MATHEMATICS

Chapter 10: Mensuration



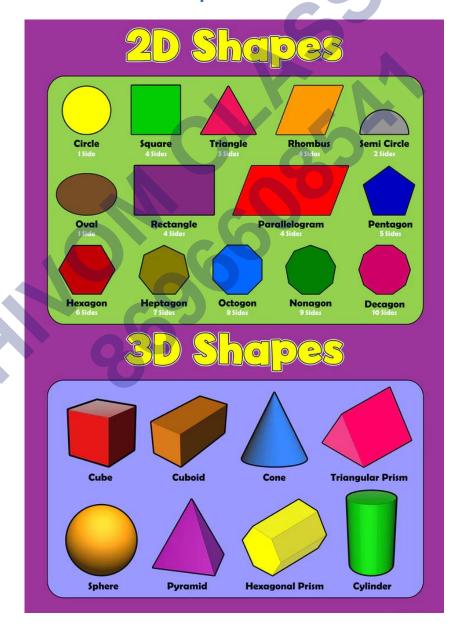
MENSURATION

Mensuration

Mensuration is the branch of mathematics that deals with the measurement of length, area or volume of various geometric shapes.

A branch of mathematics that talks about the length, volume, or area of different geometric shapes is called Mensuration. These shapes exist in 2 dimensions or 3 dimensions. Let's learn the difference between the two.

Differences Between 2D and 3D shapes



| 2D Shape | 3D Shape |
|---|--|
| If a shape is surrounded by three or more straight lines in a plane, then it is a 2D shape. | If a shape is surrounded by a no. of surfaces or planes then it is a 3D shape. |
| These shapes have no depth or height. | These are also called solid shapes and unlike 2D they have height or depth. |
| These shapes have only two dimensions say length and breadth. | These are called Three dimensional as they have depth (or height), breadth and length. |
| We can measure their area and Perimeter. | We can measure their volume, CSA, LSA or TSA. |

Mensuration in Maths-Important Terminologies

Let's learn a few more definitions related to this topic.

| Terms | Abbreviation | Unit | Definition |
|-------------------------|--------------|-----------------------------------|--|
| Area | A | m ² or cm ² | The area is the surface which is covered by the closed shape. |
| Perimeter | 86 | cm or m | The measure of the continuous line along the boundary of the given figure is called a Perimeter. |
| Volume | V | cm ³ or m ³ | The space occupied by a 3D shape is called a Volume. |
| Curved Surface Area | CSA | m ² or cm ² | If there's a curved surface, then the total area is called a Curved Surface area. Example: Sphere |
| Lateral Surface area | LSA | m ² or cm ² | The total area of all the lateral surfaces that surrounds the given figure is called the Lateral Surface area. |

| Total Surface Area | TSA | m ² or cm ² | The sum of all the curved and lateral surface areas is called the Total Surface area. |
|-----------------------|-----|-----------------------------------|---|
| Square Unit | _ | m ² or cm ² | The area covered by a square of side one unit is called a Square unit. |
| Cube Unit | _ | m ³ or cm ³ | The volume occupied by a cube of one side one unit |

Mensuration Formulas

Now let's learn all the important mensuration formulas involving 2D and 3D shapes. Using this mensuration formula list, it will be easy to solve the mensuration problems. Students can also download the mensuration formulas list PDF from the link given above. In general, the most common formulas in mensuration involve surface area and volumes of 2D and 3D figures.

