

Measures of Central Tendency

Important terms and concepts –

Average: It is a value which is typical or representative of a set of data. Averages are also called Measures of Central Tendency.

❖ Functions of Average:

- ✚ Presents complex data in a simple form.
- ✚ Facilitates comparison.
- ✚ Helps government to form policies.
- ✚ Useful in Economic analysis.

❖ Essentials of a good Average:

- ✚ Simple to calculate.
- ✚ It should be easy to understand.
- ✚ Rigidly defined.
- ✚ Based on all items of observation.
- ✚ Least affected by extreme values.
- ✚ Capable of further algebraic treatment.
- ✚ Least affected by sampling fluctuation.
- ✚ Graphic measurement possible.

❖ Types of Averages:

- ✚ Arithmetic Mean
- ✚ Median
- ✚ Mode
- ✚ Quartiles

❖ Arithmetic Mean (\bar{X}):

- ✚ It is the most common type of measures of central tendency.
- ✚ It is obtained by dividing the sum of all observation in a series by the total number of observation.

❖ Calculation of Arithmetic Mean:

Types of Series	Direct Method	Shortcut Methods	Step deviation Methods
Individual Series	$\bar{X} = \frac{\sum X}{N}$	$\bar{X} = A + \frac{\sum d}{N}$	$\bar{X} = A + \frac{\sum d'}{N} \times C$
Discrete series	$\bar{X} = \frac{\sum fx}{N}$	$\bar{X} = A + \frac{\sum fd}{N}$	$\bar{X} = A + \frac{\sum fd'}{N} \times C$
Continuous Series	$\bar{X} = \frac{\sum fm}{N}$	$\bar{X} = A + \frac{\sum fd}{N}$	$\bar{X} = A + \frac{\sum fd'}{N} \times C$

❖ Merits of Arithmetic Mean:

- ✚ Easy to calculate.
- ✚ Simple to understand.
- ✚ Based on all observations.
- ✚ Capable of further mathematical calculations.

Demerits:

- ✚ Affected by extreme values.
- ✚ Cannot be calculated in open-end series.
- ✚ Cannot be graphically ascertained.
- ✚ Sometimes misleading or absurd result.

❖ Weighted Arithmetic Mean:

Values to be arranged are given varying importance.

