain electrons from another object, then they get negative charge. It is denoted by (-q) sign. The charges were named as positive and negative by the American scientist Benjamin Franklin. If an object possesses an electric charge, it is said to be electrified or charged. When it has no charge it is said to be neutral.

Basic Properties of Electric Charge:

The important properties and characteristic of electric charge are given below.

Attraction and Repulsion: Like charges repel each other while unlike charges attract each other.

Electric Induction: When a charged object brings to contact with another uncharged, it gets opposite charge of charged object. It is called charging by induction.

Charge is Quantized: An object that is electrically charged has an excess or deficiency of some whole number of electrons. Since, electrons cannot be divided into fraction of electrons, it means that the charge of an object is a whole-number multiple of the charge of an electron. For example, it cannot have a charge equal to the charge of 0.5 or 1000.5 electrons.

Mathematically $q = \pm ne$, here $n = 1, 2, 3$ and $e = 1.6 \times 10^{-19}$ coulomb.

Electric Charge is Conserved: According to this property, "An electric charge neither can be created nor can be destroyed" i.e., total net charge of an isolated system is always conserved. Thus, when a glass rod rubbed with silk cloth, both glass rod and silk cloth acquire opposite charge in same quantity. Thus, total amount of charge remains same before rubbing as well as after rubbing.

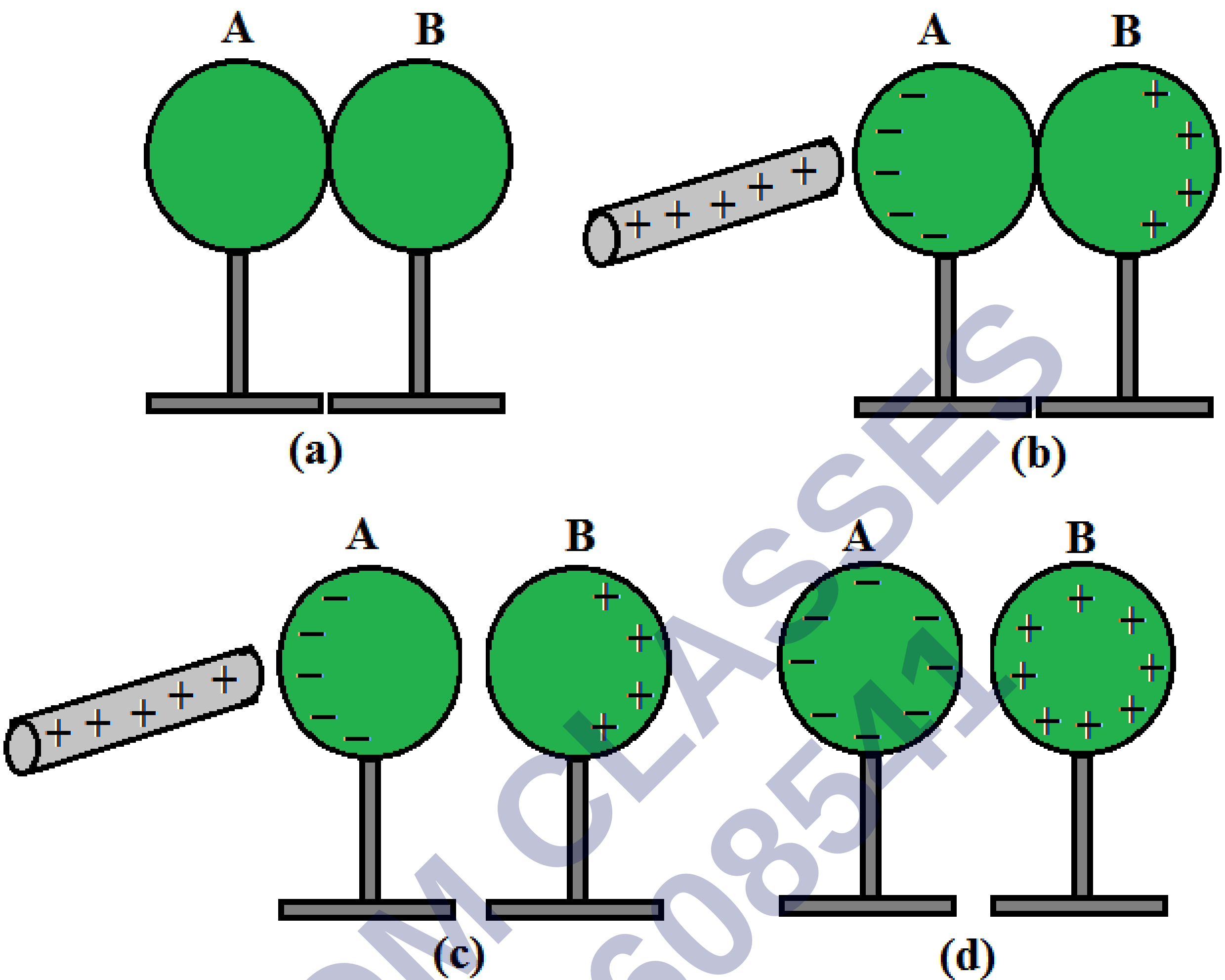
Conductors and Insulators:

Some substances easily allow passage of electricity through them while others do not. Substances which allow electricity to pass through them easily are called 'conductors. They have electrons that are free to move inside the material. Metals, human and animal bodies, earth etc. are example of conductors. Non-metals e.g., glass, plastic, wood are 'insulators' because they do not easily allow passage of electricity through them.

Most substances are either conductors or insulators. There is a third category called 'semiconductors' which are intermediate between conductors and insulators because they partially allow movement of charges through them.

Charging by Induction:

Now as we know that two oppositely charged bodies attract each other. But it also has been our observation that a charged body attracts a neutral body as well. This is explained on the basis of charging by induction. In induction process two bodies (at least one body must be charged) are brought very close, but they never touch each other.



Let us examine how a charged body attracts an uncharged body. Imagine a conducting or partially conducting body (sphere here) is kept on an insulating stand and a charged rod (positive, for example) is brought very close to it. It will attract electrons to its side and the farther end of the sphere will become positively charged as it is deficient of electrons.

Coulomb's Law:

- In 1785 Charles Coulomb (1736-1806) experimentally established the fundamental law of electric force between two stationary charged particles. He observed that An electric force between two charge particles has the following properties:
- It is directed along a line joining the two particles and is inversely proportional to the square of the separation distance r, between them.
- t is proportional to the product of the magnitudes of the charges, $|q_1|$ and $|q_2|$, of the two particles.

- It is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

From these observations, Coulomb proposed the following mathematical form for the electric force between two charges. The magnitude of the electric force F between charges q1 and q2 separated by a distance r is given by

$$F = k\frac{|q_1||q_1|}{r^2}$$

where k is a constant called the Coulomb constant. The proportionality constant k in Coulomb's law is similar to G in Newton's law of gravitation. Instead of being a very small number like G (6.67×10^{-11}), the electrical proportionality constant k is a very large number. It is approximately.

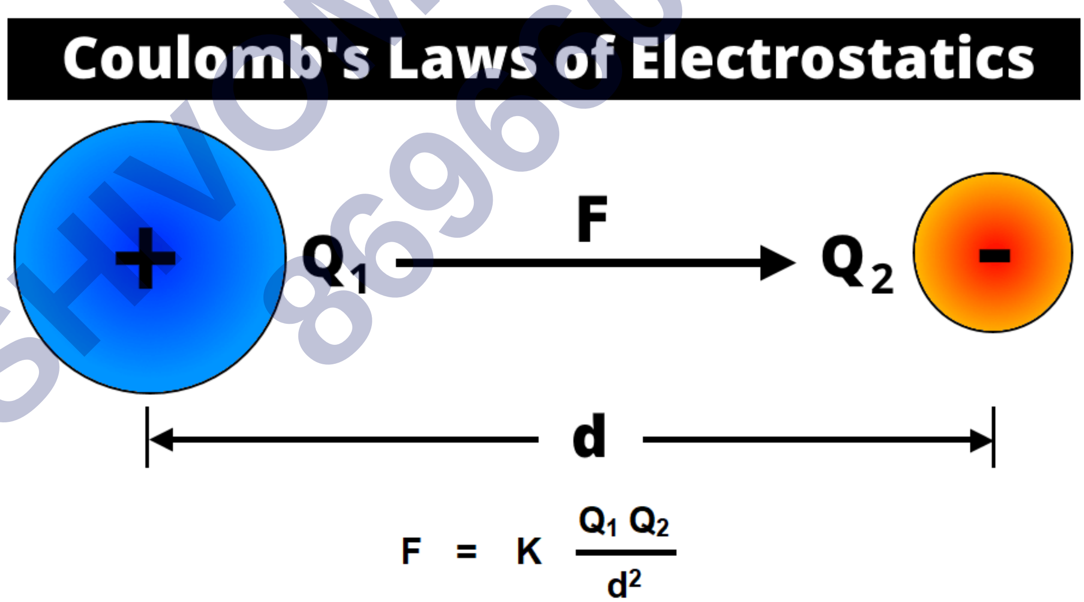
$$k = 8.9875 \times 10^{9}\ \text{N-m}^2\text{C}^{-2}$$

The constant k is often written in terms of another constant, ε0, called the permittivity of free space. It is related to k by

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$\therefore F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_1|}{r^2}$$

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times \frac{10^{-12}C^2}{Nm^2}$$



Electric Field:

A charge produces something called an electric field in the space around it and this electric field exerts a force on any charge (except the source charge itself) placed in it. The electric field has its own existence and is present even if there is no additional charge to experience the force.

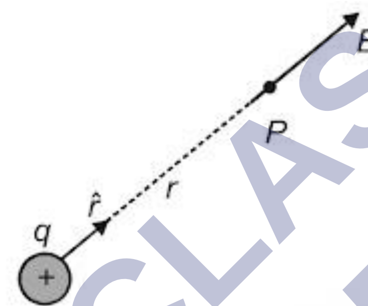
Intensity of Electric Field:

Intensity of electric field due to a charge configuration at a point is defined as the force acting on a unit positive charge at this point. Hence if a charge q experiences an electric force F at a point then intensity of electric field at this point is given as

$$E = \frac{F}{q}$$

It has S.I. units of newtons per coulomb (N/C).

