# MATHEMATICS 

Chapter 13: Direct and Inverse Proportions


## Direct and Inverse Proportions

1. If two quantities are related such that a change in one causes a corresponding change in the other, then we say that one varies with the other.

There are two types of variations/ proportions:
i. Direct variation/ proportion
ii. Indirect variation/ proportion

## Direct proportion:

If the value of a variable $x$ always increases or decreases with the respective increase or decrease in value of variable $y$, then it is said that the variables $x$ and $y$ are in direct proportion.

For example: In the table below, we have variable $y$ - Cost (in Rs) always increasing when there is an increase in variable $x$ - Weight of sugar (in kg ). Likewise if the weight of sugar reduced, the cost would also reduce. Hence the two variables are in direct proportion.


Two quantities $x$ and $y$ are said to be in direct proportion if whenever the value of $x$ increases (or decreases), then the value of $y$ increases (or decreases) in such a way that the ratio $\frac{x}{y}$ remains constant.
Example: Cost is directly proportional to the number of articles, Work done is directly proportional to the number of men working on it.
When $x$ and $y$ are in direct proportion, we have:
$\frac{\mathrm{x}_{1}}{\mathrm{y}_{1}}=\frac{\mathrm{x}_{2}}{\mathrm{y}_{2}}=\frac{\mathrm{x}_{3}}{\mathrm{y}_{3}}$
Here, $y_{1}, y_{2}, y_{3}, \ldots$ are the values of $y$ corresponding to the values $x_{1}, x_{2}, x_{3}, \ldots$ of $x$.

## Direct proportion Examples in Real Life

In our day-to-day life, we observe that the variations in the values of various quantities
depending upon the variation in values of some other quantities.


For example: if the number of individuals visiting a restaurant increases, earning of the restaurant also increases and vice versa. If more number of people are employed for the same job, the time taken to accomplish the job decreases.
Sometimes, we observe that the variation in the value of one quantity is similar to the variation in the value of another quantity that is when the value of one quantity increases then the value of other quantity also increases in the same proportion and vice versa. In such situations, two quantities are termed to exist in direct proportion.


## Some more examples are:

Speed is directly proportional to distance.
The cost of the fruits or vegetable increases as the weight for the same increases.

## Direct Proportion Symbol and Constant of Proportionality

The symbol for "direct proportional" is ' $\alpha$ ' (One should not confuse with the symbol for infinity $\infty$ ). Two quantities existing in direct proportion can be expressed as;
$x \propto y$
$x / y=k$
$x=k y$
$k$ is a non-zero constant of proportionality.

Where $x$ and $y$ are the value of two quantities and $k$ are a constant known as the constant of proportionality. If $x_{1}, y_{1}$ is the initial values and $x_{2}, y_{2}$ are the final values of quantities existing in direct proportion. They can be expressed as,
$\frac{x_{1}}{y_{1}}=\frac{x_{2}}{y_{2}}$

## Direct Proportion Formula

The direct proportion formula says if the quantity $y$ is in direct proportion to quantity $x$, then we can say $y=k x$, for a constant $k . y=k x$ is also the general form of the direct proportion equation.

$$
\begin{aligned}
& \text { Direct Proportion Formula } \\
& \text { If } \mathbf{y} \propto \mathbf{x} \text {, } \\
& \Rightarrow y=k x \text {, for a constant } k
\end{aligned}
$$

## Example: a machine manufactures 20units per hour

The units that machine manufactures is directly proportional to how many hours it has worked.

More works the machine does, more are the units manufactured; in direct proportion.
This could be written as:
Units $\propto$ Hours Worked
If it works 2 hours we get 40 Units
If it works 4 hours we get 80 Units

## Relation for Direct Proportion

Considering two variables x and y ,
$\frac{x}{y}=\mathrm{k}$ or $\mathrm{x}=\mathrm{ky}$ establishes the simple relation for direct proportion between x and y , where k is a constant.