$$XW = \frac{\sum WX}{\sum W}$$

Where Xw = Weighted Arithmetic Mean

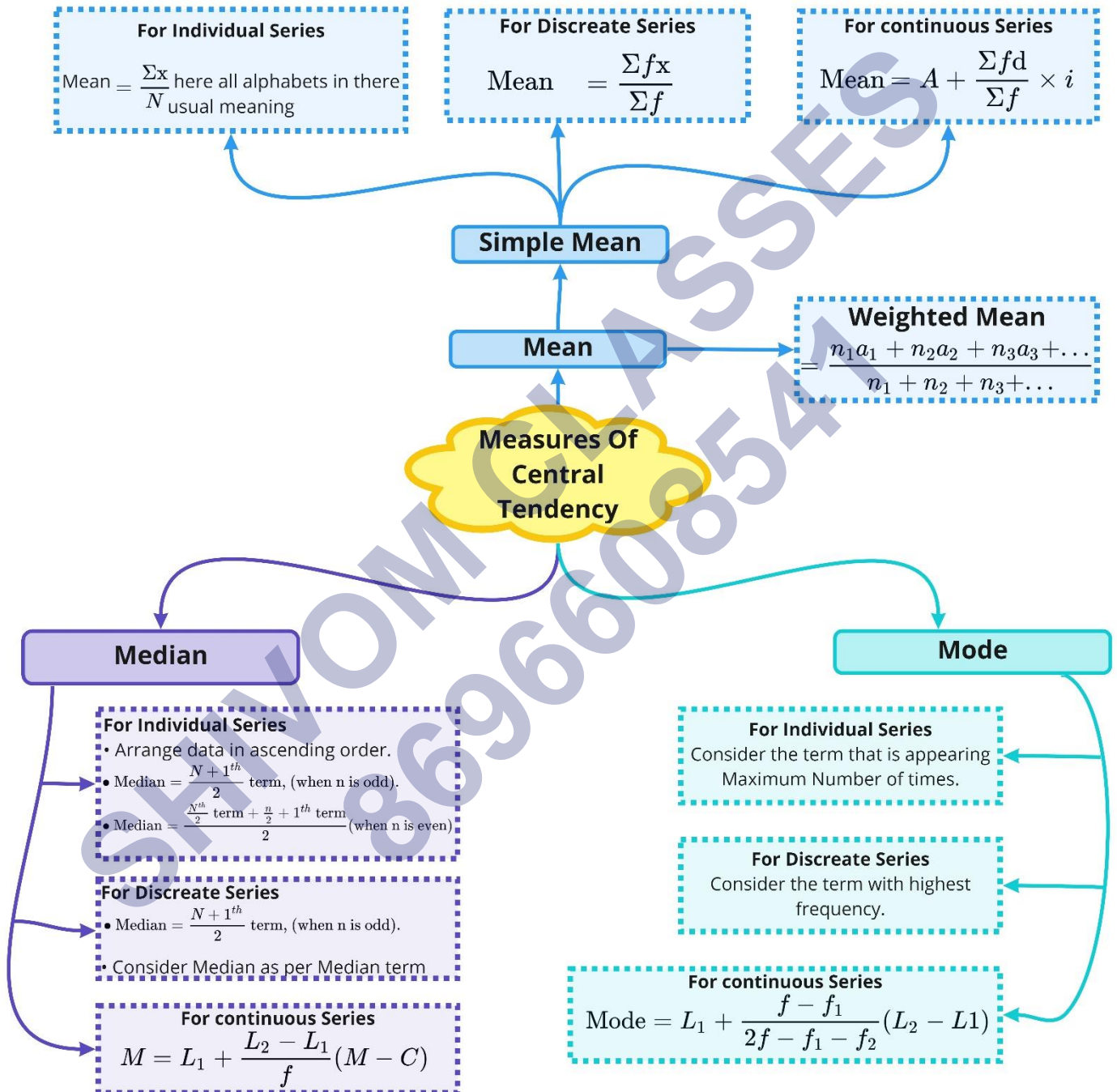
W = Weight

X = Values of the Variables

❖ Median (M):

It is defined as the middle value of the series, when the data is arranged in ascending or descending order.

Class : 11th Economics (Statistics)
Chapter-5 : Measures Of Central Tendency



Important Questions

Multiple Choice Questions-

- Which of the following is a type of mathematical average?
 - Median
 - Partition value
 - Mode
 - None of these
- Arithmetic mean of these items 5, 7, 9, 15, 20 is:
 - 10
 - 10.2
 - 11.2
 - 12
- Arithmetic mean of these items: 10, 15, X, 20, 30 is 20. Find out the missing item.
 - 10
 - 15
 - 5
 - 25
- While computing the arithmetic mean of a frequency distribution, the each value of a class is considered equal to:
 - Class mark
 - Lower limit
 - Upper limit
 - Lower class boundary
- Arithmetic mean of a series is 15 and if 5 is added in all the items of this series, the new arithmetic mean will be:
 - 5
 - 20
 - 18
 - (d)10

7. Which of the following is not a measure of central tendency?
- Mean
 - Mode
 - Standard deviation
 - Median
8. Which is not a method to find arithmetic mean?
- Direct method
 - Short-cut method
 - Step-deviation method
 - Karl Pearson's method
9. The sample mean is a:
- Parameter
 - Statistic
 - Variable
 - Constant
10. Assumed mean is taken in which method?
- Direct method
 - Step-deviation method
 - Karl Pearson's method
 - Spearman's method
11. The sum of deviations taken from mean is:
- Always equal to zero
 - Some times equal to zero
 - Never equal to zero
 - Less than zero
12. Sum of deviations of different values from arithmetic mean is always equal to:
- zero
 - one
 - less than one

d. more than one

13. Which of the following statements is always true?

- a. The mean has an effect on extreme scores
- b. The median has an effect on extreme scores
- c. Extreme scores have an effect on the mean
- d. Extreme scores have an effect on the median

14. Mean of 0.3, 5, 6, 7, 9, 12, 0.6 is:

- a. 4.9
- b. 5.7
- c. 5.6
- d. None of these

15. Simple average is sometimes called:

- a. Unweighted average
- b. Weighted average
- c. Relative average
- d. None of these

Very Short Questions:

1. Define median.
2. What is the mode?
3. Define the partition value.
4. Explain quartile.
5. What is positional average?
6. Define the central tendency.
7. What are the purpose of average is the statistical method?
8. What are the different kinds of statistical average?
9. What are the two methods that can calculate the simple arithmetic mean in case of individual series?
10. What are the methods calculating simple arithmetic mean?