| Mensuration Formulas | | | | |
|----------------------|--|----------|----------------------------|--|
| Perimeter | | Surfa | ace Area | |
| Square | P=4s | Cube | $SA = 6s^2$ | |
| Rectangle | P = 2(l+w) | Cylinder | $SA = 2\pi r h + 2\pi r^2$ | |
| Cive | umference | Cone | $SA = \pi r l$ | |
| Circle | $C = 2\pi r$ | Sphere | $SA = 4\pi r^2$ | |
| Circle | C = Zni | | | |
| | Area | Volume | | |
| Square | $A = s^2$ | Cube | $V = s^3$ | |
| Rectangle | A = h w | Cylinder | $V = \pi r^2 h$ | |
| Triangle | $A = \frac{1}{2}bh$ | Cone | $V = \frac{1}{3}\pi r^2 h$ | |
| Trapezoid | $A = \frac{1}{2} \left(b_1 + b_2 \right) h$ | Sphere | $V = \frac{4}{3}\pi r^3$ | |
| Circle | $A = \pi r^2$ | | | |
| | | | | |

Mensuration Formulas For 2D Shapes

| Shape | Area (Square units) | Perimeter (units) | Figure |
|---------------------|---|----------------------|--------------------------|
| Square | a ² | 4a | |
| Rectangle | l×b | 2 (l + b) | b |
| Circle | πr ² | 2πη | r |
| Scalene Triangle | v[s(s-a)(s-b)(s-c)], Where, s = (a+b+c)/2 | a+b+c | Area of scalene triangle |

| Shape | Area (Square units) | Perimeter (units) | Figure |
|-------------------------|---------------------|--------------------------|--------|
| Isosceles Triangle | ½ × b × h | 2a + b | a h |
| Equilateral triangle | (√3/4) × a² | 3a | a |
| Right Angle Triangle | 1/2 × b × h | b + hypotenuse + h | h |

| Shape | Area (Square units) | Perimeter (units) | Figure |
|---------------|-------------------------------------|----------------------|-----------|
| Rhombus | $\frac{1}{2} \times d_1 \times d_2$ | 4 × side | A B C C |
| Parallelogram | b × h | 2(I+b) | D C h |
| Trapezium | ½ h(a+c) | a+b+c+d | A a B b C |

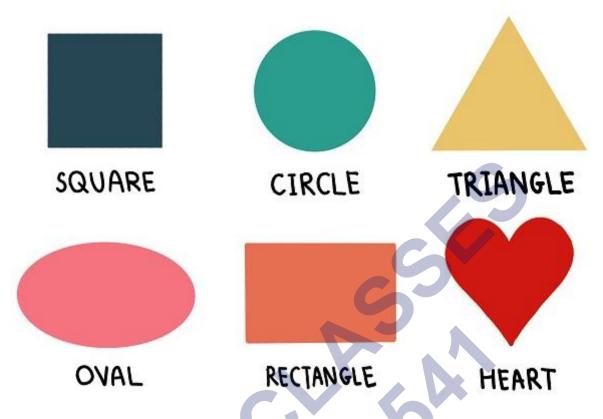
Mensuration Formulas for 3D Shapes

| Shape | Volume (Cubic units) | Curved Surface Area (CSA) or Lateral Surface Area (LSA) (Square units) | Total Surface Area (TSA) (Square units) | Figure |
|-------|----------------------------|--|---|--------|
|-------|----------------------------|--|---|--------|

| Shape | Volume (Cubic units) | Curved Surface Area (CSA) or Lateral Surface Area (LSA) (Square units) | Total Surface Area (TSA) (Square units) | Figure |
|--------|----------------------------|--|---|--------|
| Cube | a ³ | LSA = 4 a ² | 6 a ² | a |
| Cuboid | I×b×h | LSA = 2h(I + b) | 2 (lb +bh +hl) | h |
| Sphere | (4/3) π r ³ | 4 π r ² | 4 π r ² | F. |

| Shape | Volume (Cubic units) | Curved Surface Area (CSA) or Lateral Surface Area (LSA) (Square units) | Total Surface Area (TSA) (Square units) | Figure |
|------------|----------------------------|--|---|--------|
| Hemisphere | (⅔) π r³ | 2 π r ² | 3 π r ² | |
| Cylinder | $\pi r^2 h$ | 2π r h | 2πrh + 2πr² | h |
| Cone | (½) π r² h | πη | πr (r + I) | h |

Shapes

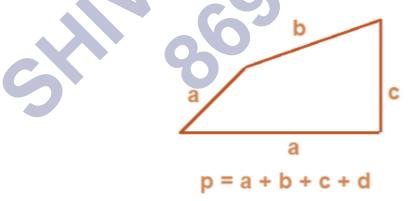


A shape is the form of an object.

