

## Gravitation

## Gravitation

When we talk about gravitation or gravity it is a naturally occurring phenomenon or a force which exists among all material objects in the universe.
Whenever we throw an object towards the sky it will fall back onto the ground.
For Example: - A ball comes down when thrown up. Rain drops fall towards the ground; Planets revolve in an elliptical orbit around sun etc.


Planets revolving in the elliptical orbit.


Rain drops falling on the earth.


Leaves fall off the tree.

There is a force due to which all things are attracted towards the earth. This force is known as Gravitation.

Gravitation is the force of attraction between all masses in the universe, especially the force of attraction exerted by the earth on all the bodies near its surface.

In this chapter we will take a look at gravitation force, its laws, and we will also study about the planetary motion.

## Gravitational Constant and Universal Law of Gravitation

We know that the universal law of gravitation was put forth by Sir Isaac Newton in 1687. It is one of the most important laws of physics. Let us know more about Newton's universal law of gravitation and gravitational constant.

## Universal Law of Gravitation

According to Newton's Universal Law of Gravitation, the force exerted between two objects by each other is given by the following relation.

$$
F_{g} \propto \frac{m_{1} \cdot m_{2}}{r^{2}}
$$

where $g$ is the gravitational force between two bodies, $m_{1}$ is the mass of one object, $m_{2}$ is the mass of the second object and $r$ is the distance between the centres of two objects.

## Gravitational Constant

The actual force exerted between two bodies can be given by the following equation
$F_{g}=G \frac{m_{1} \cdot m_{2}}{r^{2}}$
where $G$ is the universal gravitational constant with a value $\left(G=6.674 \times 10-11 \mathrm{~N} \cdot(\mathrm{~m} / \mathrm{kg})^{\wedge}\{2\}\right)$. $G$ here is an empirical constant of proportionality.

What is interesting here is that, even though it is Newton's Universal Law of Gravitation, the value of G wasn't given by him. This was calculated by Henry Cavendish in 1798 through a series of experiments and observations. The influence of the earth's core on the experiments is hypothesised to alter its rotational inertia, because of which the value of Given is not always constant throughout the globe.

Another theory regarding the universal gravitational constant (fun fact: it is also referred to as Big G ) is that, if it is true that the universe is expanding since the Big Bang, then the value of G will keep decreasing!
The universal gravitational constant is used in Newton's Universal Law of Gravitation, Einstein's General Theory of Relativity and also Kepler's Third Law of Planetary Motion to calculate the time period of a planet to complete one full revolution in its orbit.

## Acceleration due to gravity of the earth

Acceleration attained due to gravity of earth.
All the objects fall towards the earth because of gravitational pull of the earth.
And when a body is falling freely, it will have some velocity and therefore it will attain some acceleration. This acceleration is known as acceleration due to gravity.

It is a vector quantity.
Denoted by ' g '.
Its value is $9.8 \mathrm{~m} / \mathrm{s}^{2}$.
Example: Stones falling from a rock will have some velocity because of which some acceleration. This acceleration is due to the force exerted by the earth on the rocks.This is known as acceleration due to gravity.


Stones falling from rock

## Expression for Acceleration due to gravity

Consider any object of mass ' $m$ ' at a point $A$ on the surface of the earth.
The force of gravity between the body and earth can be calculated as

$$
\mathrm{F}=\mathrm{G} m \mathrm{M}_{\mathrm{e}} / \mathrm{R}_{\mathrm{e}}^{2}(1) \text { where }
$$

- $m=m a s s$ of the body
- $M_{e}=$ mass of the Earth
- $\mathrm{R}_{\mathrm{e}}=$ distance between the body and the earthis same as the radius of the earth

Newton's Second law states that
F=ma (2)
Comparing the equations (1) and (2)
$\mathrm{F}=\mathrm{m}\left(\mathrm{GmM} / \mathrm{R}_{\mathrm{e}}{ }^{2}\right)$
( $G m M_{e} / R_{e}{ }^{2}$ ) is same as $g$ (acceleration due to gravity)
Therefore, the expression for Acceleration due to gravity.

$$
\mathrm{g}=\mathrm{G} \mathrm{M}_{\mathrm{e}} / \mathrm{R}_{\mathrm{e}}{ }^{2}
$$



## Acceleration due to gravity below the surface of earth

To calculate acceleration due to gravity below the surface of the earth (between the surface and centre of the earth).
Density of the earth is constant throughout. Therefore,

$$
\rho=M_{e} /\left(4 / 3 \pi R_{e}{ }^{3}\right) \text { equation(1) }
$$

where

- $M_{e}=$ mass of the earth
- Volume of sphere $=4 / 3 \pi R^{3}{ }^{3}$
- $\mathrm{R}_{\mathrm{e}}$-radius of the earth.