Electric Field due to a Point Charge:



To determine the direction of an electric field, consider a point charge q as a source charge. This charge creates an electric field at all points in space surrounding it. A test charge q0 is placed at point P, a distance r from the source charge. According to Coulomb's law, the force exerted by q on the test charge is.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$

This force is directed away from the source charge q, since the electric field at P, the position of the test charge, is defined by

$$E = \frac{F}{q_0}$$

we find that at P, the electric field created by q is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

OR

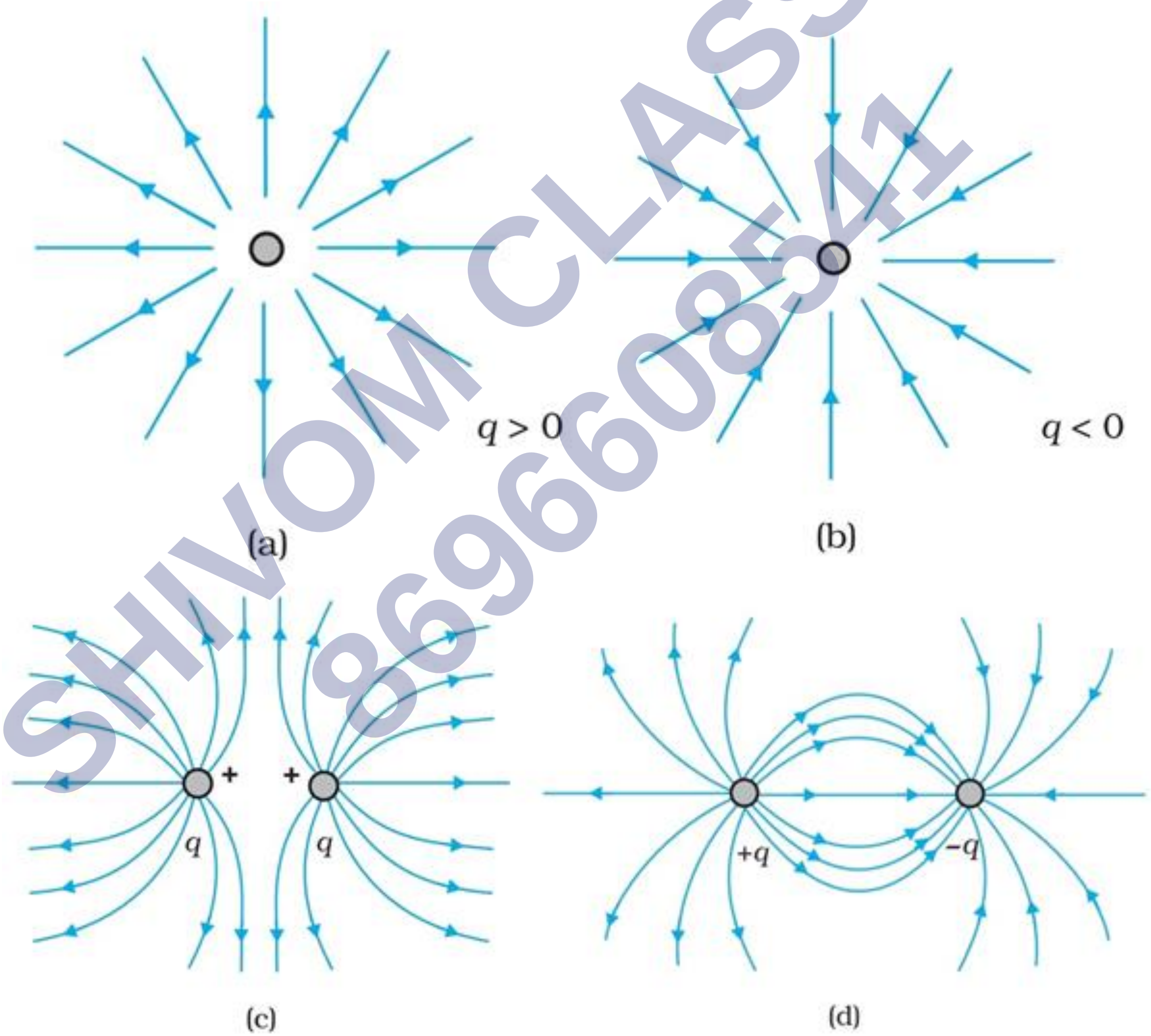
$$E = \frac{kq}{r^2}$$

Electric Field Lines:

Electric field lines are a way of pictorially mapping the electric field around a configuration of charges. An electric field line is, in general, a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point.

The field lines follow some important general properties:

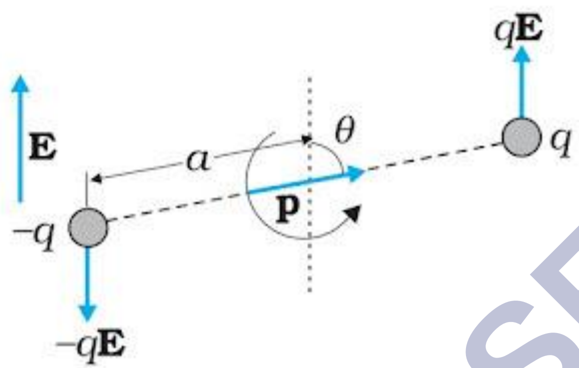
- The tangent to electric field lines at any point gives the direction of electric field at that point.
- In free space, they are continuous curves which emerge from positive charge and terminate at negative charge
- They do not intersect each other. If they do so, then it would mean two directions of electric field at the point of intersection, which is not possible.
- Electrostatic field lines do not form any closed loops. This follows from the conservative nature of electric field.



Electric Dipole:

A configuration of two charges of same magnitude q, but of opposite sign, separated by a small distance (say 2a) is called an electric dipole.

Dipole moment for an electric dipole is a vector quantity directed from the negative charge to the positive charge and its magnitude is p = q × 2a (charge × separation). The SI unit of dipole moment is C^{-m} (coulombmeter).

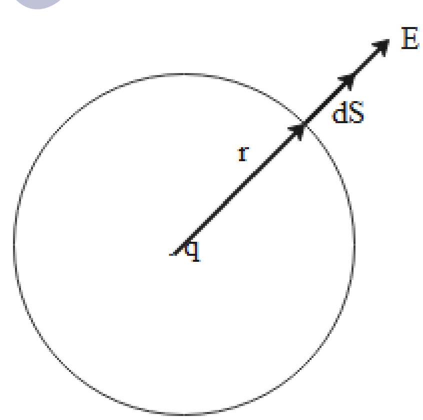


Gauss's Law:

- The flux of electric field through any closed surface S is 1/ε_0 times the
- Total charge enclosed by S.
- Electric field outside the charged shell is as though the total charge is concentrated at the center. The same result is true for a solid sphere of uniform volume charge density.
- The electric field is zero at all points inside a charged shell.

Deduction of Coulomb's law from Gauss' Law:

Consider a charge +q in place at origin in a vacuum. We want to calculate the electric field due to this charge at a distance r from the charge. Imagine that the charge is surrounded by an imaginary sphere of radius r as shown in the figure below. This sphere is called the Gaussian sphere.



Consider a small area element dS on the Gaussian sphere. We can calculate the flux through this area element due to charge as follows:

$$\oint \vec{E}.\overrightarrow{ds} = E\oint ds$$

$$\oint \vec{E}.\overrightarrow{ds} = E(4\pi r^2)$$

Using this in Gauss theorem we get

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

We know that

$$F = Eq_0$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$

This is the required Coulomb's law obtained from Gauss theorem.

The Gauss Theorem

The net flux through a closed surface is directly proportional to the net charge in the volume enclosed by the closed surface.

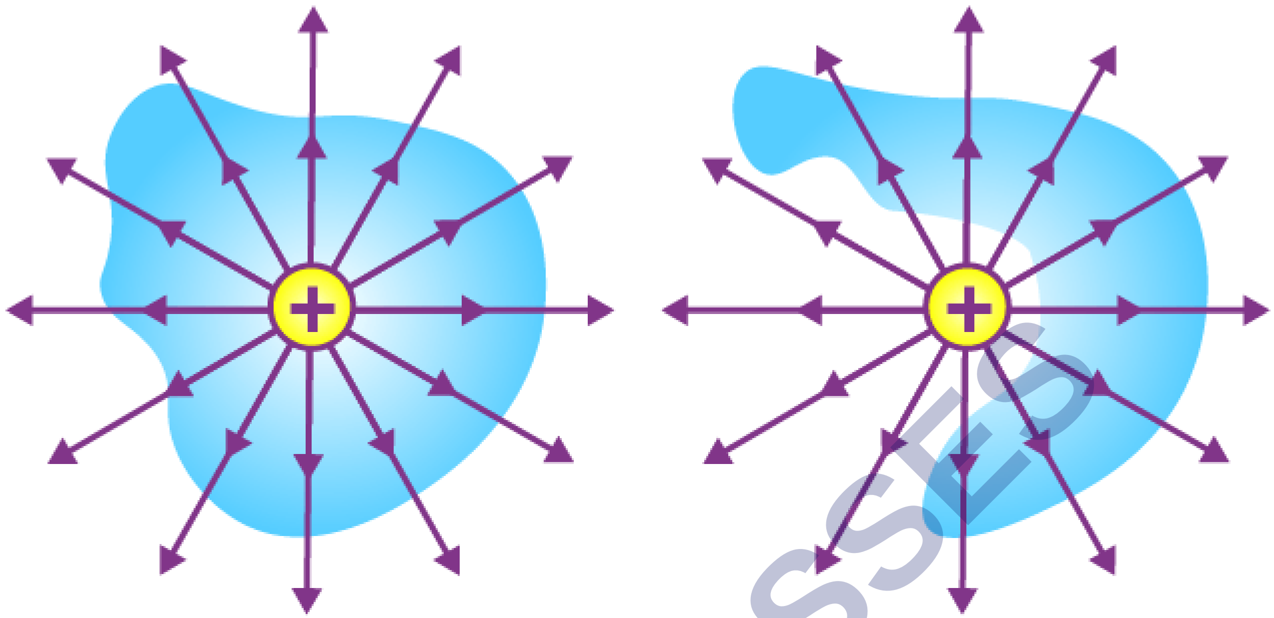
$\Phi = \rightarrow E.d \rightarrow A = q_{net}/\varepsilon_0$

In simple words, the Gauss theorem relates the 'flow' of electric field lines (flux) to the charges within the enclosed surface. If no charges are enclosed by a surface, then the net electric flux remains zero.