Examples of two-dimensional shapes are square, rectangle and triangle, and of three-dimensional shapes are cube, cuboid and sphere.

Perimeter

Perimeter is the total length or total distance covered along the boundary of a closed shape.



Perimeter of a circle is also called as the circumference of the circle.

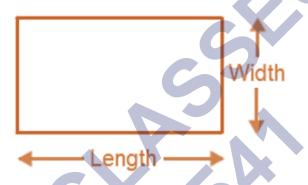
Perimeter of a Triangle



- Perimeter of triangle = Sum of lengths of all sides = a + b + c.
- If the given triangle is equilateral that is if all the sides are equal (a = b = c), then its perimeter is equal to $3 \times length$ of one side of the triangle.

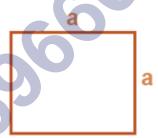
Perimeter of a Rectangle

Perimeter of the rectangle = length (I) + length (I) + width (w) + width (w) = 2 × [length (I) + width (w)]



Perimeter of a Square

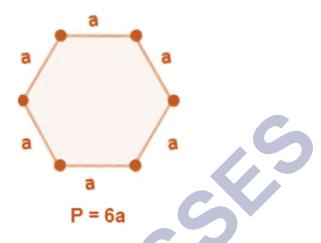
Perimeter of square = $4 \times length$ of a side = 4a



Perimeter of a 'n' sided polygon

- A polygon is a closed shape made up of line segments.
- Perimeter of n sided polygon = n × length of one side.
- Example: Length of each side of a hexagon is a cm, then:

Perimeter of the hexagon = 6a cm



Perimeter of irregular shapes

Irregular shapes are the shapes which do not have all sides and angles equal.

The perimeter of irregular shapes is equal to total length covered by the shape.

In the figure given below, perimeter is the sum of all sides.

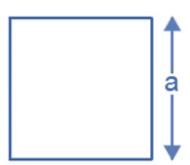


Area

Area is the total amount of surface enclosed by a closed figure.

Area of Square

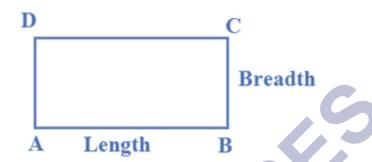
Area of a square = Side \times Side = Side² = a^2 , where a is the length of each side.



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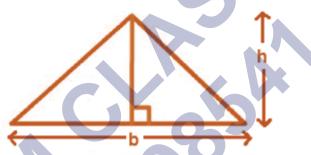
Area of Rectangle

Area = length (I) × breadth (b)



Area of a triangle

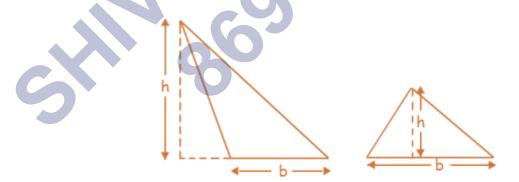
Area of triangle = $(1/2) \times base \times height = (1/2) \times b \times h$



Areas of different types of triangles

Consider an acute and an obtuse triangle.

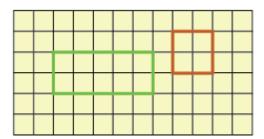
Area of each triangle = $(1/2) \times base \times height = (1/2) \times b \times h$



Visualisation of Area

In the given graph, if the area of each small square is 1 cm², then Area of rectangle = $I \times b = 5 \times 2 = 10$ cm²

Area of square = $a \times a = 2 \times 2 = 4cm^2$



Area of irregular shapes

Area of an irregular figure can be calculated:

Step 1: Divide the irregular shape into regular shapes that you can recognize (eg. triangles, rectangles, circles and squares)

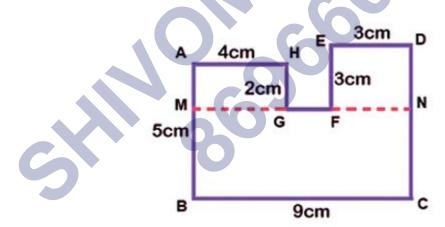
Step 2: Find the area of these individual shapes and add them. Sum will be the area of the irregular figure.