As entire mass is concentrated at the centre of the earth.
Therefore, density can be written as

$$
\rho=M_{s} /\left(4 / 3 \pi R_{s}{ }^{3}\right) \text { equation (2) }
$$

Comparing equation (1) and (2)
$M_{e} / M_{s}=R_{e}{ }^{3} / R_{s}{ }^{3}$ where $R_{s}=\left(R_{e}-d\right)^{3}$
$\mathrm{d}=$ distance of the body form the centre to the surface of the earth.
Therefore,
$M_{e} / M_{s}=\operatorname{Re}^{3} /\left(R_{e}-d\right)^{3}$
$M_{s}=M_{e}\left(R_{e}-d\right)^{3} / R_{e}{ }^{3}$ from equation (3)
To calculate Gravitational force (F) between earth and point mass $m$ at a depth $d$ below the surface of the earth.

## depth $d$ below the earth surface



Above figure shows the value of $g$ at a depth $d$. In this case only the smaller sphere of radius ( $\mathrm{Re}_{\mathrm{e}}-\mathrm{d}$ ) contributes to g .
$\mathrm{F}=\mathrm{Gm} \mathrm{Ms}_{s} /\left(\mathrm{Re}_{\mathrm{e}}-\mathrm{d}\right)^{2}$
$\mathrm{g}=\mathrm{F} / \mathrm{m}$ where $\mathrm{g}=$ acceleration due to gravity at point d below the surface of the earth.
$\mathrm{g}=\mathrm{GM} /\left(\mathrm{R}_{\mathrm{e}}-\mathrm{d}\right)^{2}$
Putting the value of $M_{s}$ from equation (3)
$=G M_{e}\left(R_{e}-d\right)^{3} / R_{e}{ }^{3}\left(R_{e}-d\right)^{2}$
$=G M_{e}\left(R_{e}-\mathrm{d}\right) / R_{e}{ }^{3}$
We know $\mathrm{g}=\mathrm{GM}_{\mathrm{e}} / \mathrm{R}_{\mathrm{e}}{ }^{2}$ equation (4)
$\mathrm{g}(\mathrm{d})=\mathrm{GM}_{\mathrm{e}} / \mathrm{Re}_{\mathrm{e}}{ }^{2}\left(1-\mathrm{d} / \mathrm{Re}_{\mathrm{e}}\right)$
From equation (4)
$g(d)=g\left(1-d / R_{e}\right)$

## Acceleration due to gravity above the surface of earth

To calculate the value of acceleration due to gravity of a point mass $m$ at a height $h$ above the surface of the earth.


Above figure shows the value of acceleration due to gravity $g$ at a height $h$ above the surface of the earth.

Force of gravitation between the object and the earth will be
$\mathrm{F}=\mathrm{GmM}_{\mathrm{e}} /\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)^{2}$ where
$\mathrm{m}=$ mass of the object, $\mathrm{R}_{\mathrm{e}}=$ radius of the earth
$\left.\mathrm{g}(\mathrm{h})=\mathrm{F} / \mathrm{m}=\mathrm{GM} / /\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)^{2}=\mathrm{GM} / \mathrm{e}_{\mathrm{e}}{ }^{2}\left(1+\mathrm{h} / \mathrm{R}_{\mathrm{e}}\right)^{2}\right]$
$h \ll R_{e}$ (as radius of the earth is very large)
By calculating we will get,

$$
g(h)=g\left(1-2 h / R_{e}\right)
$$

The value of acceleration due to gravity varies on the surface, above the surface and below the surface of the earth.