Short & Long Questions:

1. Definition Of Central Tendency?

2. Purpose And Functions Of Averages?
3. Essentials Of A Good Average?
4. Types of statistical averages?
5. Arithmetic mean?
6. Types of arithmetic mean?
7. Pocket allowance of 10 students is Rs. 15, 20, 30, 22, 25, 18, 40, 50, 55 and 65. Find out the average pocket allowance.
8. Following is the pocket allowance of 10 students. Find out arithmetic mean using Shortcut Method.

Pocket Allowance (Rs.)	15	20	30	22	25	18	40	50	55	65
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9. Weighted Arithmetic Mean?
10. Combined Arithmetic Mean?
11. Properties of Arithmetic Mean (AM)?
12. The Principal Merit of Arithmetic Mean?
13. Calculation of Arithmetic Mean for Different Series: A Glance?

ANSWER KEY

Multiple Choice Answers-

1. D
2. A
3. C
4. D
5. B
6. B
7. C
8. D
9. B
10. B
11. A
12. A
13. C

14. B
15. A

Very Short Answers:

1. Median is a value located centrally of a series in such a way that half of the value of the series is above it and the other half is below.
2. The mode is a value that frequently occurs in the series. Which means the modal value has the highest frequency in the series.
3. The value that divides the series into more than two parts is known as a partition value.
4. The end value of the statistical series when divided into four parts is known as quartile.
5. Positional average are those averages whose value is worked out on the basis of their position in the statistical series.
6. All the methods of statistical analysis by which the average of the statistical series are analysed is known as a central tendency.
7. The purpose of the average is the statistical method are
 - Brief description
 - Comparison
 - Formulation of policies
 - Statistical analysis
 - One value of all
8. The different kinds of statistical average are.
 - Mathematical average
 - Positional average
9. The two methods that can calculate the simple arithmetic mean in the case of individual series are.
 - Direct method
 - Short-cut method
10. The methods of calculating simple arithmetic mean are.
 - Individual series
 - Discrete series
 - Frequency distribution

Short & Long Answers:

1. A Central Tendency refers to an average or a central value of a statistical series. It is difficult for anyone to understand or remember a large group of raw data. One would like to know the critical value which represents all the items in a series. Such a value is called 'central tendency' or 'average value'. For instance it is very difficult to remember and understand the data concerning the income of millions of Indians. However, if it is said that in 2016-17, provisional estimates for average income of the people in India was Rs. 82,269 per annum, it will be easy for us to guess the economic condition of most of the Indians. It is this average value which is called central tendency of the series. It is also called measure of location. Thus, measures of central tendency refer to all those methods of statistical analysis by which averages of the statistical series are worked out.

Definition

According to Croxton and Cowden, "An average is a single value within the range of the data that is used to represent all of the values in the series. Since an average is somewhere within the range of data, it is sometimes called measure of central value."

According to Clark, "An average is a figure that represents the whole group."

2. Study of averages is of central significance in statistical methods. That is why Bowley defines statistics as, "A Science of Averages".

What is the Basic Purpose of Finding an Average Value of a Series?

It is to identify such a value that represents characteristics of all the items in the series.

According to Moroney, "The purpose of an average is brief and simple representation of a group of individual values so that the brain may quickly grasp the general basis of the units of the group." Some of the main purposes and functions of averages are as under:

(1) Brief Description: The main purpose of an average is to present a brief description of the principal features of the raw data. As a result, data are easily understood.

(2) Comparison: Averages help in making comparison of different sets of data. For example, a comparison of the per capita income of India and USA shows that per capita income of India is much less than the per capita income of USA. Accordingly, it is concluded that India is a poor country.

(3) Formulation of Policies: Averages help in formulation of policies. For example, in India the per capita income is Rs. 82,269 per annum which is much less than many countries in the world. Accordingly, it becomes clear for the government to focus on such economic policies as are likely to increase per capita income.