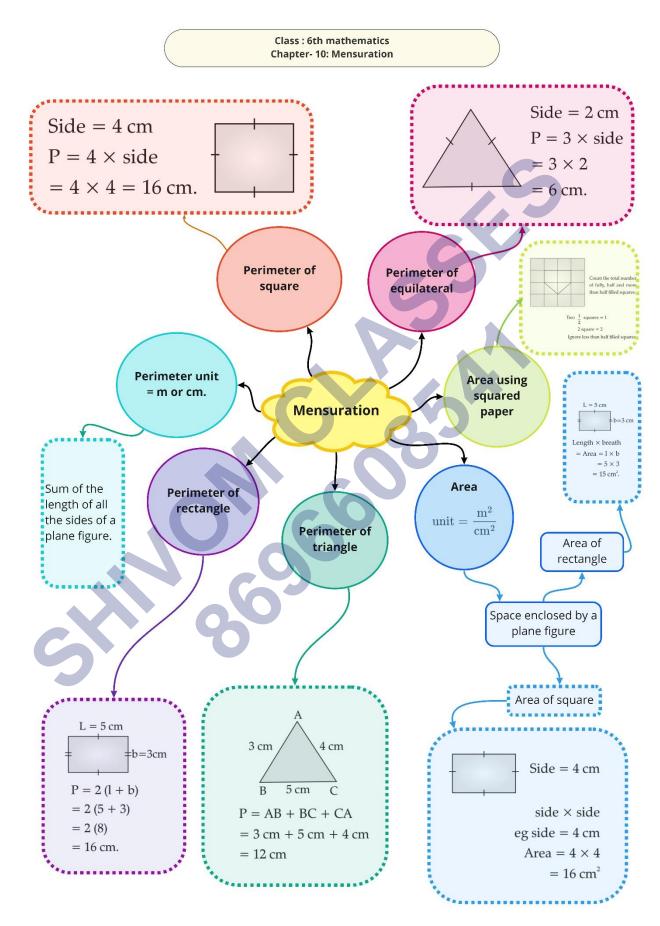
Example: Area of the given figure = Area of MNCB + Area of AMGH + Area of EFND

$$= [5 \times 9 + 4 \times 2 + 3 \times 3] \text{ cm}^2$$

$$= [45 + 8 + 9] \text{ cm}^2$$

$$= 62 \text{ cm}^2$$





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Important Questions

Multiple Choice Questions:

Question 1. Perimeter of a rectangle =

- (a) Length × Breadth
- (b) Length + Breadth
- (c) $2 \times (Length + Breadth)$
- (d) $2 \times (Length \times Breadth)$.

Question 2. Perimeter of a square =

- (a) 4 × Length of a side
- (b) 2 × Length of a side
- (c) 3 × Length of a side
- (d) $6 \times \text{Length of a side}$.

Question 3. Perimeter of an equilateral triangle

- (a) 2 × Length of a side
- (b) 3 × Length of a side
- (c) $4 \times \text{Length of a side}$
- (d) 6 x Length of a side.

Question 4. Area of a rectangle =

- (a) Length × Breadth
- (b) Length + Breadth

- (c) 2 × (Length + Breadth)
- (d) $2 \times (Length \times Breadth)$.

Question 5. Area of a square =

- (a) side × side
- (b) 4 × Length of a side
- (c) 2 × Length of a side
- (d) $6 \times \text{Length of a side}$.

