# MATHEMATICS 

Chapter 5: Data Handling


## Data Handling

1. A collection of information is called the data. Data obtained in the original form is called a raw data.
2. To draw meaningful inferences, we need to organize the data systematically.
3. The systematic arrangement of data, either in ascending or in descending order, is called an arrayed data.
4. The number of times a particular observation occurs is called its frequency.
5. The difference between the highest and lowest values of the observations in a given data is called its range.
6. When the number of observations is large, the data is usually organized into groups, called class intervals.
7. Presentation of data in the form of groups (classes) along with the frequency of each class is called grouped data or frequency distribution.
8. Frequency distributions are of two types: (i) Discrete frequency distribution (ii) Continuous or grouped frequency distribution.
9. The lower value of a class interval is called its lower limit and the upper value of the class interval is called its upper limit.
10. The difference between the upper limit and lower limit of a class interval is called the class size. The mid-value of a class interval is called its class mark.
11. In a bar graph, bars of uniform width are drawn with various heights. The height of a bar represents the frequency of the corresponding observation.
12. A histogram is a pictorial representation of the grouped data in which class intervals are generally taken along the horizontal axis and class frequencies along the vertical axis. For each class a rectangle is constructed with base as the class interval and height determined from the class frequency such that there is no gap between any two successive rectangles.
13. Pie chart represents data in relative quantities by using the area of sectors in the circle.

Central angle for a component $=\left(\frac{\text { Value of the component }}{\text { Sum of the values of all components }} * 360\right)^{0}$
14. The words probable, chance etc. are used for an event which has some chance of uncertainty. In probability, we give a numerical value to certainty and uncertainty associated with any event.
i. Experiment: An operation which can produce some well-defined outcomes it called an experiment.
ii. Random experiment: An experiment in which all possible outcomes are known and the exact outcome cannot be predicted in advance, is called a random experiment.
iii. By a trial, we mean performing a random experiment.
iv. Event: The collection of all or some of the possible outcomes is called an event.
15. Let $E$ be an event, then probability of occurrence of $E$ is defined as

$$
\text { PE }=\frac{\text { number of out comes favourable to } E}{\text { total number of possible outcomes }}
$$

16. The probability of an event always lies between 0 and 1 .

## Pie Charts

A pie chart shows the relationship between a whole circle and its parts. The circle is divided into sectors. The size of each sector is proportional to the information it represents. Pie charts are also known as circle graphs.


Composition of milk

The above pie chart shows the composition of milk.

## Creating Different Pie Charts

For creating a pie chart the following steps needs to be followed:

- For each list of item or activity calculate the fraction or part which it represents.
- Convert each fraction into degrees by multiplying it with 360 .
- Draw a circle and divide it into sectors. The central angle of each sector is equal to the fraction of $360^{\circ}$ as calculated above.


| Item | Amount | Fraction | Angle |
| :--- | :--- | :--- | :--- |
| Movie | Rs. 30 | $\frac{30}{120}=\frac{1}{4}$ | $\frac{30}{120} \times 360=90^{\circ}$ |
| Book | Rs. 50 | $\frac{50}{120}=\frac{5}{12}$ | $\frac{50}{120} \times 360=150^{\circ}$ |
| Food | Rs. 40 | $\frac{40}{120}=\frac{1}{3}$ | $\frac{40}{120} \times 360=120^{\circ}$ |

Each sector in the pie chart is proportional to the amount spent for that particular activity or item.

## Introduction to Data Handling

## Raw Data

Raw data is also known as primary data which is available in an unorganized form.

## Organisation of Raw Data

Raw data is unorganised. To draw meaningful inferences we organise data. There are various ways in which we can organise data. For example, we can organise raw data using Frequency distribution table, Bar graphs etc.