This means that the number of electric field lines entering the surface equals the field lines leaving the surface.

The Gauss theorem statement also gives an important corollary:

The electric flux from any closed surface is only due to the sources (positive charges) and sinks (negative charges) of the electric fields enclosed by the surface. Any charges outside the surface do not contribute to the electric flux. Also, only electric charges can act as sources or sinks of electric fields. Changing magnetic fields, for example, cannot act as sources or sinks of electric fields.



Gauss Law in Magnetism

The net flux for the surface on the left is non-zero as it encloses a net charge. The net flux for the surface on the right is zero since it does not enclose any charge.

Note: The Gauss law is only a restatement of the Coulombs law. If you apply the Gauss theorem to a point charge enclosed by a sphere, you will get back Coulomb's law easily.

Applications of Gauss Law

1. In the case of a charged ring of radius R on its axis at a distance x from the centre of the ring.

$$E = \frac{1}{4\pi\epsilon_0} \frac{qx}{(R^2+x^2)^{3/2}}$$

2. In the case of an infinite line of charge, at a distance, 'r'. $E = \left(\frac{1}{4} \times \pi r \varepsilon_0\right)\left(\frac{2\pi}{r}\right) = \frac{\lambda}{2\pi r \varepsilon_0}$. Where λ is the linear charge density.

3. The intensity of the electric field near a plane sheet of charge is $E = \frac{\sigma}{2\varepsilon_0 K}$, where σ = surface charge density.

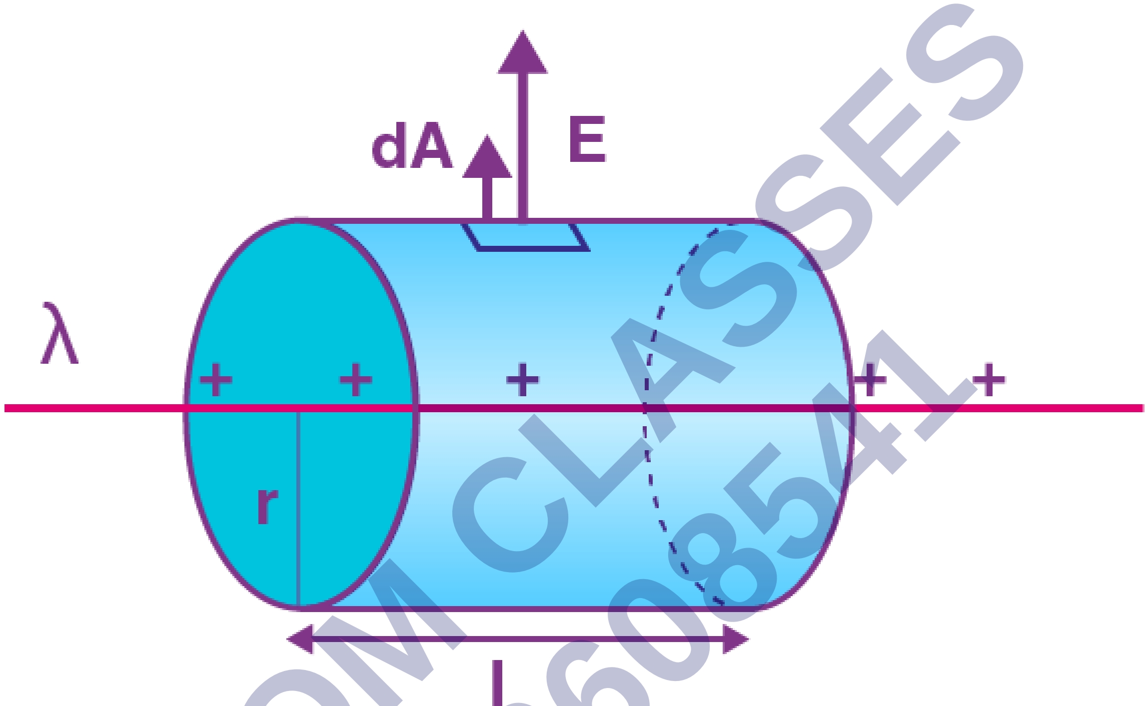
4. The intensity of the electric field near a plane charged conductor $E = \frac{\sigma}{K\varepsilon_0}$ in a medium of dielectric constant K. If the dielectric medium is air, then $E_{air} = \frac{\sigma}{\varepsilon_0}$.

5. The field between two parallel plates of a condenser is $E = \frac{\sigma}{\varepsilon_0}$, where σ is the surface charge density.

Electric Field due to Infinite Wire – Gauss Law Application

Consider an infinitely long line of charge with the charge per unit length being λ. We can take advantage of the cylindrical symmetry of this situation. By symmetry, The electric fields all point radially away from the line of charge, and there is no component parallel to the line of charge.

We can use a cylinder (with an arbitrary radius (r) and length (l)) centred on the line of charge as our Gaussian surface.



Applications of Gauss Law – Electric Field due to Infinite Wire

As you can see in the above diagram, the electric field is perpendicular to the curved surface of the cylinder. Thus, the angle between the electric field and area vector is zero and cos θ = 1

The top and bottom surfaces of the cylinder lie parallel to the electric field. Thus the angle between the area vector and the electric field is 90 degrees, and cos θ = 0.

Thus, the electric flux is only due to the curved surface

According to Gauss Law,

$\Phi = \rightarrow E.d \rightarrow A$

$\Phi = \Phi_{curved} + \Phi_{top} + \Phi_{bottom}$

$\Phi = \rightarrow E \,.\, d \rightarrow A = \int E \,.\, dA \cos 0 + \int E \,.\, dA \cos 90° + \int E \,.\, dA \cos 90°$

$\Phi = \int E \,.\, dA \times 1$

Due to radial symmetry, the curved surface is equidistant from the line of charge, and the electric field on the surface has a constant magnitude throughout.

$\Phi = \int E \,.\, dA = E \int dA = E \,.\, 2\pi rl$

The net charge enclosed by the surface is:

$q_{net} = \lambda.l$

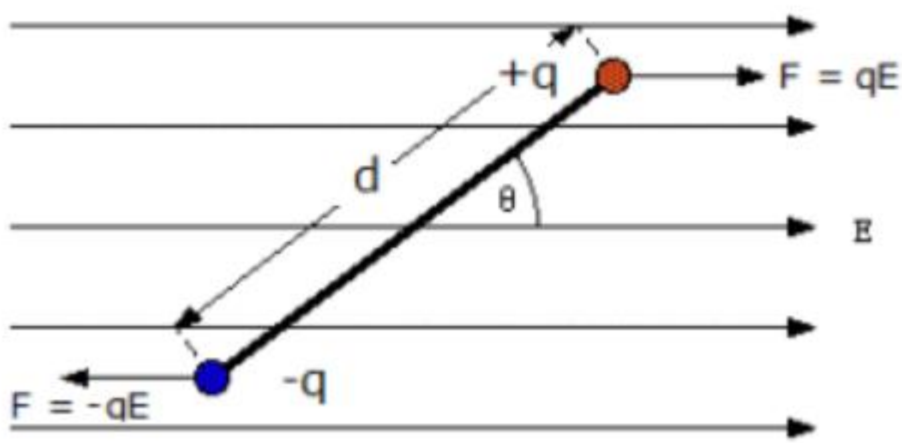
Using Gauss theorem,

$$\Phi = E \times 2\pi rl = \frac{q_{net}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$$E \times 2\pi rl = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi r\varepsilon_0}$$

Dipole Placed in Uniform External Field:



Since the impact of an external electric field on charges is already known to us; a dipole too will experience some form of force when introduced to an external field. It is interesting to learn that, a dipole placed in an external electric field acquires a rotating effect. This rotating effect is termed as ‘torque’ felt by the dipole. Excitingly, the net torque can be calculated on the opposite charges present in a dipole for estimating the overall rotation.

Torque on dipole:

Consider a dipole located in the same position ‘E’ to calculate the torque received by the dipole when positioned outside. The compulsory charge will be placed below the ‘qE’ magnitude as you go up, while the negative charge will be placed below the ‘qE’ magnitude as you go down.

Since the absolute power is zero, it can be seen that the dipole is in the equation at the moment. But what is the rotation rate? In this case, the dipole may remain stable but rotates at a certain angular velocity. This fact has been demonstrated by experimentation, and it shows that both electrostatic forces (qE) act as clock-related torque.

As a result, when a dipole is inserted into the same external electrical circuit, it rotates. Torque always works with external force applied which will be in pairs. Moreover, its size is a result of its strength and arm. The arm can be thought of as the distance between the point of force applied and the point at which rotation occurs at the dipole.

Torque

Torque (τ) = Force × distance separating forces

Torque is a vector whose direction is determined by the force acting on the axis. The magnitude of the torque vector is determined as follows:

$T = F\ r\ \sin\theta$

Which means,

F - force acting on the axis

r - temporary arm length

θ - angle between force vector and temporary arm

τ - is the vector of torque

Derivation of Torque

Consider a dipole with the angles of + q and q forming a dipole because they are separated by a distance of d. Positioned in the same electric field of power E, the dipole axis forms an θ angle with an electric field.

Charging power, F = ± q E

Elements of power perpendicular to dipole, F = ± q E sinθ

Since 'qd' is the magnitude of the dipole moment (p), and the direction of the dipole moment ranges from positive to negative; torque is the product of a dipole moment cross and an electric field. When the direction of the electric field is positive, the torque is in the clock (therefore negative) in the image above.