Question 6. Perimeter of a regular pentagon =

- (a) 4 × Length of a side
- (b) $3 \times \text{Length of a side}$
- (c) 6 × Length of a side
- (d) $5 \times \text{Length of a side.}$

Question 7. Perimeter of a regular hexagon =

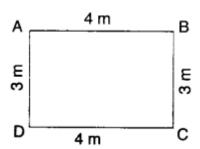
- (a) 3 × Length of a side
- (b) 4 × Length of a side
- (c) $5 \times \text{Length of a side}$
- (d) $6 \times \text{Length of a side}$.

Question 8. Apala went to a park 20 m long and 10 m wide. She took one complete round of it. The distance covered by her is:

- (a) 30 m
- (b) 60 m

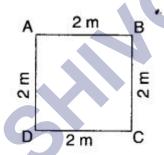
- (c) 20 m
- (d) 10 m.

Question 9. The perimeter of the figure is



- (a) 12m
- (b) 14m
- (c) 24 m
- (d) 7 m.

Question 10. The perimeter of the figure is

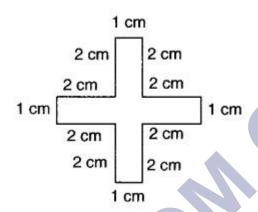


- (a) 8m
- (b) 16m
- (c) 4m
- (d) none of these.

Question 11. A page is 25 cm long and 20 cm wide. Find the perimeter of this page.

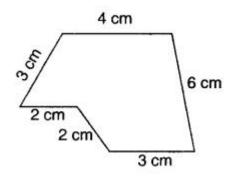
- (a) 90 cm
- (b) 45 cm
- (c) 500 cm
- (d) 5 cm.

Question 12. The perimeter of the figure is



- (a) 5 cm
- (b) 10 cm
- (c) 15 cm
- (d) 20 cm.

Question 13. The perimeter of the figure is



- (a) 20 cm
- (b) 10 cm
- (c) 24 cm
- (d) 15 cm.

Question 14. Meenu wants to put a lace border all around a rectangle table cover 2 m long and 1 m wide. Find the length of the lace required by Meenu.

- (a) 3 m
- (b) 4 m
- (c) 5 m
- (d) 6m.

Question 15. Find the perimeter of a rectangle whose length and breadth are 9 cm and 1 cm respectively,

- (a) 10 cm
- (b) 20 cm
- (c) 30 cm
- (d) 40 cm.

Match The Following:

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| | Column I | | Column II |
|----|----------|----|-----------|
| 1. | 8 | Α. | 15 |
| 2. | 3 | В. | 8 |
| 3. | 5 5 | C. | 24 |
| 4. | 3 3 2 | D. | 12 |

Fill in the blanks:

- **1.** _____ is a rectangle whose all sides are equal.
- **2.** The amount of surface enclosed by a figure is called its _____.
- **3.** For fencing the plot, we need to calculate its _____.
- **4.** _____ is the sum of all sides.

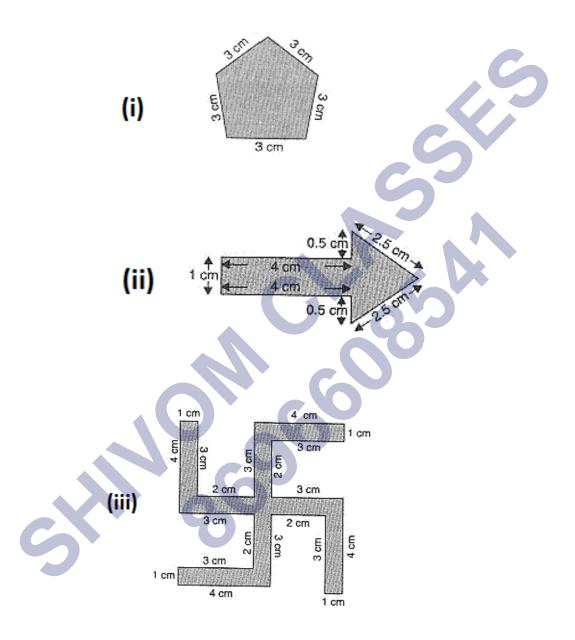
True /False:

- **1.** For tiling a rectangular Plot, we must calculate its area.
- 2. Length and breadth of a Rectangle are 1.5m and 1m. Area is 1.5m².
- **3.** The Perimeter of a square is 4 times the length of the side.
- **4.** To find the length of fencing the square playground we must find its Perimeter.