## Pictographs and Bar Graphs

## Pictographs

A pictograph is the pictorial representation of data using symbols.
For example, If 10 Apples were sold in January, 40 were sold in February, 25 were sold in March, and 20 were sold in April. We can represent the given data as a pictograph as given below:

## Apples Sold



## Scale Factor

The scale factor is the ratio of the length of a side of one figure to the length of the corresponding side of the other figure. The scale factor is used in making maps. The scale of a map is the ratio of a distance on the map to the corresponding distance on the ground.

## Bar Graphs

A bar graph is a representation of data using a rectangular bars that are having heights that are proportional to the values that are represented by them.

For example, the bar graph below shows the sale of cars of various brands in the month of April:

## Multiple Bar Graphs

Multiple bar graphs is a bar graph which is used for comparing more than one kind of information.

Example:


The above figure is a double bar graph. It shows the number of cup of coffees sold in cafes and canteens for the months January, February, March, April and May, June and July.

## Grouped Data

## Frequency

Frequency is the number of times that a particular observation/ event occurs.

## Grouped Frequency Distribution

In a grouped frequency distribution, a large amount of raw data is represented by making groups or class intervals and obtain a frequency distribution of the number of observations falling in each group.

The marks scored (out of 100) by the students of class 10th are given below:
93,98,87,65,75,77,67,88,67,97,72,73,75,90.
The above data can be represented as a frequency distribution table as:

| Marks Frequency |  |
| :---: | :---: |
| $60-70$ | 3 |
| $70-80$ | 2 |
| $80-90$ | 4 |
| $90-100$ |  |

Here, 60-70, 70-80, 80-90, 90-100 are the class intervals.
In the class interval 60-70, 60 is the lower limit and 70 is the upper limit.
Class width or size $=$ upper limit - lower limit

## Note:

The value equal to the upper limit is not included in that class interval. It is included in the next higher-class interval. For example, here in the case of 90 , it is not included in the class interval 80-90, it is included in the class interval 90-100.

## Tally Marks

Tally marks are used to represent and count data. In tally marks, one vertical line is made for each count for the first four numbers and the fifth number is represented by a diagonal
line across the previous four. The table below shows the tally marks for the numbers 1 to 10.

| 1 | I | 6 | HHI |
| :--- | :--- | :--- | :--- |
| 2 | II | 7 | HH II |
| 3 | III | 8 | HIIII |
| 4 | IIII | 9 | HIIII |
| 5 | HI | 10 | HHIH |

Tally marks representation for 1 to 10

## Histogram

A Histogram is a type of bar diagram, where:

- The horizontal axis represents the class intervals.
- The frequency of the class interval is represented by the height of the bars.
- Since there is no gap between the class intervals, there is no gap between the bars.
- Below is the Histogram representing the distribution of marks scored out of 20 by 24 students.


Marks distribution of a class in a test.

## Plot Histogram

You need to follow the below steps to construct a histogram.

- Begin by marking the class intervals on the $X$-axis and frequencies on the $Y$-axis.
- The scales for both the axes have to be the same.
- Class intervals need to be exclusive.
- Draw rectangles with bases as class intervals and corresponding frequencies as heights.
- A rectangle is built on each class interval since the class limits are marked on the horizontal axis, and the frequencies are indicated on the vertical axis.
- The height of each rectangle is proportional to the corresponding class frequency if the intervals are equal.
- The area of every individual rectangle is proportional to the corresponding class frequency if the intervals are unequal.

Although histograms seem similar to graphs, there is a slight difference between them. The histogram does not involve any gaps between the two successive bars.

## Use Histogram

The histogram graph is used under certain conditions. They are:

- The data should be numerical.
- A histogram is used to check the shape of the data distribution.
- Used to check whether the process changes from one period to another.
- Used to determine whether the output is different when it involves two or more processes.
- Used to analyse whether the given process meets the customer requirements.


## Equally Likely Outcomes

## Random Experiment

A random experiment is an experiment for which the outcome cannot be predicted with certainty. Example: Rolling a dice.