So if x and y are in direct proportion, it can be said that $\frac{x_{1}}{y_{1}}=\frac{x_{2}}{y_{2}}$ where y 1 and y 2 correspond to respective values of $\mathrm{x}_{1}$ and $\mathrm{x}_{2}$.

## Direct Proportion Graph

The graph of direct proportion is a straight line with an upward slope. Look at the image given below. There are two points marked on the $x$-axis and two on the $y$-axis, $x_{1}<x_{2}$ and $y_{1}<y_{2}$ where and. If we increase the value of $x$ from $x_{1}$ to $x_{2}$, we observe that the value of $y$ is also increased from. $Y_{1}$ to $y_{2}$ Thus, the line $y=k x$ represents direct proportionality graphically.


## Inverse Proportion:

If the value of variable $x$ decreases or increases upon corresponding increase or decrease in the value of variable $y$, then we can say that variables $x$ and $y$ are in inverse proportion.

For example: In the table below, we have variable y : Time taken (in minutes) reducing proportionally to the increase in value of variable x : Speed (in km/hour). Hence the two variables are in inverse proportion.


Two quantities $x$ and $y$ are said to be indirect proportion if whenever the value of $x$ increases (or decreases), then the value of $y$ decreases (or increases) in such a way that xy remains constant.
Example: The time taken to finish a work is inversely proportional to the number of persons working at it, the time taken by any vehicle in covering a certain distance is inversely proportional to the speed of the car.
When $x$ and $y$ are in inverse proportion, then
$x_{1} \times y_{1}=x_{2} \times y_{2}=x_{3} \times y_{3}$, and so on.
Here, $y_{1}, y_{2}, y_{3}, \ldots$ are the values of $y$ corresponding to the values $x_{1}, x_{2}, x_{3}, \ldots$ of $x$.
2. Map is a miniature representation of a large regions.

Hence, we can say that scale of a map is based on the concept of direct variation.
The scale shows a relationship between actual length and the length represented on the map. Thus, it is the ratio of the distance between two points on the map to the actual distance between two points on the large region.
3. Suppose A can finish a piece of work in n days. Then, work done by A in 1 day $=\frac{1}{\mathrm{n}}$.
4. When a person A completes $\left(\frac{1}{\mathrm{n}}\right)^{\text {th }}$ part of the work in one day, then A will take n days to complete the work.
5. In a cistern there are two pipes. The inlet is the pipe that fills the cistern and the outlet is the pipethat empties the cistern.
6. When an inlet fills the cistern in ' $n$ ' hours, then it will fill up $\frac{1}{n}$ th part of the cistern in onehour.
7. When an outlet empties the cistern in ' $n$ ' hours, then it will empty out $\frac{1}{n}$ th part of the cistern inone hour.
8. Multiplicative inverse of a number is called the reciprocal of a number.

## Time and Work

It is important to establish the relationship between time taken and the work done in any given problem or situation. If time increases with increase in work, then the relation is directly proportional. In such a case we will use $\frac{x_{1}}{y_{1}}=\frac{x_{2}}{y_{2}}$ to arrive at our solution.
However, if they are inversely proportional, we will use the relation $\frac{x_{1}}{x_{2}}=\frac{y_{2}}{y_{1}}$ to arrive at our answer.

For example: In the table below, we have the number of students $(x)$ that took a certain number of days ( $y$ ) to complete a fixed amount of food supplies. Now we have to calculate the number of days it would take for an increased number of students to finish the identical amount of food.

| Number of students | 100 | 125 |
| :--- | :--- | :--- |
| Number of days | 20 | y |

We know that with greater number of people, the time taken to complete the food will be lesser, therefore we have an inverse proportionality relation between x and y here.
Hence by applying the formula, we have:
$\frac{100}{125}=\frac{y}{20} \Rightarrow y=\frac{20.100}{125}=16$ days

## Constant of Proportionality

When two variables are directly or indirectly proportional to each other, then their relationship can be described as $y=k x$ or $y=k / x$, where $k$ determines how the two variables are related to one another. This $k$ is known as the constant of proportionality.