So,

$\tau = -\, pE \sin\theta$

An incorrect sign indicates that the torque is in the clockwise direction.

Stable and unstable equilibrium:

Stable equilibrium

"A body is said to be in stable equilibrium if after a slight tilt it returns to its previous position."stable equilibrium state

Consider a book lying on the table. Tilt the book slightly about its one edge by lifting it from the opposite side. It returns to its previous position when sets free. Such a state of the body is called a stable equilibrium.

When a body is in stable equilibrium, its center of gravity is at the lowest position. When it is tilted, its center of gravity rises. It returns to its stable equilibrium as long as the center of gravity acts through the base of the body.

Examples of stable equilibrium:

- Chair lying on the floor
- The heavy base of the vehicle
- Table lying on the ground
- Cone lying on its base by lowering its center of gravity

- Bottle lying on its base

Unstable equilibrium:

“If a body does not return to its previous position when sets free after the slightest tilt is said to be in unstable equilibrium.”unstable equilibrium state

Take a pencil and try to keep it in the vertical position on its tip. Whenever you leave it, the pencil topples over about its tip and falls down. This is called an unstable equilibrium. In an unstable equilibrium, a body may be made to stay only for a moment. Thus a body is an unstable equilibrium.

The center of gravity of the body is at its highest position in the state of unstable equilibrium. As the body topples over about its base (tip), its center of gravity moves towards its lower position and does not return to its previous position.

Example of unstable equilibrium:

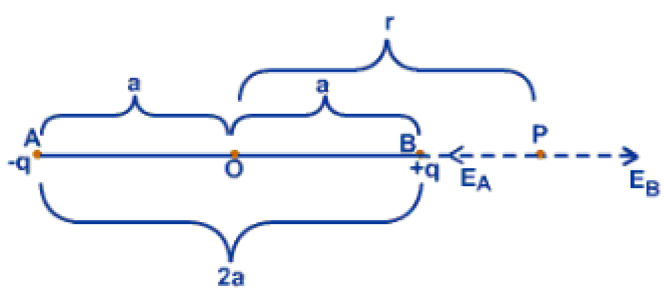
- When the ice cream cone is made to rest on its apex on a book, the movement of the book will disturb the position of the ice cream cone. This is an example of unstable equilibrium.
- When the weather changes from freezing to hot to freezing rapidly and without reason, this is an example of a time when it is unstable. When a person has a bad temper that can explode or flare up with no provocation at all, this is an example of a person who would be described as unstable. Not firmly placed; unsteady.

Electric Field on axial and equatorial line

Axial line: Axial line is the line which is passing through the positive and negative charges and the point lies on that line is called the axial point.

Electric field on axial line of dipole is given by:

$$\vec{E}_{ax} = \frac{2\vec{p}}{4\pi\varepsilon_o r^3} \text{ for } r >> a$$

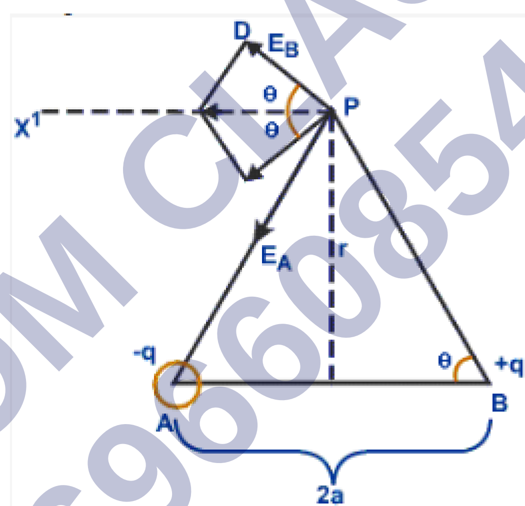


Equatorial line: Equatorial line is the perpendicular line to the line passing through the positive and negative charges and the point lies on that line is known as the equatorial point.

Electric field on equatorial line of dipole is given by:

$$\vec{E}_{ax} = -\frac{\vec{p}}{4\pi\varepsilon_o r^3} \text{ for } r >> a$$

$$\Rightarrow \vec{E}_{ax} = -2\vec{E}_{eq}$$



Relation between electric field at axial and equatorial line

Electric field due to an electric dipole at points situated at a distancer r along its axial line is given as,

$$E_{equ.} = \frac{2p}{4\pi \in_0 r^3} \cdots (i)$$

Electric field due to an electric dipole at points situated at a distance r along its equatorial plane is given as,

$$E_{equ.} = \frac{p}{4\pi \in_0 r^3} \cdots (ii)$$

From (i) and (ii)

$$\frac{E_{axial}}{E_{equ.}} = 2$$

Therefore, ratio is 2 : 1

Electric Flux

The total number of electric field lines passing a given area in a unit time is defined as the electric flux. However, we note that there is no flow of a physically observable quantity like in the case of liquid flow. Coming to the definition, Electric flux Δθ through an area element ΔS is defined by

$\Delta\theta = E.\Delta S = E\ \Delta S\ \cos\theta$

This is proportional to the number of field lines cutting the area element. The angle θ here is the angle between E and ΔS. In a closed surface, where the convention is already stated, θ is the angle between E and the outward normal to the area element. To calculate the total flux through any given surface divide the surface into small area elements, calculate the flux at each element and add them up. Thus, the total flux θ through a surface S is $\theta \sim \Sigma\ E.\ \Delta S$. The approximation symbol is used because the electric field E is taken to be constant over the small area element.

Electric Dipole

It is a pair of equal or opposite charges A and -B which are separated by distance 2x. The dipole moment vector (let's assume it as p) has a magnitude 2Ax and is in the direction of the dipole axis from -B to A

Electric field intensity on the axes

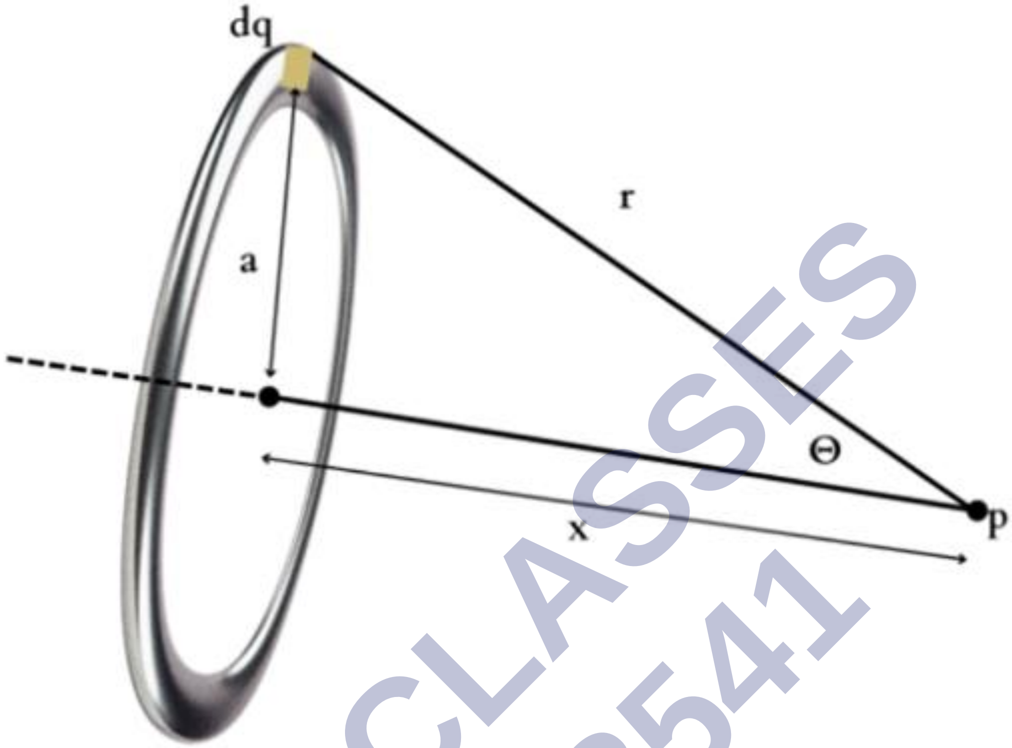
An electric field gets associated with any charged point in space. Such an electric field is often quantified using terms such as the electric field strength and intensity. The electric field can be present in any object irrespective of how the charge is distributed.

For instance, an electric field gets created along with the spherical, linear, and planar distribution of charges. In all the above cases, the electric field has a magnitude and a direction. Likewise, when a ring is charged either uniformly or non-uniformly, an electric field gets created along the axis of the ring, the magnitude, and direction of which gets determined by the charge on the ring itself.

In this article, you will learn about the axis of a uniformly charged ring and the electric field due to the ring. It is significant to understand the nature and magnitude of the electric field at various points along the axis to understand the force it would exert on any unit positive charge kept nearby.

Deriving the Circular Ring Formula:

Consider the following figure as a charged ring whose axis is subjected to an electric field of varying intensity from the centre of the charged ring.



Electric Field at P due to an Infinitesimal Charge dq on a Charged Ring

Let's now derive the equation to find the electric field along the axis at a distance of x from the centre of the charged ring. Here, $r = \sqrt{x^2 + a^2}$ is the distance of point p from the arc element dq.

According to the principle of superposition, the total electric field at point p (along the axis of the charged ring) is the vector sum of individual electric fields due to all the point charges.

According to Gauss Law, the electric field caused by a single point charge is as follows:

$$\vec{E} = \frac{1}{4\,\pi\,\epsilon_0} \times \frac{q}{r^2}\,\hat{r}$$

The electric field at point p due to the small point charge dq which is at a radius of a from the centre of the charged ring can be written as:

$$\overrightarrow{dE} = \frac{1}{4\,\pi\,\epsilon_0} \times \frac{dq}{r^2}\,\hat{r}$$

It is important to note that since there is a corresponding piece of point charge on the opposite side of the ring, the y components of the electric field will get nulled throughout.