## Experiment and Outcomes

An Experiment is any procedure that can be infinitely repeated and has a well-defined set of possible outcomes, known as the sample space.

Each outcome of an experiment or a collection of outcomes make an event.
For example, rolling of a die is an experiment. Getting $1,2,3$, or getting even numbers when a die is rolled is an event.

## Equally Likely Outcomes

Equally likely outcomes are those which have the same chance of occurring.
Example: Tossing a coin - the probability of getting a head and probability of getting a tail is equal.

## Probability of an Event

Probability is the likelihood of occurrence of an event. An event is a set of outcomes of an experiment.

When the outcomes of an experiment are equally likely, the probability of an event is given by:
$P(E)=\frac{\text { number of outcomes that make an event }}{\text { total number of outcomes of the experiment }}$
Outcomes of an experiment are equally likely if each has the same chance of occurring.

## Experimental Probability

When we toss a coin we get only one outcome either a head or a tail. The probability of getting a head or a tail is 0.5 .

However, if a coin is tossed ten times its not necessary that we will get a head five times and a tail five times.

So, based on what we observe as the outcomes of our trials, we find the experimental or empirical probability.

Experimental or empirical probability:
$P(E)=\frac{\text { number of trials where the event occurred }}{\text { total number of trials }}$
where $E$ is any event.
There are two approaches to study probability:
Experimental Probability
Theoretical Probability

## Experimental Probability

Experimental probability, also known as Empirical probability, is based on actual experiments and adequate recordings of the happening of events. To determine the occurrence of any event, a series of actual experiments are conducted. Experiments which do not have a fixed result are known as random experiments. The outcome of such experiments is uncertain. Random experiments are repeated multiple times to determine
their likelihood. An experiment is repeated a fixed number of times and each repetition is known as a trial. Mathematically, the formula for the experimental probability is defined by;

Probability of an Event $P(E)=$ Number of times an event occurs / Total number of trials.

## Theoretical Probability

In probability, the theoretical probability is used to find the probability of an event. Theoretical probability does not require any experiments to conduct. Instead of that, we should know about the situation to find the probability of an event occurring. Mathematically, the theoretical probability is described as the number of favourable outcomes divided by the number of possible outcomes.

Probability of Event $P(E)=$ No. of. Favourable outcomes/ No. of. Possible outcomes.


## Important Questions

## Multiple Choice Questions:

Question 1.Numbers 1 to 10 are written on ten separates slips (one number on one slip), kept in a box and mixed well. One slip is chosen from the box without looking in to it. What is the probability of getting a number greater than 6 ?
(a) 1
(b) 0
(c) $\frac{1}{2}$
(d) $\frac{1}{10}$

Question 2. There are 2 Red, 3 Blue and 5 Black balls in a bag. A ball is drawn from the bag without looking in to the bag. What is the probability of getting a non-blue ball?
(a) $\frac{7}{10}$
(b) $\frac{3}{5}$
(c) $\frac{2}{5}$
(d) None of these

Question 3. A coin is tossed. Which of the following is the probability of getting a head or tail?
(a) 0
(b) 1
(c) $\frac{1}{2}$
(d) None of these

Question 4. There are 2 Red, 3 Blue and 5 Black balls in a bag. A ball is drawn from the bag without looking in to the bag. What is the probability of getting a non-red ball?
(a) $\frac{3}{5}$
(b) $\frac{4}{5}$
(c) $\frac{2}{5}$
(d) None of these

Question 5. The central total angle in a pie chart is
(a) $180^{\circ}$
(b) $210^{\circ}$
(c) $360^{\circ}$
(d) None of these

Question 6. There are 2 Red, 3 Blue and 5 Black balls in a bag. A ball is drawn from the bag without looking in to the bag. What is the probability of getting a blue ball?
(a) $\frac{3}{5}$
(b) $\frac{2}{5}$
(c) $\frac{3}{10}$
(d) None of these