Constant of proportionality is the constant value of the ratio between two proportional quantities. Two varying quantities are said to be in a relation of proportionality when, either their ratio or their product yields a constant. The value of the constant of proportionality depends on the type of proportion between the two given quantities: Direct Variation and Inverse Variation.

Direct Variation: The equation for direct proportionality is $y=k x$, which shows as $x$ increases, $y$ also increases at the same rate. Example: The cost per item(y) is directly proportional to the number of items $(x)$ purchased, expressed as $y \propto x$

Inverse Variation: The equation for the indirect proportionality is $y=k / x$, which shows that as $y$ increases, $x$ decreases and vice-versa. Example: The speed of a moving vehicle ( $y$ ) inversely varies as the time taken ( $x$ ) to cover a certain distance, expressed as $y \propto 1 / x$

In both the cases, $k$ is constant. The value of this constant is called the coefficient of proportionality. The constant of proportionality is also known as unit rate.


## Use of Constant Proportionality

We use constant of proportionality in mathematics to calculate the rate of change and at the same time determine if it is direct variation or inverse variation that we are dealing with. Let us assume that the cost of 2 apples $=\$ 20$. We determine that the cost of 1 apple $=$ $\$ 10$. We have found the Constant of Proportionality for the cost of an apple is 2.

If we want to draw a picture of the Taj Mahal by sitting in front of it on a piece of paper by looking at the real image in front of us, we should maintain a proportional relationship between the measures of length, height, and width of the building. We need to identify the constant of proportionality to get the desired outcome. Based on this, we can draw the
monument with proportional measurements. For instance, if the height of the dome is 2 meters then in our drawing we can represent the same dome with height 2 inches. Similarly, we can draw other parts. In such scenarios, we use constant of proportionality.

Working with proportional relationships allows one to solve many real-life problems such as:

Adjusting a recipe's ratio of ingredients
Quantifying chance like finding odds and probability of events
Scaling a diagram for drafting and architectural uses
Finding percent increase or percent decrease for price mark-ups
Discounts on products based on unit rate

## Identifying The Constant of Proportionality

We shall now learn how to identify the constant of proportionality (unit rate) in tables or graphs. Examine the table below and determine if the relationship is proportional and find the constant of proportionality.


We infer that as the number of days increases, the ariticles written also increases. Here we identify that it is in direct proportion. We apply the equation $y=k x$. To find the constant of proportionality we determine the ratio between the number of articles and the number of days. We need to evaluate for $k=y / x$
$y / x=3 / 1=9 / 3=15 / 5=18 / 6=3$
From the result of the ratios of $y$ and $x$ for the given values, we can observe that the same value is obtained for all the instances. The Constant of Proportionality is 3.

If we plot the values from the above table onto a graph, we observe that the straight line that passes through the origin shows a proportional relationship. The constant of proportionality under the direct proportion condition is the slope of the line when plotted for two proportional constants $x$ and $y$ on a graph.

# Constant of Proportionality Graph 


where $k=$ constant of proportionality

## Direct Proportion Vs Inverse Proportion

There are two types of proportionality that can be established based on the relation between the two given quantities. Those are direct proportion and inverse proportional. Two quantities are directly proportional to each other when an increase or decrease in one leads to an increase or decrease in the other. While on the other hand, two quantities are said to be in inverse proportion if an increase in one quantity leads to a decrease in the other, and vice-versa. The graph of direct proportion is a straight line while the inverse proportion graph is a curve. Look at the image given below to understand the difference between direct proportion and inverse proportion.