Hence, only the x component of the electric field will be significant in deriving the total electric field at point p due to a charged ring.
However, we can see that the tiny x component forms an angle Θ at point p along the axis of the charged ring. Hence the electric field equation will be adjusted while considering this angle and hence becomes:

$$dE_x = \frac{1}{4\,\pi\,\epsilon_0} \times \frac{q}{r^2}\,\cos\Theta$$

Since the axis forms a right angle with the distance from dq to point p and Θ is unknown, we replace r and cosΘ with the known distances x and a. The above equation thus becomes:

$$dE_x = \frac{1}{4\,\pi\,\epsilon_0} \times \frac{dq}{(x^2+a^2)}\,\frac{x}{\sqrt{(x^2+a^2)}}$$

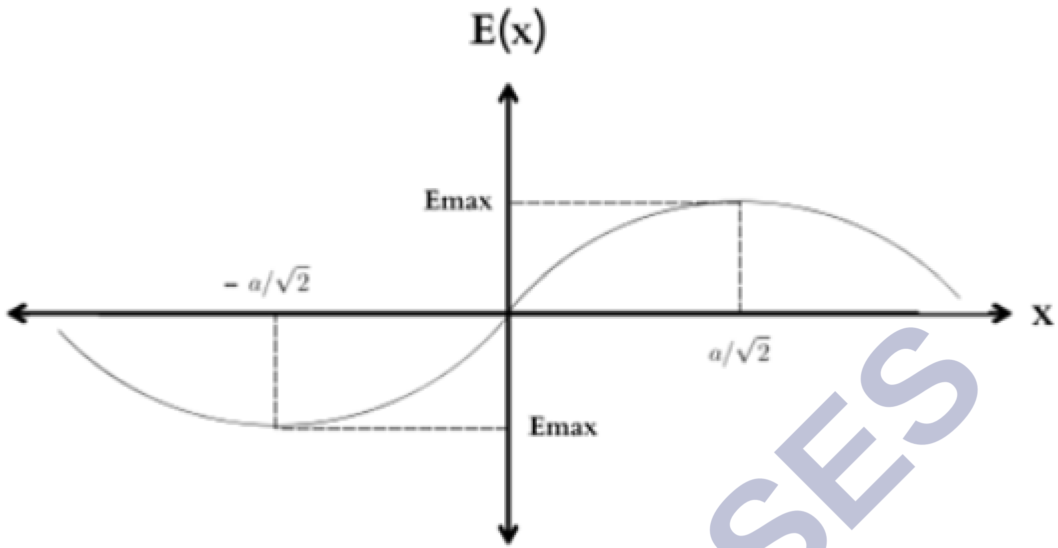
Upon simplification,

$$dE_x = \frac{1}{4\,\pi\,\epsilon_0} \times \frac{x\,dq}{(x^2+a^2)^{\frac{3}{2}}}$$

Now, by integrating the above equation with respect to dq, the total electric field at point p on the axis of a charged ring is given by the following equation:

$$E_x = \frac{1}{4\,\pi\,\epsilon_0} \times \frac{x\,Q}{(x^2+a^2)^{\frac{3}{2}}}$$

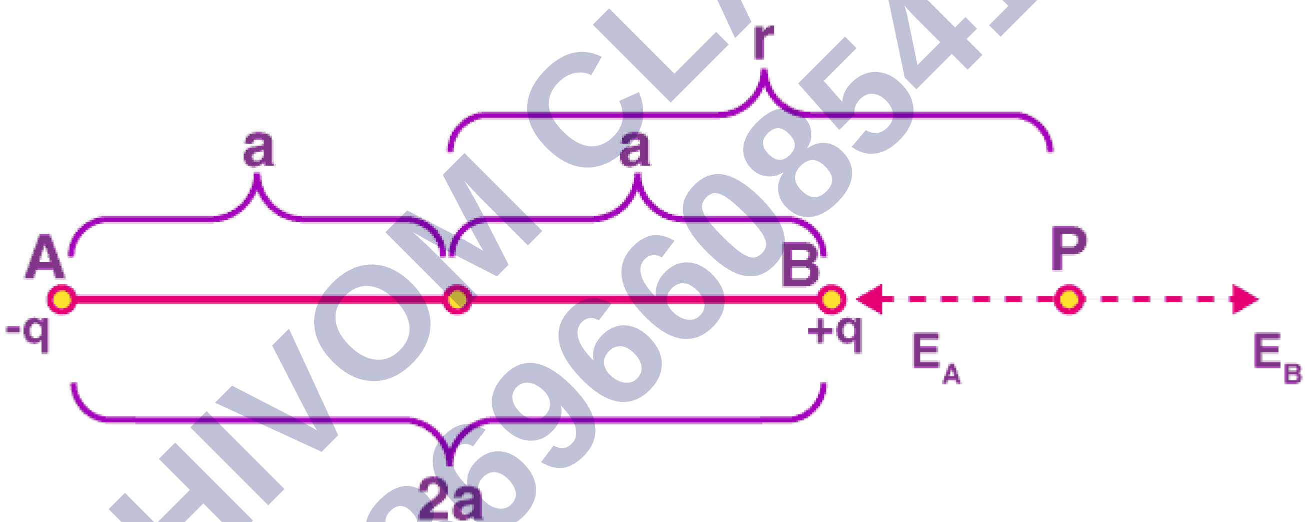
Note: The above equation holds good only for finding the electric field on any point on the x-axis.
Let us now draw a graph that closely represents the relationship between the electric field along the axis of a charged ring and the distance from the centre of the charged ring.
The electric field is zero at the centre and increases to a maximum on either side of the ring, and then gradually falls back to zero as x approaches infinity. The magnitude of the electric field will be the same due to symmetry and uniformity in the distribution of charge, and the closest graphical representation or the graph of a uniformly charged ring is drawn below:



What is an Electric Dipole?

An electric dipole is defined as a pair of opposite charges q and –q separated by a distance d.

Derivation of Electric Field Intensity for points on the Axial Line of a Dipole



Consider a system of charges -q and +q separated by a distance 2a. Let "P" be any point on the axial line where the electric field intensity needs to be determined.

Electric Field at P (E_B) due to +q is given as follows:

$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2}$$
$$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$$

The electric field at P (E_A) due to -q is given as follows:

$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{(AP)^2}$$
$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$$

The following equation gives the net field at point P:

$$E_P = E_B - E_A$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2} - \frac{q}{(r+a)^2}$$

Simplifying the above equation, we get

$$E_p = \frac{q}{4\pi\epsilon_0} \times \frac{2r}{(r^2-a^2)^2}$$

Further simplifying, we get

$$E_p = \frac{2kpr}{(r^2-a^2)^2}$$

In the equation, p = 2aq and

$$k = \frac{1}{4\pi\epsilon_0}$$

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Chapter- 1 : Electric Charges and Fields

Electric Charges and Fields

Electric Charges

- Two kinds of charges + ve and – ve
- S.I unit is coulomb (C)
  1 C = charge flowing through
- a wire in 1 sec if current in it is 1A
  1e = 1.602 × 10^{-19}C

Conservation of charge

It is not possible to create or destroy net charge of an isolated system.

Quantization of charge

All observable charges must be integral multiple of e. i.e.
Q = ne
where n = integers

Coulomb's Law

- Force between two charged particle

$\vec{F} = \frac{k q_1 q_2 \vec{r}}{r^3} = \frac{k q_1 q_2 \hat{r}}{r^2}$, force is attractive when q_1 and q_2 are of opposite sign, otherwise repulsive

$|\vec{F}| = \frac{k q_1 q_2}{r^2}$

$k = \frac{1}{4\pi\varepsilon_0} \approx 9\times10^9 Nm^2C^{-2}$

- Force between multiple charges

$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \ = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{r}_{1i}$

where $\hat{r}_{1i}$ is the unit vector along charge particle under consideration
$\hat{r}_{1i}$ = distance between I and i^{th} particle

"The electric force with which two charges attracts or repel one another are not affected by the presence of other charges". It is known as Superposition Principle.

Electric Flux

$\Delta\phi = \vec{E}\Delta\vec{S}$, $\Delta\vec{S}$ = area vector
E = electric field
$\phi = \int \vec{E}.\vec{ds}$

Gauss's Theorem

$\oint \vec{E}.\vec{ds} = q_{in}/\varepsilon_0 = \phi$

where, q_{in} = net charge enclosed
ϕ = flux through a closed surface

Application of Gauss's Law

- Electric field due to uniformly charged sphere at an
- Outside point
  $E = Q/4\pi\varepsilon_0 r^2$
  Internal point
  $E = Qr/4\pi\varepsilon_0 R^3$
- On the surface
  $E = Q/4\pi\varepsilon_0 R^2$

- Field due to linear charge distribution
  $E = \lambda/2\pi\varepsilon_0 r$,
  λ= linear charge density
- Field due to a plane sheet of charge
  $E = \sigma/2\varepsilon_0$,
  σ = charge per unit area
- Field due to a charged conducting plate
  $E = \sigma/\varepsilon_0$,
  σ = charge per unit area
- Field inside a conductor = 0
- Electric field due to a thin spherical shell at a point
  Outside the shell, $E = Q/4\pi\varepsilon_0 r^2$
  Inside the shell, E = 0
  On the surface $E = Q/4\pi\varepsilon_0 R^2$
- Electric field due to uniformly charged share at an
  Outside point
  $E = Q/4\pi\varepsilon_0 r^2$
  Internal point
  $E = Qr/4\pi\varepsilon_0 R^3$
  On the surface
  $E = Q/4\pi\varepsilon_0 R^2$

Electric field

Electric field lines

- Always normal to conducting surface
- Lines originating from +ve charge
- Terminating at -ve charge
- Never intersect each other
- Never from closed loop
- Are imaginary lines.