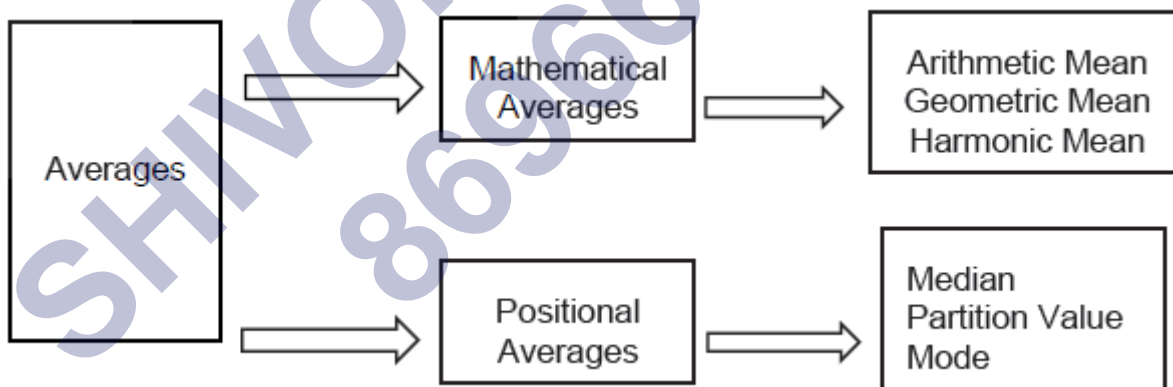
(4) Statistical Analysis: Averages constitute the basis of statistical analysis. For example, if one knows the average marks secured by the students of a class in their different subjects, one can easily analyse the subjects in which the students are weak.

(5) One Value for All: Averages represent the universe or the mass of statistical data. One value represents all values of the series. Accordingly, conclusion can be drawn in respect of the universe as a whole.

3. A good and satisfactory average should have the following features:

- 1) **Clear and Stable Definition:** A good and a satisfactory average should be clear and stable in definition.
- 2) **representative:** An average value should be representative of the entire mass of data. It should be based on all the observations of the series.
- 3) **Simplicity:** Simplicity is another essential feature of a good average. It must be so simple that it is easily worked out.
- 4) **Certainty:** A good average must be certain in character. Only then an average value can be used as the basis of statistical analysis.
- 5) **Absolute Number:** A good average should be an absolute number. A percentage or a relative value does not serve as a good average.
- 6) **Least Effect of a Change in the Sample:** An average of a series should be least affected by a change in the sample on which the average is based.
- 7) **Algebraic Treatment:** A good average should be capable of further mathematical or algebraic treatment.

4. TYPES OF STATISTICAL AVERAGES



Averages are broadly classified into two categories:

- (1) Mathematical Averages, and
- (2) Positional Averages.

In the present chapter we discuss arithmetic mean.

5. **ARITHMETIC MEAN-** Arithmetic Mean is a simple average of all items in a series. It is the simplest measure of central tendencies. The arithmetic mean of a series is simply called 'Mean'. If, for example, Ram plays 5 matches, Shyam 6, Mohan 7, Kishan 8 and Ravi 9 matches a week, the average number of matches played by Ram, Shyam, Mohan, Kishan and Ravi would be determined as under:

Number of Matches = $5 + 6 + 7 + 8 + 9 = 35$

Number of Boys = 5

Mean = Total Value of the Items/Number of Items = $35/5 = 7$

What is the basic difference between Simple Arithmetic Mean and Weighted Arithmetic Mean?

In simple arithmetic mean, all items of the series are taken as of equal importance. In the weighted average, on the other hand, different items are taken as of different importance;

accordingly, weights are accorded to different items depending on their relative importance.

Definition

Arithmetic Mean or Mean is the number which is obtained by adding the values of all the items of a series and dividing the total by the number of items.

In the words of H. Secrist, "The Arithmetic Mean is the amount secured by dividing the sum of value of the items in a series by their numbers."

FORMULA

Arithmetic mean is generally written as \bar{X} . It may be expressed in the form of following formula:

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \dots + X_n}{N} = \frac{\sum X}{N}$$

Here, X_1, X_2, X_3, X_n are the values of different items in the series. Thus, X_1 = matches played by Ram, 5; X_2 = matches played by Shyam, 6 and X_3 = matches played by Mohan, 7, etc.

N = Total number of items (in the above example, it is 5 comprising of five boys, Ram, Shyam, Mohan, Kishan and Ravi).

Σ is a sign called Sigma. It refers to the sum total of the values of different items in the series.

6. Arithmetic mean is of two types:

(1) Simple Arithmetic Mean: In it, all items of a series are given equal importance.

(2) Weighted Arithmetic Mean: In it, different items of a series are accorded different weights in accordance with their relative importance.