## Direct Proportion Vs Inverse Proportion

Direct Proportion
$\mathbf{y} \propto \mathbf{C} \mathbf{x}$
$\mathbf{y}=\mathbf{k x}$ for a constant $\mathbf{k}$
Inverse Proportion
$y \propto \frac{1}{x}$
$y=\frac{k}{x}$ for a constant $k$


## Important Questions

## Multiple Choice Questions:

Question 1. 10 meters of cloth cost Rs 1000 . What will 4 meters cost?
(a) Rs 400
(b) Rs 800
(c) Rs 200
(d) Rs 100.

Question 2. 15 books weigh 6 kg . What will 6 books weigh ?
(a) 1.2 kg
(b) 2.4 kg
(c) 3.8 kg
(d) 3 kg .

Question 3. A horse eats 18 kg of com in 12 days ? How much does he eat in 9 days ?
(a) 11.5 kg
(b) 12.5 kg
(c) 13.5 kg
(d) 14.5 kg .

Question 4.8 g of sandal wood cost Rs 40 . What will 10 g cost ?
(a) Rs 30
(b) Rs 36
(c) Rs 48
(d) Rs 50.

Question 5. 20 trucks can hold 150 metric tonnes. How much will 12 trucks hold ?
(a) 80 metric tonnes
(b) 90 metric tonnes
(c) 60 metric tonnes
(d) 40 metric tonnes.

Question 6. 120 copies of a book cost Rs 600. What will 400 copies cost ?
(a) Rs 1000
(b) Rs 2000
(c) Rs 3000
(d) Rs 2400.

Question 7. The rent of 7 hectares is Rs 875 . What is the rent of 16 hectares ?
(a) Rs 2000
(b) Rs 1500
(c) Rs 1600
(d) Rs 1200.

Question 8. A boy runs 1 km in 10 minutes. How long will he take to ran 600 m ?
(a) 2 minutes
(b) 3 minutes
(c) 4 minutes
(d) 6 minutes.

Question 9. A shot travels 90 m in 1 second. How long will it take to go 225 m ?
(a) 2 seconds
(b) 2.5 seconds
(c) 4 seconds
(d) 3.5 seconds.

Question 10. 3 knives cost Rs 63. What will 17 knives cost?
(a) Rs 357
(b) Rs 375
(c) Rs 537
(d) Rs 573.

## Very Short Questions:

1. A train is moving at a uniform speed of $100 \mathrm{~km} / \mathrm{h}$. How far will it travel in 20 minutes?
2. Complete the table if $x$ and $y$ vary directly.

| $\boldsymbol{x}$ | 3.5 | 4 | 7.5 | - |
| :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | - | 8 | - | 15 |

3. If the cost of 20 books is ₹ 180 , how much will 15 books cost?
4. If $x_{1}=5, y_{1}=7.5, x_{2}=7.5$ then find $y_{2}$ if $x$ and $y$ vary directly.
5. If 3 kg of sugar contains $9 \times 10^{8}$ crystals. How many sugar crystals are there in 4 kg of sugar?
6. If 15 men can do a work in 12 days, how many men will do the same work in 6 days?

Short Questions:

1. A train travels 112 km in 1 hour 30 minutes with a certain speed. How many kilometres it will travel in 4 hours 45 minutes with the same speed?
2. The scale of a map is given as $1: 50,000$. Two villages are 5 cm apart on the map. Find the actual distance between them.
3. 8 pipes are required to fill a tank in 1 hr 20 min . How long will it take if only 6 pipes of the same type are used?
4. 15 men can build a wall in 42 hours, how many workers will be required for the same work in 30 hours?
5. The volume of a gas $V$ varies inversely as the pressure $P$ for a given mass of the gas. Fill in the blank spaces in the following table:

|  |  |  |  |  |  |  | (A) | (B) | (C) |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (D) | (D) | (F) | (G) |  |  |  |  |  |  |
| Volume (in cm ${ }^{3}$ ) | - | 48 | 60 | - | 100 | - | 200 |  |  |
| Pressure (in atmosphere) | 2 | - | $\frac{3}{2}$ | 1 | - | $\frac{1}{2}$ | - |  |  |

6. The cost of 5 metres of cloth is ₹ 210 . Tabulate the cost of $2,4,10$ and 13 metres of cloth of the same type.
7. Six pumps working together empty a tank in 28 minutes. How long will it take to empty the tank if 4 such pumps are working together?