Question 7. 18 out of 36 people love reading, so reading in the pie chart will be represented by
(a) 36 degree sector
(b) quarter sector
(c) semi circular sector
(d) None of these

Question 8. When a die is thrown, total number of possible outcomes is $\qquad$ .
(a) 6
(b) 36
(c) 2
(d) None of these

Question 9. The pie-chart is divided into
(a) circles
(b) squares
(c) sectors
(d) segments

Question 10. The class mark of 95-100 is
(a) 95.5
(b) 97.5
(c) 95
(d) 100

## Very Short Questions:

1. In the class interval 5-10, find the
(i) lower limit
(ii) upper limit
(iii) class mark
(iv) class size
2. A group of 20 students recorded their heights (in cm ). The data received were as given below. What is the range?
$150,120,112,160,155,151,158,142,148,149,161,165,140,157,156,146$, 148, 153, 138, 135
3. In the given pie chart, which colour is most popular? Which colour is the least popular?

4. A die is thrown once. Find the probability of getting a number greater than 4.
5. A class consists of 21 boys and 9 girls. A student is to be selected for social work. Find the probability that
(i) a girl is selected
(ii) a boy is selected

## Short Questions :

1. The following pie chart depicts the percentage of students, nationwide. What is the percentage of
(i) Indian students
(ii) African students?

2. Fill in the blanks:

| Weights in kg | Class-mark |
| :--- | :---: |
| $10-15$ | - |
| $15-20$ | - |
| $20-25$ | - |
| $25-30$ | - |
| $30-35$ | - |
| $35-40$ | - |

3. Construct a frequency table for the following marks obtained by 50 students using equal Intervals taking 16-24 (24 not included) as one of the classintervals.
$52,16,18,20,42,48,39,38,54,58,47,37,25,16,42,49,36,35,53,21,30$,
$43,56,34,33,17,22,24,37,41,40,50,54,56,54,36,38,42,44,56,17,18$, $22,24,17,48,58,23,29,58$.
4. The double bar graph shows the average monthly temperatures of two cities over 4 months period. Read the graph carefully and answer the questions given below:
(i)What does each 1 cm block on the vertical axis represent?

(ii) What was the average monthly temperature in Dehradun in
(a) March
(b) April
(c) May
(d) June?
(iii) What was the average monthly temperature in Delhi for the whole 4 months?
(iv) In which month was the difference between the temperature of Delhi and

Dehradun maximum and how much?
5. The following table represents the number of students in a school playing six different games.

| Games | Number of students |
| :--- | :---: |
| Hockey | 175 |
| Football | 200 |
| Cricket | 150 |
| Tennis | 50 |
| Squash | 75 |
| Badminton | 40 |

Present the above information on a bar graph.

## Long Questions :

1. Prepare a grouped frequency table for the given histogram.

2. A bag contains 144 coloured balls represented by the following table. Draw a pie chart to show this information.

| Colour | Number of balls |
| :--- | :---: |
| Red | 12 |
| Yellow | 18 |
| Blue | 28 |
| Green | 42 |
| White | 44 |

3. Mrs. Verma spends her allowance in the following way.

| Items | Percent |
| :---: | :---: |
| Lunch | $25 \%$ |
| Hobby | $20 \%$ |
| Recreations | $40 \%$ |
| Saving | $15 \%$ |
| Total | $100 \%$ |

Represent the above information by a pie chart.
4. What is the probability of getting a marble which is not red from a bag containing 3 black, 8 yellow, 2 red and 5 white marbles?
5. From a well shuffled deck of 52 playing cards, a card is selected at random. Find the probability of getting
(i) a black card
(ii) a black king
(iii) an ace
(iv) a card of diamond
6. The frequency table of the weights (in kilograms) of the students is given alongside:
Answer the questions with reference to the given table:

| Class intervals (Wt. in .kg) | Frequency (Number of Students) |
| :---: | :---: |
| $40 ? 45$ | 8 |
| $45 ? 50$ | 15 |
| $50 ? 55$ | 14 |
| $55 ? 60$ | 9 |
| $60 ? 65$ | 3 |