## Inertial and Gravitational Mass

Inertial mass is a mass parameter giving the inertial resistance to acceleration of the body when responding to all types of force. Gravitational mass is determined by the strength of the gravitational force experienced by the body when in the gravitational field g.
Inertial Mass: - Inertial mass is defined as the mass of body by virtue of inertia of mass.
By Newton's Law F=ma
$m=\frac{F}{a}$ where $m=$ inertial mass (as it is because of inertia of a body)
Gravitational Mass: -Gravitational mass is defined as the mass of the body by virtue of the gravitational force exerted by the earth.
By Gravitation Force of attraction
$F=\frac{G m M}{r^{2}}$
$\mathrm{M}=\frac{F r^{2}}{G M}$ where
$M=$ mass of the object
$F=$ force of attraction exerted by the earth
$R=$ distance between object and earth
$\mathrm{M}=$ mass of the earth
Experimentally, Inertial mass = Gravitational mass

## Gravitational Potential Energy

- Potential energy is due to the virtue of position of the object.
- Gravitational Potential Energy is due to the potential energy of a body arising out of the force of gravity.
- Consider a particle which is at a point $P$ above the surface of earth and when it falls on the surface of earth at position $Q$, the particle is changing its position because of force of gravity.
- The change in potential energy from position $P$ to $Q$ is same as the work done by the gravity.
- It depends on the height above the ground and mass of the body.


Stationary roller-coaster
Expression for Gravitational Potential Energy
Case1:- ' g ' is constant.
Consider an object of mass ' $m$ ' at point $A$ on the surface of earth.
Work done will be given as:
$\mathrm{W}_{\mathrm{BA}}=\mathrm{FX}$ displacement where F = gravitational force exerted towards the earth)
$=m g\left(h_{2}-h_{1}\right)$ (body is brought from position $A$ to $\left.B\right)$
$=\mathrm{mgh}_{2}-\mathrm{mgh}_{1}$
$W_{A B}=V_{A}-V_{B}$
where
$V_{A}=$ potential energy at point $A$
$V_{B}=$ potential energy at point $B$
From above equation we can say that the work done in moving the particle is just the difference of potential energy between its final and initial positions.
Case2:-'g' is not constant.
Calculate Work done in lifting a particle from $r=r_{1}$ to $r=r_{2}\left(r_{2}>r_{1}\right)$ along a vertical path,
We will get, $W=V\left(r_{2}\right)-V\left(r_{1}\right)$
In general the gravitational potential energy at a distance ' $r$ ' is given by :
where
$\mathrm{V}(\mathrm{r})=$ potential energy at distance ' r '
$\mathrm{V}_{\mathrm{o}}=A t$ this point gravitational potential energy is zero.
Gravitational potential energy is $\alpha$ to the mass of the particle.

## Gravitational Potential

Gravitational Potential is defined as the potential energy of a particle of unit mass at that point due to the gravitational force exerted byearth.
Gravitational potential energy of a unit mass is known as gravitational potential.
Mathematically:
$\mathrm{G}_{\text {potential }}=-\frac{G M}{R}$

## Planetary Motion

Ptolemy was the first scientist who studied the planetary motion.
He gave geocentric model. It means all the planets, stars and sun revolve around the earth and earth is at the centre.
Heliocentric model was proposed by some Indian astronomers.
According to which all planets revolve around the sun.
Nicholas proposed the Nicholas Copernicus model according to which all planets move in circles around the sun.
After Nicholas one more scientist named Tycho Brahe did lot of observations on planets.
Finally came Johannes Kepler who used Tycho Brahe observations, and he gave Kepler's 3 laws of Gravitation.
These 3 laws became the basis of Newton's Universal law of Gravitation.


## Kepler's 1st Law: Law of Orbits

Statement: - The orbit of every planet is an ellipse around the sun with sun at one of the two foci of ellipse.


Whenever a planet revolves around sun it traces an ellipse around the sun. The closest point is $P$ and the farthest point is $A, P$ is called the perihelion and $A$ the aphelion. The semi major axis is half the distance AP.