## Long Questions :

1. Mohit deposited a sum of $₹ 12000$ in a Bank at a certain rate of interest for 2 years and earns an interest of ₹ 900 . How much interest would be earned for a deposit of ₹ 15000 for the same period and at the same rate of interest?
2. A garrison of 120 men has provisions for 30 days. At the end of 5 days, 5 more men joined them. How many days can they sustain on the remaining provision?
3. In a scout camp, there is food provision for 300 cadets for 42 days. If 50 more persons join the camp, for how many days will the provision last?
4. If two cardboard boxes occupy 500 cubic centimetres space, then how much space is required to keep 200 such boxes?
5. Under the condition that the temperature remains constant, the volume of gas is inversely proportional to its pressure. If the volume of gas is 630 cubic centimetres at a pressure of 360 mm of mercury, then what will be the pressure of the gas if its volume is 720 cubic centimetres at the same temperature?

## Answer Key-

## Multiple Choice Questions:

1. (a) $a^{m+n}$
2. (a) Rs 400
3. (b) 2.4 kg
4. (c) 13.5 kg
5. (d) Rs 50 .
6. (b) 90 metric tonnes
7. (b) Rs 2000
8. (a) Rs 2000
9. (d) 6 minutes.
10. (b) 2.5 seconds
11. (a) Rs 357

## Very Short Answer:

1. Let the distance travelled by train in 20 minutes be $x \mathrm{~km}$.

| Distance travelled (in km) | 100 | $x$ |
| :--- | :---: | :---: |
| Time taken (in minutes) | 60 | 20 |

Since the speed is uniform, the distance travelled will be directly proportional to time.

$$
\begin{array}{rlrl}
\therefore & \frac{100}{60} & =\frac{x}{20} \\
\Rightarrow & & 60 \times x & =20 \times 100 \\
& \therefore & x & =\frac{20 \times 100}{60}=\frac{100}{3} \mathrm{~km} \\
& & =33 \frac{1}{3} \mathrm{~km}
\end{array}
$$

Hence, the required distance is $33 \frac{1}{3} \mathrm{~km}$.
2. Let the blank spaces be filled with $a, b$ and $c$.

| $\boldsymbol{x}$ | 3.5 | 4 | 7.5 | $c$ |
| :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | $a$ | 8 | $b$ | 15 |

Since $x$ and $y$ vary directly.

$$
\begin{aligned}
& \therefore \quad \frac{x_{1}}{y_{1}}=\frac{x_{2}}{y_{2}} \\
& \Rightarrow \quad \frac{3.5}{a}=\frac{4}{8} \\
& \Rightarrow \quad 4 \times a=3.5 \times 8 \\
& \therefore \quad a=\frac{3.5 \times 8}{4}=7 \\
& \frac{4}{8}=\frac{7.5}{b} \\
& \Rightarrow \quad 4 \times b=8 \times 7.5
\end{aligned}
$$

Hence, the required values are $a=7, b=15$ and $c=7.5$.
3. Let the required cost be $₹ x$.

Here, the two quantities vary directly.

| Number of books | 20 | 15 |
| :--- | :---: | :---: |
| Cost (in ₹) | 180 | $x$ |

$$
\begin{aligned}
& & \frac{20}{180} & =\frac{15}{x} \\
\Rightarrow & & 20 \times x & =15 \times 180 \\
& \therefore & x & =\frac{15 \times 180}{20}=₹ 135
\end{aligned}
$$

Hence, the required cost $=₹ 135$.
4. Since $x$ and $y$ vary directly.