(i) How many classes are there in the given table?
(ii) What is the frequency of the class 50 to 55 ?
(iii) Which class has the maximum frequency?
(iv) Which is the class with frequency 9 ?
(v) What is the lower limit of the class 50 to 55 ?
(vi) What is the upper limit of the class 55 to 60 ?
(vii) In which class will a student weighing 57 kg be included?
(viii) How many student weights are given in the table?
(ix) State the number of students weighing less than 56 kg ?
(x) What is the maximum weight of the student which can be included in the given table?

## Answer Key-

## Multiple Choice Questions:

1. (d) $\frac{1}{10}$
2. (a) $\frac{7}{10}$
3. (b) 1
4. (b) $\frac{4}{5}$
5. (c) $360^{\circ}$
6. (c) $\frac{3}{10}$
7. (c) semi circular sector
8. (a) 6
9. (c) sectors
10. (b) 97.5

## Very Short Answer:

1. (i) lower limit $=5$
(ii) upper limit $=10$
(iii) Class mark $=\frac{5+10}{2}=\frac{15}{2}=7.5$
(iv) Class size $=10-5=5$
2. The minimum height $=112 \mathrm{~cm}$

Maximum height $=165 \mathrm{~cm}$
Range $=$ Maximum height - Minimum height $=165 \mathrm{~cm}-112 \mathrm{~cm}=47 \mathrm{~cm}$
3. Red colour is the most popular and the blue colour is the least popular.
4. Number greater than $4=5,6$
$n(E)=2$
Sample space $n(S)=6$
Probability of getting a number greater than 4
$=\frac{n(E)}{n(S)}=\frac{2}{6}=\frac{1}{3}$
Where re(E): Number of favourable outcomes
$n(S)$ : Total number of outcomes.
5. Sample space $n(S)=21+9=30$

Number of girls $n(E)=9$
(i) Probability of selecting a girl
$=\frac{n(E)}{n(S)}=\frac{9}{30}=\frac{3}{10}$
(ii) Probability of selecting a boy
$=\frac{n(E)}{n(S)}=\frac{21}{30}=\frac{7}{10}$

## Short Answer :

1. (i) Percentage of Indian students $=\frac{180 \times 100}{360}=50 \%$
(ii) Percentage of African students $=\frac{45 \times 100}{360}=12 \frac{1}{2} \%$
2. Class-marks are

Class-mark

$$
\begin{aligned}
& \frac{15+20}{2}=\frac{35}{2}=17.5 \\
& \frac{20+25}{2}=\frac{45}{2}=22.5 \\
& \frac{25+30}{2}=\frac{55}{2}=27.5 \\
& \frac{30+35}{2}=\frac{65}{2}=32.5 \\
& \frac{35+40}{2}=\frac{75}{2}=37.5
\end{aligned}
$$

$\therefore$ Value of the blank spaces are

| Weights in kg | Class-mark |
| :--- | :---: |
| $10-15$ | 12.5 |
| $15-20$ | 17.5 |
| $20-25$ | 22.5 |
| $25-30$ | 27.5 |
| $30-35$ | 32.5 |
| $35-40$ | 37.5 |

3. 

| Classinterval | Tally marks | Frequency |
| :---: | :---: | :---: |
| $16-24$ $24-32$ $32-40$ $40-48$ $48-56$ $56-64$ |  <br> H <br> * H <br> *1\|| <br> \|H III <br> HIII |  |
| Total |  | 50 |