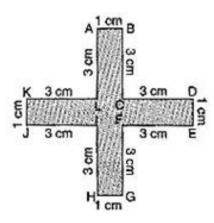
Very Short Questions:

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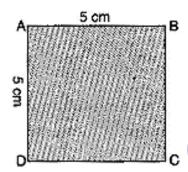
- 1. Find the area of the rectangle whose side are:3 cm and 4 cm
- 2. Find the area of the rectangle whose side are:12 m and 21 m
- **3.** Find the perimeter of each of the following figures given below?



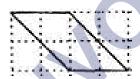
4. Find the perimeter of the following figure?



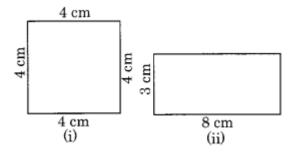
5. Find the perimeter of the following figure given below.



6. Find the area of the following figure:

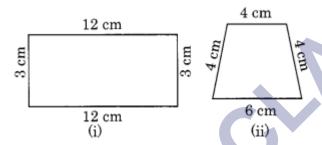


- 7. The length of rectangle is thrice its breadth and its perimeter is 48cm. Find length and breadth of rectangle.
- 8. The perimeter of a square is 64 cm. Find the length of each side.
- **9.** Length and breadth of a rectangular table-top are 36 cm and 24 cm respectively. Find its perimeter.
- **10.** Which of the following figure has greater perimeter?



Short Questions:

1. How much distance will you have to travel in going around each of the following figures?

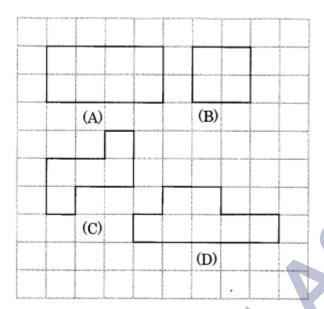


- 2. Find the perimeter of a square whose side is 15 cm.
- **3.** Find the cost of fencing a rectangular park 300 m long and 200 m wide at the rate of ₹4 per metre.
- 4. Find the area of a square field whose each side is 150 m.
- **5.** Length and breadth of a rectangular paper are 22 cm and 10 cm respectively. Find the area of the paper.
- **6.** Find the length of a rectangle given that its perimeter is 880 m and breadth is 88 m.
- **7.** How many trees can be planted at a distance of 6 metres each around a rectangular plot whose length is 120 m and breadth is 90 m?

Long Questions:

1. A rectangular park is 30 metres long and 20 metres broad. A steel wire fence is put up all around it. Find the cost of putting the fence at the rate of ₹15 per metre.

2. Find the area of the figures A, B, C and D drawn on a squared paper in the following figure by counting squares.



- **3.** A rectangle and a square have the same perimeter 100 cm. Find the side of the square. If the rectangle has a breadth 2 cm less than that of the square. Find the breadth, length and area of the rectangle.
- **4.** Fencing the compound of a house costs ₹5452. If the rate is ₹94 per metre, find the perimeter of the compound. If the breadth is 10 m, find its length.

Assertion and Reason Questions:

1.) Assertion (A) – Perimeter of a rectangle = $2 \times (\text{Length} \times \text{Breadth})$.