$$
\begin{array}{ll}
\therefore & \frac{x_{1}}{y_{1}}=\frac{x_{2}}{y_{2}} \\
\Rightarrow & \frac{5}{7.5}=\frac{7.5}{y_{2}} \\
\Rightarrow & 5 \times y_{2}=7.5 \times 7.5 \\
\therefore & y_{2}=\frac{7.5 \times 7.5}{5}=11.25
\end{array}
$$

Hence, the required value is 11.25 .
5. Let the required number of crystals be $x$.

| Sugar (in kg) | 3 | 4 |
| :--- | :---: | :---: |
| Number of crystals | $9 \times 10^{8}$ | $x$ |

Since the two quantities are directly proportional to each other.

$$
\begin{aligned}
& \therefore \quad \frac{x_{1}}{y_{1}}=\frac{x_{2}}{y_{2}} \\
& \Rightarrow \quad \frac{3}{9 \times 10^{8}}=\frac{4}{x} \\
& \Rightarrow \quad 3 \times x=4 \times 9 \times 10^{8} \\
& \therefore \quad x=\frac{4 \times 9 \times 10^{8}}{3} \\
& =12 \times 10^{8}=1.2 \times 10^{9}
\end{aligned}
$$

Hence, the required number of crystals $=1.2 \times 109$.
6. Let the required number of men be $x$.

Less days $\rightarrow$ more men.
Thus the two quantities are inversely proportional to each other.

| Men | 15 | $x$ |
| :--- | :---: | :---: |
| days | 12 | 6 |

$$
\left.\begin{array}{rlrl} 
& x_{1} y_{1} & =x_{2} y_{2} \\
\Rightarrow & 15 \times 12 & =x \times 6 \\
& \therefore & & x
\end{array}\right)=\frac{15 \times 12}{6}=30
$$

Hence, the required number of days $=30$.

## Short Answer:

1. Let the required distance be xkm .

More distance $\rightarrow$ more time
Thus, the two quantities are directly proportional.

| Distance (in km) | 112 | $x$ |
| :--- | :--- | :---: |
| Time (in hours) | $\frac{3}{2} \mathrm{~h}$ | $\frac{19}{4} \mathrm{~h}$ |

$$
\left[\begin{array}{rl}
\because 1 \mathrm{hr} 30 \mathrm{~min} & =\frac{3}{2} \mathrm{~h} \\
4 \mathrm{hrs} 45 \mathrm{~min} & =\frac{19}{4} \mathrm{~h}
\end{array}\right]
$$

$$
\begin{array}{rlrl}
\frac{x_{1}}{y_{1}} & =\frac{x_{2}}{y_{2}} \\
\Rightarrow \quad & \frac{112}{x} & =\frac{\frac{3}{2}}{\frac{19}{4}} \\
\Rightarrow \quad & \frac{3}{2} x & =112 \times \frac{19}{4} \\
\therefore \quad & x & =112 \times \frac{19}{4_{q 2}} \times \frac{22}{3}=\frac{1064}{3} \\
& =354.6 \mathrm{~km} .
\end{array}
$$

Hence, the required distance $=354.6 \mathrm{~km}$.
2. Let the map distance be $x \mathrm{~cm}$ and actual distance be y .
$1: 50,000=x: y$

$$
\Rightarrow \quad \frac{1}{50,000}=\frac{x}{y}
$$

Since

$$
x=5, \text { So } \frac{1}{50,000}=\frac{5}{y}
$$

$$
\therefore \quad y=5 \times 50,000
$$

$$
=2,50,000 \mathrm{~cm}
$$

$$
=250 \mathrm{~km}
$$

Hence, the required distance $=250 \mathrm{~km}$.
3. Let the required time be ' $t$ ' hours.

| Number of pipe | 8 | 6 |
| :--- | :---: | :---: |
| Time (in hours) | $\frac{4}{3}$ | $t$ |

$$
\left[\because 1 \mathrm{hr} 20 \mathrm{~min}=\frac{4}{3} \mathrm{~h}\right]
$$