Electric field due to point charge

$\vec{E} = \frac{\vec{F}}{q} = k\frac{Q}{r^2}$ (N/C)

$\because$ unit = N/C

- Due to discrete distribution of charges
  $\vec{E} = \sum_i \vec{E}_i$
- Due to continuous distribution of charges
  $\vec{E} = k\int\frac{dQ}{r^3}\vec{r}$
  $|\vec{E}| = k\int\frac{dQ}{r^2}$

Electric Dipole, $\vec{p} = q\vec{d}$

- Electric field due to dipole at axial position
  $\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2P}{r^3}\right)\hat{r}$
- Electric field due to dipole at equatorial position
  $|E| = \frac{1}{4\pi\varepsilon_0}\left(\frac{P}{r^3}\right)$
- Torque on an electric dipole placed in an electric field $(\vec{E})$
  $\vec{\tau} = \vec{p}\times\vec{e} = pE\sin\theta$

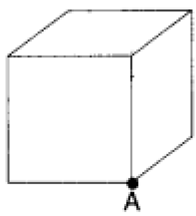
Important Questions

Multiple Choice questions-

1. The surface considered for Gauss's law is called

(a) Closed surface
(b) Spherical surface
(c) Gaussian surface
(d) Plane surface

2. The total flux through the faces of the cube with side of length a if a charge q is placed at corner A of the cube is



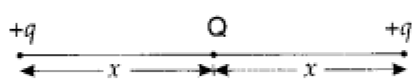
(a) $\frac{q}{8\varepsilon_0}$ (b) $\frac{q}{4\varepsilon_0}$

(c) $\frac{q}{2\varepsilon_0}$ (d) $\frac{q}{\varepsilon_0}$

3. Which of the following statements is not true about Gauss's law?

(a) Gauss's law is true for any closed surface.
(b) The term q on the right side of Gauss's law includes the sum of all charges enclosed by the surface.
(c) Gauss's law is not much useful in calculating electrostatic field when the system has some symmetry.
(d) Gauss's law is based on the inverse square dependence on distance contained in the coulomb's law

4. A charge Q is placed at the center of the line joining two point charges +q and +q as shown in the figure. The ratio of charges Q and q is



(a) 4
(b) 1/4
(c) -4
(d) -1/4

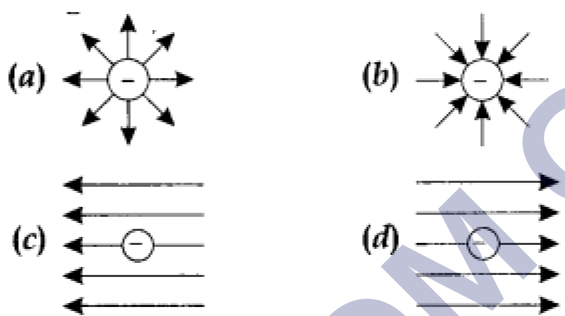
5. The force per unit charge is known as

(a) electric flux
(b) electric field
(c) electric potential
(d) electric current

6. Electric field lines provide information about

(a) field strength
(b) direction
(c) nature of charge
(d) all of these

7. Which of the following figures represent the electric field lines due to a single negative charge?



8. The SI unit of electric flux is

(a) $N\ C^{-1}\ m^{-2}$
(b) $N\ C\ m^{-2}$
(c) $N\ C^{-2}\ m^{2}$
(d) $N\ C^{-1}\ m^{2}$

9. The unit of electric dipole moment is

(a) newton
(b) coulomb
(c) farad
(d) Debye

10. Consider a region inside which, there are various types of charges but the total charge is zero. At points outside the region

(a) the electric field is necessarily zero.
(b) the electric field is due to the dipole moment of the charge distribution only.

(c) the dominant electric field is inversely pro-portional to r3, for large r (distance from ori-gin).

(d) the work done to move a charged particle along a closed path, away from the region will not be zero.

Very Short:

1. What is the value of the angle between the vectors $\vec{P}$ and $\vec{E}$ for which the potential energy of an electric dipole of dipole moment $\vec{P}$, kept in an external electric field $\vec{E}$, has maximum value.
2. Define electric field intensity at a point.
3. Two equal point charges separated by 1 m distance experience force of 8 N. What will be the force experienced by them, if they are held in water, at the same distance? (Given: K_{water} = 80) (CBSE AI 2011C)
4. A charge 'q' is placed at the centre of a cube of side l. What is the electric flux passing through each face of the cube? (CBSE AI 2012) (CBSE Sample Paper 2019)
5. Why do the electric field lines not form closed loops? (CBSE AI 2012C)
6. Two equal balls having equal positive charge 'q' coulomb are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two? (CBSE AI 2014)
7. What is the electric flux through a cube of side l cm which encloses an electric dipole? (CBSE Delhi 2015)
8. Why are electric field lines perpendicular at a point on an equipotential surface of a conductor? (CBSE AI 2015C)
9. What is the amount of work done in moving a point charge Q. around a circular arc of radius 'r' at the centre of which another point charge 'q' is located? (CBSE AI 2016)
10. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased? (CBSE Delhi 2016)

Short Questions:

1.

(a) Electric field inside a conductor is zero. Explain.

(b) The electric field due to a point charge at any point near it is given as

$$\mathbf{E} = \lim_{q \to 0} \frac{F}{q}$$

what is the physical significance of this limit?

2. Define the electric line of force and give its two important properties.

3. Draw electric field lines due to (i) two similar charges, (ii) two opposite charges, separated by a small distance.

4. An electric dipole is free to move in a uniform electric field. Explain what is the force and torque acting on it when it is placed

(i) parallel to the field

(ii) perpendicular to the field

5. A small metal sphere carrying charge +Q. is located at the centre of a spherical cavity in a large uncharged metallic spherical shell. Write the charges on the inner and outer surfaces of the shell. Write the expression for the electric field at the point P_1 (CBSE Delhi 2014C)

6. Two-point charges q and -2q are kept 'd' distance apart. Find the location of the point relative to charge 'q' at which potential due to this system of charges is zero. (CBSE Al 2014C)

7. Two small identical electrical dipoles AB and CD, each of dipole moment 'p' are kept at an angle of 120° as shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to the electric field ($\vec{E}$) directed along +X direction, what will be the magnitude and direction of the torque acting on this? (CBSE Delhi 2011)

8. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on (i) the inner surface, and (ii) the outer surface? (NCERT Exemplar)

Long Questions:

1.

(a) State Gauss theorem in electrostatics. Using it, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

(b) How is the field directed if (i) the sheet is positively charged, (ii) negatively charged? (C8SE Delhi 2012)

2. Use Gauss's law to derive the expression for the electric field ($\vec{E}$) due to a straight uniformly charged infinite line of charge λ Cm^{-1}. (CBSE Delhi 2018)

Assertion and Reason Questions-

1. For two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

a) Both A and R are true, and R is the correct explanation of A.

b) Both A and R are true, but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false and R is also false.

Assertion (A): The electric flux emanating out and entering a closed surface are 8 × 10^3 and 2 × 10^3Vm respectively. The charge enclosed by the surface is 0.053μC.

Reason (R): Gauss's theorem in electrostatics may be applied to verify.

2. For two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

a) Both A and R are true, and R is the correct explanation of A.

b) Both A and R are true, but R is not the correct explanation of A.

c) A is true but R is false.

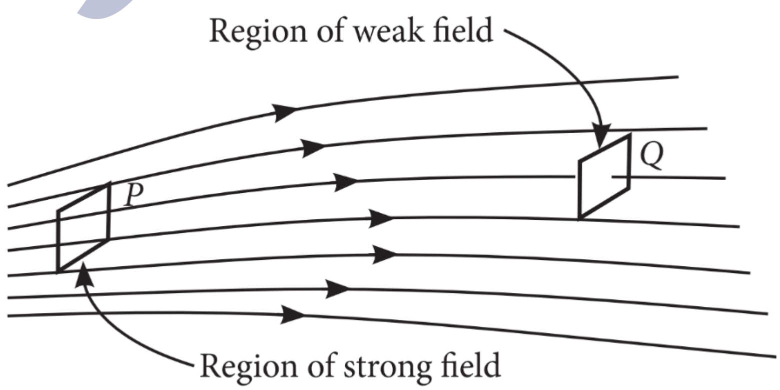
d) A is false and R is also false.

Assertion (A): Charge is quantized.

Reason (R): Charge which is less than l C is not possible.

Case Study Questions-

1. Electric field strength is proportional to the density of lines of force i.e., electric field strength at a point is proportional to the number of lines of force cutting a unit area element placed normal to the field at that point. As illustrated in the given figure, the electric field at P is stronger that at Q.



(i) Electric lines of force about a positive point charge are:

a) Radially outwards.

b) Circular clockwise.

c) Radially inwards.

d) Parallel straight lines.

(ii) Which of the following is false for electric lines of force?

a) They always start from positive charges and terminate on negative charges.

b) They are always perpendicular to the surface of a charged conductor.

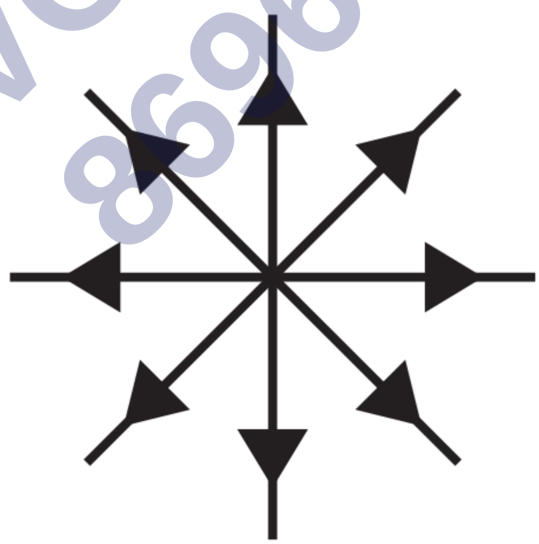
c) They always form closed loops.

d) They are parallel and equally spaced in a region of uniform electric field.