Methods of Calculating Simple Arithmetic Mean

We know, there are three types of statistical series:

(1) Individual Series

(2) Discrete Series

(3) Frequency Distribution.

Arithmetic mean may be calculated with respect to these series using different methods.

Calculation of Simple Arithmetic Mean in Case of Individual Series

In the case of individual series, arithmetic mean may be calculated by two methods:

- (1) Direct Method
 (2) Short-cut Method.
 (1) Direct Method

Following steps are involved in this method:

- (i) Add up values of all the items of a series (ΣX);
 (ii) Find out total number of items in the series (N); and
 (iii) Divide the total of value of all the items (ΣX) with the number of items (N). The resultant value would be the arithmetic mean. Thus,

FORMULA

$$\bar{X} = \Sigma X / N \text{ OR } \bar{X} = \text{TOTAL VALUE OF THE ITEMS} / \text{NUMBER OF ITEMS}$$

7. Average pocket allowance of the 10 students = Rs. 34.

Pocket Allowance (₹) (X)
15
20
30
22
25
18
40
50
55
65
$\Sigma X = 340$

Short-cut Method

This method is used when the size of items is very large.

The use of short-cut method involves the following steps:

- Before finding an actual average, some value in the series is taken as 'assumed average'. Assumed average should preferably be the middle item of the series. It facilitates the calculation of deviation from the assumed average.
- Assumed average is generally taken by dividing by 2, the total of maximum and minimum values of the items. It is always a complete number. In statistics, assumed average is often denoted as A' .

- iii. Deviations of different values from the assumed average are found and each deviation is written against the concerned value in the series. Thus, d (deviation) = $X - A$

Where, X is the actual value in the series and A is the assumed average. If value of the item (X) is less than the assumed average (A), then ' d ' or $X - A$ would be negative. Thus,

in the earlier illustration, if A is 40, ' d ' corresponding to 1st value in the series (15) would be, $15 - 40 = (-) 25$. Likewise corresponding to 8th value in the series (50) it would be, $50 - 40 = (+) 10$.

While noting down the deviation against a particular item, sign of the deviation (+) or (-) must also be specified. Thus, ' d ' of the first value would be written as, -25 and of the 8th value as, $+10$.

- iv. Find the sum/total of all deviations. Add up positive deviations and negative deviations separately; and then find out the difference. If the sum of negative deviations is more than that of positive deviations then net sum of all the deviations will be negative and vice versa.
- v. Divide the net sum of the deviations by the number of items in the series. If the dividend is positive (+) then it gets added to A , the assumed average. And, if the dividend is negative (-), then it gets subtracted from A , the assumed average. The value, thus, obtained would be the actual average of the series.

FORMULA

$$\bar{X} = A + \frac{\sum D}{N}$$

(Here, \bar{X} = Arithmetic mean; A = Assumed average; $\sum d$ = Net sum of the deviations of the different values from the assumed average; N = Number of items in the series.)

8. (Assumed Average, $A = 40$)

Number of Student	Pocket Allowance (₹) (X)	Deviation from the Assumed Average ($d=x-a$), ($A=40$)
1	15	$15 - 40 = -25$
2	20	$20 - 40 = -20$
3	30	$30 - 40 = -10$
4	22	$22 - 40 = -18$
5	25	$25 - 40 = -15$
6	18	$18 - 40 = -22$
7	40	$40 - 40 = -0$
8	50	$50 - 40 = 10$
		-110
		50

9	55	$55 - 40 = 15$
10	65	$65 - 40 = 25$
N = 10		$\sum d = -110 + 50 = -60$

The sum of '+' deviations = + 50

The sum of deviations = -110

The net sum of deviations, $\sum d = -110 + 50 = -60$

Dividing the aggregate of deviations ($\sum d$) by the number of items (N),
 $\sum d/N = -60/10 = -6$

Substituting this value of $\sum d/N$ in the following formula:

$$X = A + \sum D/N$$

We have,

$$X = 40 + (-) 6 = 40 - 6 = 34$$

Arithmetic Mean = Rs. 34.

Calculation of Simple Arithmetic Mean in Case of Discrete Series or Frequency Array
 Individual series do not have frequencies of the items. These series show only values of different items.

In discrete series, there are frequencies corresponding to different items in the series.

There are three methods of calculating mean of the discrete series.

That discrete series (also called frequency array) do not have class intervals. An item in the series does not assume any range of values but each item has corresponding frequency.

- (1) Direct Method
- (2) Short-cut Method
- (3) Step-deviation Method.