Less number of pipes $\rightarrow$ more time

$$
\begin{aligned}
\therefore & & x_{1} y_{1} & =x_{2} y_{2} \\
\Rightarrow & & 8 \times \frac{4}{3} & =6 \times t \\
\Rightarrow & & t & =\frac{8 \times 4}{6 \times 3}=\frac{16}{9} \mathrm{~h} \\
& & & 1 \frac{7}{9} \mathrm{~h} .
\end{aligned}
$$

Hence, the required time $=1 \frac{7}{9} \mathrm{~h}$.
4. Let the required number of workers be $x$.

The number of workers, faster will they do the work.
So, the two quantities are inversely proportional.

| Number of hours | 42 | 30 |
| :--- | :---: | :---: |
| Number of men | 15 | $x$ |

$\mathrm{x}_{1} \mathrm{y}_{1}=\mathrm{x}_{2} \mathrm{y}_{2}$
$\Rightarrow 42 \times 15=30 \times x$
$\Rightarrow \mathrm{x}=21$
Hence, the required number of men $=21$
5. Since volume and pressure are inversely proportional.
$P V=K$
From (C) $60 \times \frac{3}{2}=\mathrm{K} \therefore \mathrm{K}=90$
From (A)

$$
\mathrm{P}=2, \mathrm{~K}=90
$$

$$
\therefore \quad \mathrm{V}=\frac{\mathrm{K}}{\mathrm{P}}=\frac{90}{2}=45 \mathrm{~cm}^{3}
$$

From (B)

$$
\mathrm{V}=48, \mathrm{~K}=90
$$

$\therefore \quad \mathrm{P}=\frac{\mathrm{K}}{\mathrm{V}}=\frac{90}{48}$

$$
=\frac{15}{8} \mathrm{~atm}
$$

From (D)

$$
\mathrm{P}=1, \mathrm{~K}=90
$$

$$
\mathrm{V}=\frac{\mathrm{K}}{\mathrm{P}}=\frac{90}{1}=90 \mathrm{~cm}^{3}
$$

$$
\begin{aligned}
& \text { From (E) } \\
& \mathrm{V}=100, \mathrm{~K}=90 \\
& \therefore \quad \mathrm{P}=\frac{\mathrm{K}}{\mathrm{~V}}=\frac{90}{100} \\
& =\frac{9}{10} \mathrm{~atm} \\
& \text { From (F) } \\
& \mathrm{P}=\frac{1}{2}, \mathrm{~K}=90 \\
& \therefore \quad \mathrm{~V}=\frac{\mathrm{K}}{\mathrm{P}}=\frac{90}{\frac{1}{2}} \\
& =180 \mathrm{~cm}^{3}
\end{aligned}
$$

From (G)

$$
\mathrm{V}=200, \mathrm{~K}=90
$$

$$
\begin{aligned}
\therefore \quad \mathrm{P} & =\frac{\mathrm{K}}{\mathrm{~V}}=\frac{90}{200} \\
& =\frac{9}{20} \mathrm{~atm}
\end{aligned}
$$

Hence, the completed table is

| (A) |  |  |  |  |  |  |  | (B) | (C) |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Volume (in cm ${ }^{3}$ ) | 45 | 48 | 60 | 90 | 100 | 180 | 200 |  |  |
| Pressure (in atmosphere) | 2 | $\frac{15}{8}$ | $\frac{3}{2}$ | 1 | $\frac{9}{10}$ | $\frac{1}{2}$ | $\frac{9}{20}$ |  |  |

6. Let the length of the cloth be $x \mathrm{~m}$ and its cost be ₹ y . We have the following table.

| $\boldsymbol{x}$ | 2 | 4 | 5 | 10 | 13 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | $y_{1}$ | $y_{2}$ | 210 | $y_{3}$ | $y_{4}$ |

Since $x$ and $y$ are directly proportional.