4. (i) 1 cm block on vertical axis $=10^{\circ} \mathrm{C}$
(ii) The average monthly temperature in Dehradun in the month of
(a) March was $25^{\circ} \mathrm{C}$
(b) April was $34^{\circ} \mathrm{C}$
(c) May was $40^{\circ} \mathrm{C}$
(d) June was $36^{\circ} \mathrm{C}$
(iii) The average monthly temperature in Delhi in the 4 months

$$
\begin{aligned}
& \begin{array}{r}
\text { Temperature of } \\
(\text { March }+ \text { April }+ \text { May }+ \text { June })
\end{array} \\
= & \frac{30^{\circ} \mathrm{C}+40^{\circ} \mathrm{C}+45^{\circ} \mathrm{C}+50^{\circ} \mathrm{C}}{4} \\
= & \frac{30^{\circ}}{4}=41.25^{\circ} \mathrm{C} \\
= & \frac{165^{\circ} \mathrm{C}}{4}
\end{aligned}
$$

(iv) Difference between the average monthly temperature of Delhi and Dehradun was maximum in the month of June, i.e. $\left(50^{\circ}-36^{\circ}\right)=14^{\circ} \mathrm{C}$.
5.


## Long Answer:

1. 

| Salary (in thousand ₹) | Number of Employee |
| :---: | :---: |
| $15-20$ | 35 |
| $20-25$ | 30 |
| $25-30$ | 45 |
| $30-35$ | 40 |
| $35-40$ | 10 |
| $40-45$ | 15 |

2. 

| Colour | Number of balls | Central Angle |
| :--- | :---: | :---: |
| Red | 12 | $\frac{12}{144} \times 360^{\circ}=30^{\circ}$ |
| Yellow | 18 | $\frac{18}{144} \times 360^{\circ}=45^{\circ}$ |
| Blue | 28 | $\frac{28}{144} \times 360^{\circ}=70^{\circ}$ |
| Green | 42 | $\frac{42}{144} \times 360^{\circ}=105^{\circ}$ |
| White | 44 | $\frac{44}{144} \times 360^{\circ}=110^{\circ}$ |



Pie chart
3.

| Items | Percent | Central angle |
| :--- | :---: | :---: |
| Lunch | $25 \%$ | $\frac{25}{100} \times 360^{\circ}=90^{\circ}$ |
| Hobby | $20 \%$ | $\frac{20}{100} \times 360^{\circ}=72^{\circ}$ |
| Recreations | $40 \%$ | $\frac{40}{100} \times 360^{\circ}=144^{\circ}$ |
| Saving | $15 \%$ | $\frac{15}{100} \times 360^{\circ}=54^{\circ}$ |


4. Total number of balls $=3$ black +8 yellow +2 red +5 white $=18$
$n(S)=18$
Number of the balls which are not red $=3+8+5=16$
$n(E)=16$
Probability $=\frac{n(E)}{n(S)}=\frac{16}{18}=\frac{8}{9}$
5. Here, $n(S)=52$
(i) Total number of black card $=26$
$n(E)=26$
Probability of getting a black card $=\frac{n(E)}{n(S)}=\frac{26}{52}=\frac{1}{2}$
(ii) Number of black king = 2
$n(E)=2$
Probability of getting a black king $=\frac{n(E)}{n(S)}=\frac{2}{52}=\frac{1}{26}$
(iii) Number of aces $=4$
$n(E)=4$
Probability of getting an ace $=\frac{n(E)}{n(S)}=\frac{4}{52}=\frac{1}{13}$
(iv) Number of diamond cards $=13$
$n(E)=13$
Probability of getting a card of diamond $=\frac{n(E)}{n(S)}=\frac{13}{52}=\frac{1}{4}$
6. (i) 5
(ii) 14
(iii) 45 to 50
(iv) 55 to 60
(v) 50
(vi) 60
(vii) 55 to 60
(viii) Total number of students $=8+15+14+9+3=49$ students
(ix) Number of students weighing less than $56 \mathrm{~kg}=8+15+14=37$ students
(x) 65 kg