Reason (R) – Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once

- a) Both A and R are true and R is the correct explanation of A
- b) Both A and R are true but R is not the correct explanation of A
- c) A is true but R is false
- d) A is false but R is true
- 2.) Assertion (A) Perimeter of a square = $4 \times \text{Length of a side}$

Reason (R) – Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once

- a) Both A and R are true and R is the correct explanation of A
- b) Both A and R are true but R is not the correct explanation of A
- c) A is true but R is false
- d) A is false but R is true

ANSWER KEY -

Multiple Choice questions:

- 1. (c) $2 \times (Length + Breadth)$
- 2. (a) $4 \times \text{Length of a side}$
- 3. (b) $3 \times \text{Length of a side}$
- **4.** (a) Length × Breadth
- **5.** (a) side \times side
- **6.** (d) $5 \times \text{Length of a side}$.
- 7. (d) $6 \times \text{Length of a side}$.
- **8.** (b) 60 m

Hint:

Distance covered = 2(20 + 10) = 60 m

9. (b) 14m

Hint:

Perimeter = 2(4 + 3) = 14m

10. (a) 8m

Hint:

Perimeter = $4 \times 2 = 8m$

11. (a) 90 cm

Hint:

Perimeter = 2(25 + 20) = 90 cm

12. (d) 20 cm.

Hint:

Perimeter =
$$4(1 + 2 + 2) = 20$$
cm

13. (a) 20 cm

Hint:

Perimeter =
$$4 + 3 + 2 + 2 + 3 + 6 = 20$$
 cm

14. (d) 6m.

Hint:

Length of the lace =
$$2(2 + 1) = 6 \text{ m}$$

15. (b) 20 cm

Hint:

Perimeter =
$$2(9 + 1) = 20 \text{ cm}$$

Match The Following:

| | Column I | | Column II |
|----|----------|----|-----------|
| 1. | 8 | C. | 24 |
| 2. | 3 | D. | 12 |
| 3. | 5 5 | А. | 15 |

| | Column I | | Column II |
|----|----------|----|-----------|
| 4. | 3 3 | В. | 8 |

Fill in the blanks:

- **1. Square** is a rectangle whose all sides are equal.
- 2. The amount of surface enclosed by a figure is called its <u>area</u>.
- **3.** For fencing the plot, we need to calculate its **Perimeter**.
- 4. <u>Perimeter</u> is the sum of all sides.

True /False:

- **1.** True
- **2.** True
- 3. True
- 4. True

Very Short Answer:

- 1. Area of the rectangle = Length \times Breadth = 3×4 cm = 12 sq cm
- 2. Area of the rectangle = Length \times Breadth = 12 m \times 21 m = 252 sq m
- 3. (i) Perimeter = 3 cm + 3 cm + 3 cm + 3 cm + 3 cm
 - = 15 cm
 - (ii) Perimeter = 1 cm + 4 cm + 0.5 cm + 2.5 cm + 2.5 cm + 0.5 cm + 4 cm

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$$= 15 cm$$

$$= 52 cm$$

4. Perimeter

$$= 28 cm$$

5. Perimeter

$$= AB + BC + CD + DA$$

$$= 5 \text{ cm} + 5 \text{ cm} + 5 \text{ cm} + 5 \text{ cm} = 20$$

6. Full-filled squares = 2

Area covered by full squares = 2×1 sq unit = 2 sq units

Area covered by half squares = 4×12

7. Let the breadth of rectangle = b

Perimeter of rectangle =
$$2 \times (3b + b)$$

$$48 = 2 \times (3b + b)$$

$$\frac{48}{2} = 4b$$

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$$24 = 4b$$

$$\frac{24}{4} = b$$

$$\Rightarrow$$
 6 = b \Rightarrow breadth = 6cm

length =
$$3b = 3 \times b = 18cm$$

8. Perimeter of the square = 64 cm

... Length of its side =
$$\frac{\text{Perimeter}}{\text{Number of sides}}$$

= $\frac{64}{4}$ = 16 cm.