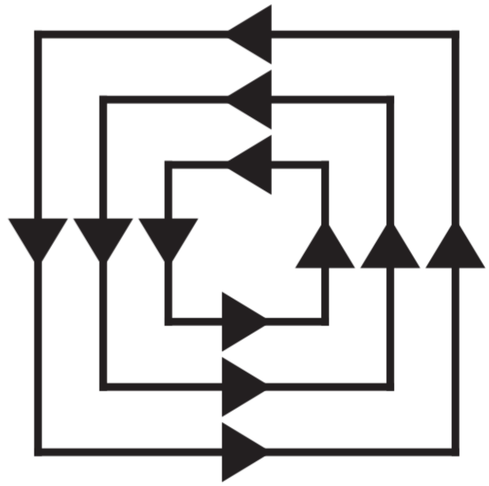
(iii) Which one of the following pattern of electric line of force in not possible in filed due to stationary charges?

a)

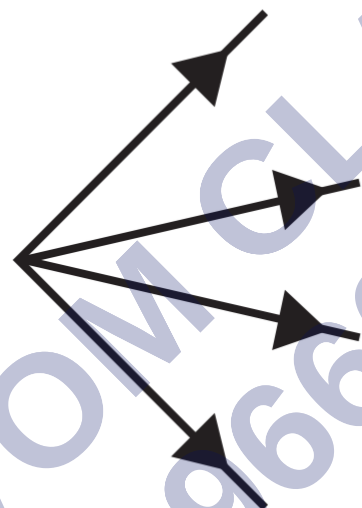
b)



c)



d)



(iv) Electric lines of force are curved:

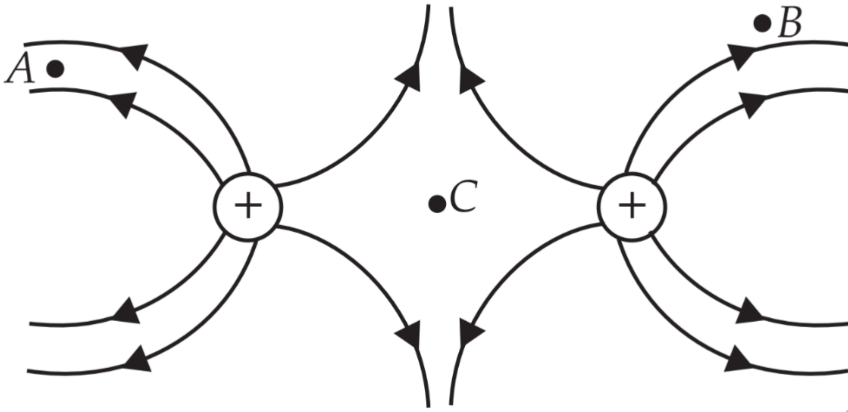
a) In the field of a single positive or negative charge.

b) In the field of two equal and opposite charges.

c) In the field of two like charges.

d) Both (b) and (c).

(v) The figure below shows the electric field lines due to two positive charges. The magnitudes E_A, E_B and E_C of the electric fields at points A, Band C respectively are related as:



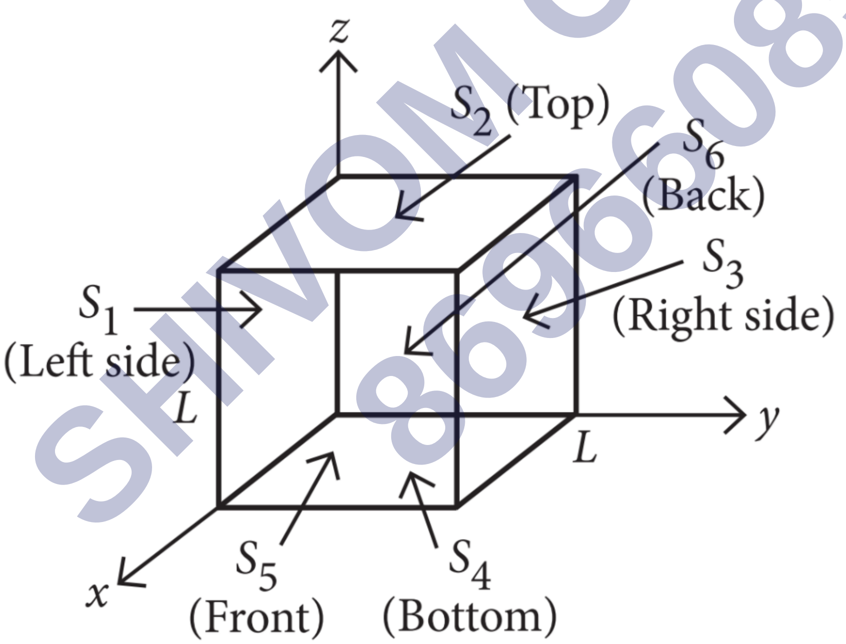
a) $E_A > E_B > E_C$

b) $E_B > E_A > E_C$

c) $E_A = E_B > E_C$

d) $E_A > E_B = E_C$

2. Net electric flux through a cube is the sum of fluxes through its six faces. Consider a cube as shown in figure, having sides of length L = 10.0cm. The electric field is uniform, has a magnitude $E = 4.00 \times 10^3 N\ C^{-1}$ and is parallel to the xy plane at an angle of 37º measured from the + x - axis towards the + y - axis.



(i) Electric flux passing through surface S_6 is:

a) $-24N\ m^2 C^{-1}$

b) $24N\ m^2 C^{-1}$

c) $32N\ m^2 C^{-1}$

d) $-32N\ m^2 C^{-1}$

(ii) Electric flux passing through surface S_1 is:

a) $-24 \text{N m}^2 \text{C}^{-1}$

b) $24 \text{N m}^2 \text{C}^{-1}$

c) $32 \text{N m}^2 \text{C}^{-1}$

d) $-32 \text{N m}^2 \text{C}^{-1}$

(iii) The surfaces that have zero flux are:

a) S_1 and S_3

b) S_5 and S_6

c) S_2 and S_4

d) S_1 and S_2

(iv) The total net electric flux through all faces of the cube is:

a) $8 \text{N m}^2 \text{C}^{-1}$

b) $-8 \text{N m}^2 \text{C}^{-1}$

c) $24 \text{N m}^2 \text{C}^{-1}$

d) Zero.

(v) The dimensional formula of surface integral $\oint \vec{E} . d\vec{S}$ of an electric field is:

a) $[M L^2 T^{-2} A^{-1}]$

b) $[M L^3 T^{-3} A^{-1}]$

c) $[M L^{-1} T^3 A^{-3}]$

d) $[M L^{-3} T^{-3} A^{-1}]$

✓ Answer Key:

Multiple Choice Answers-

1. Answer: c
2. Answer: a
3. Answer: c
4. Answer: d
5. Answer: b
6. Answer: d
7. Answer: b
8. Answer: d
9. Answer: d

10. Answer: c

Very Short Answers:

1. Answer:

   P.E. = -pEcos θ

   P.E. is maximum when cos θ = - 1, i.e.

   θ = 180°

2. Answer: Electric field intensity at a point is defined as the force experienced by a unit test charge placed at that point. Mathematically

   we have

   $$\vec{E} = \lim_{\delta q \to 0} \frac{\vec{F}}{\delta q}$$

3. Answer: The force in water is given by

   $F_W = \frac{F_{air}}{K} = \frac{8}{80} = 0.1$ N

4. Answer: $\Phi = q/6\varepsilon_0$

5. Answer: It is due to the conservative nature of the electric field.

6. Answer: It decreases because force $\propto = \frac{1}{k}$ and k > 1.

7. Answer: Zero

8. Answer: So that no net force acts on the charge at the equipotential surface, and it remains stationary.

9. Answer: Zero.

10. Answer: No change, as flux does not depend upon the size of the Gaussian surface.

Short Questions Answers:

1. Answer:

   (a) By Gauss theorem $\oint \vec{E} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$. Since there is no charge inside a conductor therefore in accordance with the above equation the electric field inside the conductor is zero.

   (b) It indicates that the test charge should be infinitesimally small so that it may not disturb the electric field of the source charge.

2. Answer:

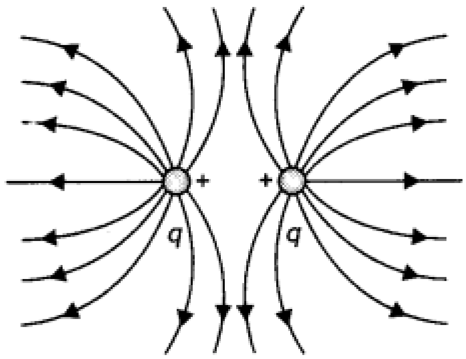
   It is a line straight or curved, a tangent to which at any point gives the direction of the electric field at that point.

(a) No two field lines can cross, because at the point of intersection two tangents can be drawn giving two directions of the electric field which is not possible.

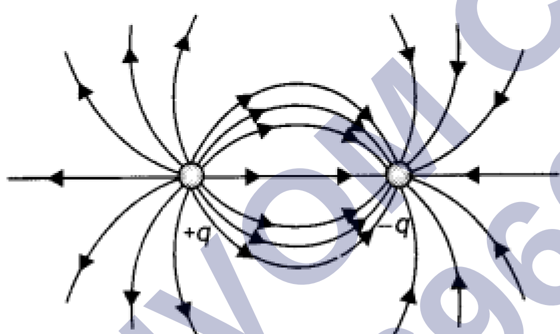
(b) The field lines are always perpendicular to the surface of a charged conductor.

3. Answer:

(a) The diagram is as shown.



(b) The diagram is as shown.

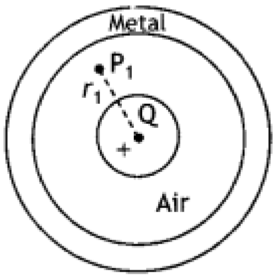


4. Answer:

(i) When an electric dipole is placed parallel to a uniform electric field, net force, as well as net torque acting on the dipole, is zero and, thus, the dipole remains in equilibrium.

(ii) When the dipole is placed perpendicular to the field, two forces acting on the dipole form a couple, and hence a torque acts on it which aligns its dipole along the direction of the electric field.

5. Answer:



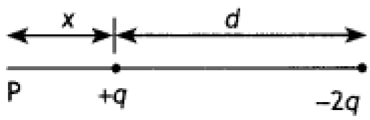
Charge on inner surface − Q.