$$
\begin{aligned}
\therefore & \frac{2}{y_{1}} & =\frac{5}{210} \\
\Rightarrow & 5 \times y_{1} & =2 \times 210 \\
\Rightarrow & y_{1} & =\frac{2 \times 210}{5} \\
\Rightarrow & y_{1} & =₹ 84
\end{aligned}
$$

$$
\begin{array}{lrl}
\text { Similarly, } \frac{4}{y_{2}} & =\frac{5}{210} \\
\Rightarrow & 5 \times y_{2} & =4 \times 210 \\
\Rightarrow & y_{2} & =\frac{4 \times 210}{5} \\
\Rightarrow & y_{2} & =₹ 168 \\
& \frac{10}{y_{3}} & =\frac{5}{210} \\
\Rightarrow & 5 \times y_{3} & =10 \times 210 \\
\Rightarrow & y_{3} & =\frac{10 \times 210}{5}=₹ 420 \\
\Rightarrow & \frac{13}{y_{4}} & =\frac{5}{210} \\
\Rightarrow & 5 \times y_{4} & =13 \times 210 \\
\Rightarrow & y_{4} & =\frac{13 \times 210}{5}=₹ 546
\end{array}
$$

7. Let the required time be t minutes.

| Number of pumps | 6 | 4 |
| :--- | :---: | :---: |
| Time (in minutes) | 28 | $x$ |

Less pump $\rightarrow$ More time
Since there is an inverse variation.
$x_{1} y_{1}=x_{2} y_{2}$
$6 \times 28=4 \times x$
$x=42$
Hence, the required time $=42$ minutes.

## Long Answer:

1. Let the required amount of interest be ₹ $x$.

| Deposit (in ₹) | 12000 | 15000 |
| :--- | :---: | :---: |
| Interest (in ₹) | 900 | $x$ |

Since there is a direct variations.

$$
\begin{aligned}
\therefore & \frac{x_{1}}{y_{1}} & =\frac{x_{2}}{y_{2}} \\
\Rightarrow & \frac{12000}{900} & =\frac{15000}{x} \\
\Rightarrow & 12000 \times x & =900 \times 15000 \\
\Rightarrow & x & =\frac{900 \times 15000}{12000} \\
\Rightarrow & x & =1125
\end{aligned}
$$

Hence, the required amount of interest = ₹ 1125 .
2. Let the number of days be $x$.

| Number of men | 120 | 125 |
| :--- | :---: | :---: |
| Number of days | 25 | $x$ |

$[\because$ Remaining days $=30-5=25$ ]
[Total men = $120+5=125$ ]
Since there is an inverse variation.
$\mathrm{x}_{1} \mathrm{y}_{1}=\mathrm{x}_{2} \mathrm{y}_{2}$
$120 \times 25=125 \times x$
$x=24$
Hence, the required number of days be 24 .
3. More the persons, the sooner would be the provision exhausted. So, this is a case of inverse proportion.
Let the required number of days be $x$.
Hence, $300 \times 42=(300+50) \times x$
$300 \times 42=350 \times x$
$x=36$
4. As the number of boxes increases, the space required to keep them also increases.

So, this is a case of direct proportion.

| Number of boxes | 2 | 200 |
| :--- | :---: | :---: |
| Space occupied <br> (in cubic centimetres) | 500 | $x$ |


Thus, the required space is 50,000 cubic centimetres.
5. Given that, at constant temperature pressure and volume of a gas are inversely proportional.
Let the required pressure be $x$.

| Volume of gas <br> (in cubic centimetres) | 630 | 720 |
| :--- | :---: | :---: |
| Pressure of gas <br> (in mm) | 360 | $x$ |

Then, $\quad 630 \times 360=720 \times x$

$$
\begin{array}{rlrl} 
& & \frac{630 \times 360}{720} & =x \\
\therefore & x & =315
\end{array}
$$

Therefore, the required pressure is 315 mm of mercury.