- 9. Length of the rectangular table-top = 36 cm and its breadth = 24 cm.
- **10.** ∴ Perimeter of the table-top = 2 [length + breadth]

$$= 2 \times 60 \text{ cm} = 120 \text{ cm}.$$

Fig. (i) Perimeter of the square = $4 \times \text{side}$

$$= 4 \times 4$$
 cm $= 16$ cm

Fig. (ii) Perimeter of the rectangle

$$= 2 \times 11 \text{ cm} = 22 \text{ cm}$$

: Rectangle has greater perimeter than the square.

Short Answer:

1. Distance travelled in going around Fig. (i)

$$= 12 \text{ cm} + 3 \text{ cm} + 12 \text{ cm} + 3 \text{ cm} = 30 \text{ cm}$$

Distance travelled in going around Fig. (ii)

$$= 6 \text{ cm} + 4 \text{ cm} + 4 \text{ cm} + 4 \text{ cm} = 18 \text{ cm}$$

2. Side of the square = 15 cm

$$\therefore$$
 Perimeter of the square = 15 cm \times 4 = 60 cm

3. Length of the park = 300 m

Breadth =
$$200 \text{ m}$$

∴ Perimeter of the park = 2 [length + breadth]

$$= 2 [300 m + 200 m]$$

$$= 2 \times 500 \text{ m} = 1000 \text{ m}.$$

Cost of fencing the rectangular park = 1000 × 4 = ₹ 4000

4. Side of the square field = 150 m

∴ Area of the square field = Side × Side

$$= 150 \text{ m} \times 150 \text{ m}$$

$$= 22500 \text{ sq m}.$$

5. Length of the rectangular paper = 22 cm

∴ Area of the rectangular paper = length × breadth

$$= 22 \text{ cm} \times 10 \text{ cm}$$

$$= 220 \text{ sq cm}$$

6. Perimeter of the rectangle = 2 [length + breadth]

length + breadth =
$$880 \div 2 = 440$$

: Length = 440 m
$$- 88$$
 m = 352 m

Hence, the required length = 352 m.

7. Length of the rectangular plot = 120 m

Breadth =
$$90 \text{ m}$$

∴ Perimeter of the rectangular plot

$$= 2 [120 m + 90 m]$$

$$= 2 \times 210 \text{ m} = 420 \text{ m}$$

Now distance between two trees = 6 m

 \therefore Number of trees around the rectangular plot = 420 m \div 6 m = 70

Long Answer:

1. Length of the rectangular park = 30 m

∴ Perimeter of the rectangular park = 2(length + breadth)

∴ Cost of fencing all around the park = ₹15 × 100 = ₹1500

2. (A) Counting the squares, we have 8 squares

(B) Counting the squares, we have 4 squares

- (C) Counting the squares, we have 5 squares
- ∴ Area = 5 sq units
- (D) Counting the squares, we have 7 squares
- \therefore Area = 7 sq units
- **3.** Perimeter of the square = 100 cm

Perimeter 100

Side of the square =
$$\frac{\text{Perimeter}}{4} = \frac{100}{4}$$

25 cm.

 \therefore Breadth of the rectangle = 25 cm - 2 cm = 23 cm

Now perimeter of the rectangle = 100 cm

∴ 2 [length + breadth] = 100

length + breadth = $100 \div 2 = 50$ cm

But breadth = 23 cm

 \therefore Length = 50 cm - 23 cm = 27 cm

Now, Area of the rectangle

= length \times breadth = 27 cm \times 23 cm

= 621 sq cm.

4. Cost of fencing the compound = ₹5452

and the rate of fencing = ₹94 per metre

 \therefore Perimeter of the compound = 5452 \div 94 = 58 metres

Now breadth of the compound = 10 m.

2 [length + breadth] = 58 m

- \therefore length + breadth = 58 + 2 m = 29 m
- ∴ Length of the compound = 29 m 10 m = 19 m.

Assertion and Reason Answers:

- 1) d) A is false but R is true
- 2) d) A is false but R is true