Charge on outer surface + Q,

Electric field at point P = E = $k\frac{Q}{r_1^2}$

6. Answer:

Let the potential be zero at point P at a distance x from charge q as shown



Now potential at point P is

$$V = \frac{kq}{x} + \frac{k(-2q)}{d+x} = 0$$

Solving for x we have

x = d

7. Answer:

The resultant dipole moment of the combination is

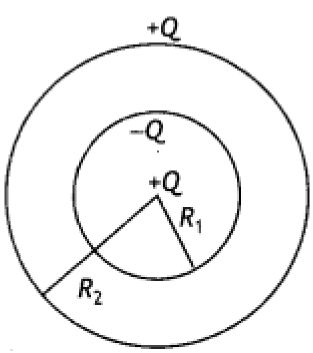
$$P_R = \sqrt{p^2 + p^2 + 2p^2 \cos 120^\circ} = p$$

since cos 120° = -1/2

This will make an angle of 30° with the X-axis, therefore torque acting on it is

τ=PE sin 30° = pE/2 (Along Z-direction)

Answer: The induction of charges is as shown.



Therefore, surface charge density on the inner and the outer shell is on the outer surface is

$$\sigma_{inner} = \frac{-Q}{4\pi R_1^2}$$

$$\sigma_{outer} = \frac{+Q}{4\pi R_2^2}$$

Long Questions Answers:

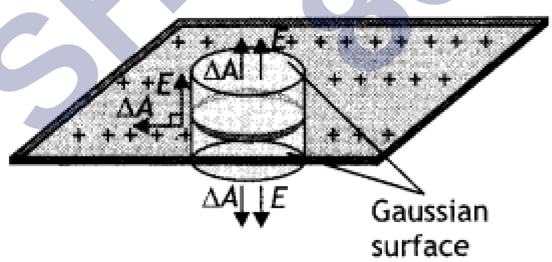
1. Answer:

It states, "The net electric flux through any Gaussian surface is equal to $\frac{1}{\varepsilon_0}$ times the net electric charge enclosed by the surface.

Mathematically, $\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

Consider an infinite plane sheet of charge. Let a be the uniform surface charge density, i.e. the charge per unit surface area. From symmetry, we find that the electric field must be perpendicular to the plane of the sheet and that the direction of E on one side of the plane must be opposite to its direction on the other side as shown in the figure below. In such a case let us choose a Gaussian surface in the form of a cylinder with its axis perpendicular to the sheet of charge, with ends of area A.

The charged sheet passes through the middle of the cylinder's length so that the cylinder's ends are equidistant from the sheet. The electric field has a normal component at each end of the cylinder and no normal component along the curved surface of the cylinder. As a result, the electric flux is linked with only the ends and not the curved surface.



Therefore, by the definition of electric flux, the flux Linked with the Gaussian surface is given by

$\Phi = \oint_A \vec{E} \cdot \vec{\Delta} A$

$\Phi = E_A + E_A = 2E_A$ ... (1)

But by Gauss's Law

$\Phi = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$ [$\because$ q = σA] ... (2)

From equations (1) and (2), we have

$2E_A = \frac{\sigma A}{\varepsilon_0}$ ... (3)

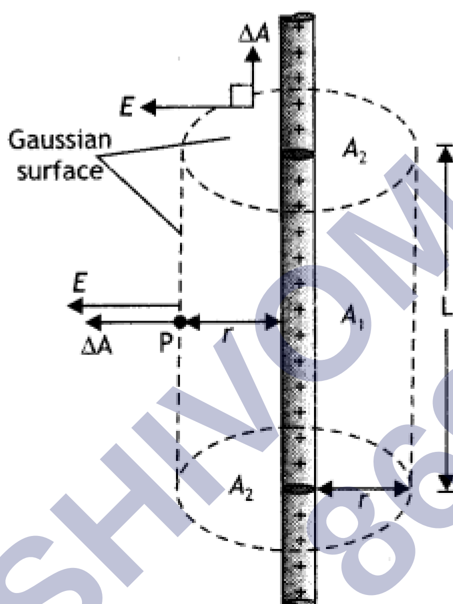
$E = \frac{\sigma}{2\varepsilon_0}$ .... (4)

(b)

(i) directed outwards

(ii) directed inwards.

2. Answer:



Consider an infinitely Long, thin wire charged positively and having uniform Linear charge density λ. The symmetry of the charge distribution shows that must be perpendicular to the tine charge and directed outwards. As a result of this symmetry, we consider a Gaussian surface in the form of a cylinder with arbitrary radius r and arbitrary Length L. with its ends perpendicular to the wire as shown in the figure. Applying Gauss's theorem to curved surface ΔA_1 and circular surface ΔA_2.

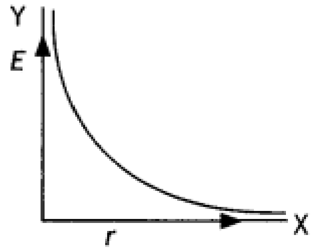
Φ EΔA1 cos 0°+ EΔA2 cos 90° = $\frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$ [$\because \lambda = \frac{q}{e}$]

Or

$E \cdot 2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$

This is the expression for the electric field due to an infinitely long thin wire.

The graph is as shown.



Assertion and Reason Answers-

1. (a) Both A and R are true, and R is the correct explanation of A.

Explanation:

According to Gauss's theorem in electrostatics, $\phi = \frac{q}{\epsilon_0}$

$$\phi = \frac{q}{\epsilon_0} = 8.85 \times 10^{-12}[8 \times 10^3 - 2 \times 10^3]$$

$$= 53.10 \times 10^{-9}\mathrm{C} = 0.053\mu\mathrm{C}.$$

2. (c) A is true but R is false.

Explanation:

The charge q on a body is given as q = ne where n is any integer positive or negative. The charge on the electron is q = 1.6 × 10^{-19}C which is less than 1C.

Case Study Answers-

1. Answer :

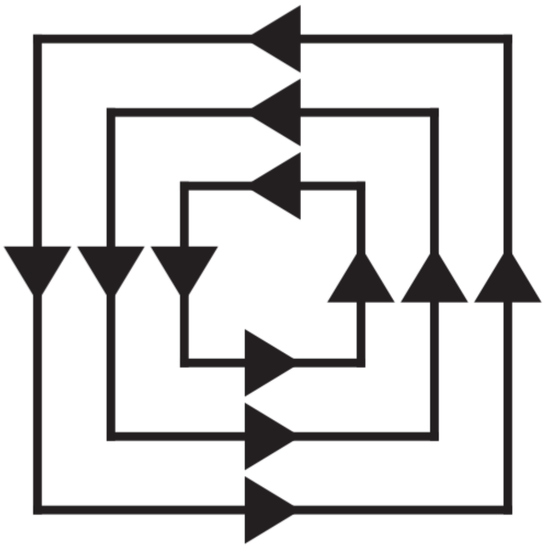
(i) (a) Radially outwards.

(ii) (c) They always form closed loops.

Explanation:

Electric lines of force do not form any closed loops.

(iii) (c)



Explanation:

Electric field tines can't be closed.

(iv) (d) Both (b) and (c).

(v) (a) $E_A > E_B > E_C$

2. Answer :

i. (d) -32N m^2 C^{-1}

Explanation:

Electric flux, $\phi = \vec{E} \cdot \vec{A} = EA\cos\theta$.

Where $\vec{A} = A\widehat{n}$

For electric flux passing through $S_6, \widehat{n}_{S_6} = -\hat{i}$ (Back)

$$\therefore \phi_{S_6} = -(4 \times 10^3 \text{ N C}^{-1})(0.1\text{m})^2 \cos 37^\circ$$

-32N m^2 C^{-1}

ii. (a) $-24 \mathrm{N\ m^2\ C^{-1}}$

Explanation:

For electric flux passing through $\mathrm{S_1}, \widehat{\mathrm{n}}_{\mathrm{s_1}} = -\hat{\mathrm{j}}$ (Left)

$$\therefore \phi_{\mathrm{s_1}} = -(4 \times 10^3\ \mathrm{N\ C^{-1}})(0.1\mathrm{m})^2 \cos(90^\circ - 37^\circ)$$

$-24\ \mathrm{N\ m^2\ C^{-1}}$

iii. (c) S_2 and S_4

Explanation:

Here, $\widehat{\mathrm{n}}_{\mathrm{s_2}} = +\widehat{\mathrm{k}}$ (Top)

$$\therefore \phi_{\mathrm{s_2}} = -(4 \times 10^3\ \mathrm{N\ C^{-1}})(0.1\mathrm{m})^2 \cos 90^\circ = 0$$

$$\widehat{\mathrm{n}}_{\mathrm{s_3}} = +\hat{\mathrm{j}}\ \text{(Right)}$$

$$\widehat{\mathrm{n}}_{\mathrm{s_4}} = -\widehat{\mathrm{k}}\ \text{(Bottom)}$$

$$\therefore \phi_{\mathrm{s_4}} = -(4 \times 10^3\ \mathrm{N\ C^{-1}})(0.1\mathrm{m})^2 \cos 90^\circ = 0$$

And, $\widehat{\mathrm{n}}_{\mathrm{s_5}} = -\hat{\mathrm{i}}$ (Fornt)

$$\therefore \phi_{\mathrm{s_5}} = -(4 \times 10^3\ \mathrm{N\ C^{-1}})(0.1\mathrm{m})^2 \cos 37^\circ$$

$= 32 \mathrm{N\ m^2\ C^{-1}}$

S_2 and S_4 surface have zero flux.

iv. (d) Zero.

Explanation:

As the field is uniform, the total flux through the cube must be zero,

i.e., any flux entering the cube must leave it.

v. (b) $[\mathrm{M\ L^3\ T^{-3}\ A^{-1}}]$

Explanation:

Surface integral $\oint \vec{\mathrm{E}} \cdot \mathrm{d}\vec{\mathrm{S}}$ is the net electric flux over a closed surface S.

$$\therefore [\phi_{\mathrm{E}}] = [\mathrm{M\ L^3\ T^{-3}\ A^{-1}}]$$