Direct method of calculating mean of the discrete series involves the following steps:

(i) Values of the various items in the series are indicated by X, and their frequencies by 'f'.

(ii) Each item is multiplied by its frequency to get 'fX'. These multiples are added to get $\sum fX$. That is,

$$\sum fX = f_1X_1 + f_2X_2 + \dots + f_nX_n$$

(iii) Frequencies are added up to get $\sum f$. That is,

$$\sum f = f_1 + f_2 + f_3 + \dots + f_n$$

(iv) $\sum fX$ is divided by $\sum f$ to obtain the mean, X.

FORMULA

$$\bar{X} = \frac{\sum fX}{\sum f}$$

9. We have so far discussed Simple Arithmetic Mean. In simple arithmetic mean, all items of a series are taken as of equal significance. But sometimes we may give greater significance to some items and less to others. As household may, for example, give more significance to food, less to clothes and still less to entertainment. When different items of a series are weighed according to their relative importance, the average of such series is called Weighted Arithmetic Average. A Weighted Arithmetic Average is, thus, the mean of weighted items.

Calculation of Weighted Mean

Calculation of weighted mean involves the following steps:

- (i) Different items are weighed according to their significance. Weights are indicated by 'W'.
- (ii) Items (X) are multiplied by their corresponding weights (W) and added up to get ΣWX .
- (iii) ΣWX is divided by the sum total of weights, i.e., ΣW , to get the mean value, that is,

FORMULA

$$\bar{X}_w = \frac{\Sigma WX}{\Sigma W}$$

Here, \bar{X}

w indicates weighted average.

10. Given the mean values of two or more parts of a series and the number of items in each part, one can get Combined Arithmetic Mean, or mean of the series as a whole. The following formula is used for the estimation of combined arithmetic mean:

FORMULA

$$\bar{X}_{1,2} = \frac{\bar{X}_1 N_1 + \bar{X}_2 N_2}{N_1 + N_2}$$

Here, $\bar{X}_{1,2}$ = Combined arithmetic mean of parts 1 and 2 of a series;

\bar{X}_1 = Arithmetic mean of part 1 of the series;

\bar{X}_2 = Arithmetic mean of part 2 of the series;

N_1 = Number of items in part 1 of the series;

N_2 = Number of items in part 2 of the series.

Likewise, when there are more than 2 parts of a series, the following formula is used to work out Combined Arithmetic Mean. *

FORMULA

$$\bar{X}_{1,2,3,\dots,n} = \frac{\bar{X}_1 N_1 + \bar{X}_2 N_2 + \dots + \bar{X}_n N_n}{N_1 + N_2 + \dots + N_n}$$

11. (i) The sum of deviations of the items from AM is always zero.
 (ii) The sum of squared deviations of the items from AM is minimum.
 (iii) If each item of a series is increased, decreased, multiplied or divided by some constant then AM also increases, decreases, multiplies or is divided by the same constant.
 (iv) The product of the AM and the number of items on which mean is based is equal

to the sum of all given items.

(v) if each item of the original series is replaced by the actual mean, then the sum of these substitutions will be equal to the sum of the individual items.

12. That it is based on all items of the series and is capable of further algebraic treatment.

The Principal Demerit of Arithmetic Mean

End-use of the goods is the principal basis of classifying the goods as intermediate goods and final goods.

Merits and Demerits of Arithmetic Mean

- 13.

Series	Methods of Calculation of Arithmetic Mean
1. Individual Series	(a) Direct Method: Formula: $\bar{X} = \frac{\sum X}{N}$
	(b) Short-cut Method: Formula: $\bar{X} = A + \frac{\sum d}{N}$
2. Discrete Series/Frequency Array	(a) Direct Method: Formula: $\bar{X} = \frac{\sum fX}{\sum f}$
	(b) Short-cut Method: Formula: $\bar{X} = A + \frac{\sum fd}{\sum f}$
	(c) Step-deviation Method: Formula: $\bar{X} = A + \frac{\sum fd'}{\sum f} \times C$
3. Frequency Distribution Series	(a) Direct Method: Formula: $\bar{X} = \frac{\sum fm}{\sum f}$
	(b) Short-cut Method: Formula: $\bar{X} = A + \frac{\sum fd}{\sum f}$
	(c) Step-deviation Method: Formula: $\bar{X} = A + \frac{\sum fd'}{\sum f} \times C$