## Kepler's 1st law Vs. Copernicus Model

According to Copernicus planets move in circular motion whereas according to Kepler planets revolve in elliptical orbit around the sun.
Copernicus model is based on one special case because circle is a special case of ellipse whereas Kepler's laws aremore of ageneral form.
Kepler's law also tells us about the orbits which planets follow.
To Show ellipse is a special form of Circle
Select two points $F_{1}$ and $F_{2}$.
Take a piece of string and fix its ends at $F_{1}$ and $F_{2}$.
Stretch the string taut with the help of a pencil and then draw a curve by moving the pencil keeping the string taut throughout. Fig. (a).
The resulting closed curve is an ellipse. For any point $T$ on the ellipse, the sum of distances from $F_{1}$ and $F_{2}$ is a constant. $F_{1}, F_{2}$ are called the foci.
Join the points $F_{1}$ and $F_{2}$ and extend the line to intersect the ellipse at points $P$ and $A$ as shown in Fig. (a).
The centre point of the line PA is the centre of the ellipse $O$ and the length $\mathrm{PO}=\mathrm{AO}$, which is also known as the semimajor axis of the ellipse.
For a circle, the two foci merge onto one and the semi-major axis becomes the radius of the circle.


Fig(a)
A string has its ends fixed at $F_{1}$ and $F_{2}$. The tip of the pencil holds the string taut and is moved around and we will get an ellipse.

## Kepler's 2nd law: Law of Areas

Statement: The line that joins a planet to the sun sweeps out equal areas in equal intervals of time.

Area covered by the planet while revolving around the sun will be equal in equal intervals of time. This means the rate of change of area with time is constant.
Suppose position and momentum of planet is denoted by ' $r$ ' and ' $p$ ' and the time taken will be $\Delta \mathrm{t}$.
$\Delta A=\frac{1}{2} x r x v \Delta t$ (where $v \Delta \mathrm{t}$ is distance travelled by a planet in $\Delta \mathrm{t}$ time.)
$\frac{\Delta A}{\Delta t}=\frac{1}{2}(r x v)$
where
(Linear momentum) $p=m v$ or we can write as
$v=\frac{p}{m}$
$=\frac{1}{2} m(r \times p)$
$=\frac{1}{2} \frac{L}{2} m$ where $\mathrm{L}=$ angular momentum (It is constant for any central force)
$\frac{\Delta A}{\Delta t}=$ constant (This means equal areas are covered in equal intervals of time).


## Kepler's 3rdLaw: Law of periods

## Statement:

According to this law the square of time period of a planet is $\alpha$ to the cube of the semi-major axisof its orbit.
Suppose earth is revolving around the sun then the square of the time period (time taken to
complete one revolution around sun) is $\propto$ to the cube of the semi major axis.
It is known as Law of Periods as it is dependent on the time period of planets.
Derivation of 3rd Law: assumption: The path of the planet is circular.
Let $\mathrm{m}=$ mass of planet
$\mathrm{M}=$ mass of sun
According to Newton's Law of Gravitation:
$F=\frac{G M m}{r^{2}}$
$F_{c}=\frac{m v^{2}}{r}$
Where,
$\mathrm{F}_{\mathrm{c}}=$ =centripetal force which helps the planet to move around sun in elliptical order.
$\mathrm{F}=\mathrm{F}_{\mathrm{c}}$
$\frac{G M m}{r^{2}}=\frac{m v^{2}}{r}$ where $\mathrm{r}=$ radius of the circle
$\frac{G M}{r}=v^{2}(1)$
$v=2 \frac{\pi r}{T}$
Squaring both the sides the above equation
$v^{2}=4 \pi^{2} r^{2} / T^{2}$
putting the value (1)
$\frac{G M}{r}=4 \pi r^{2} / T^{2}$
$T^{2}=\left(4 \pi^{2} r^{3} / G M\right)$ where $\left(4 \pi^{2} / G M\right)=$ constant
$\mathrm{T}^{2}=\mathrm{r}^{3}$ (In ellipse semi-major axis is same as radius of the circle)

## Escape Velocity

Escape velocity is the minimum velocity that a body must attain to escape the gravitational field of the earth.
Suppose if we throw a ball, it will fall back. This is happening due to the force of gravitation exerted on the ball by the surface of the earth due to which the ball is attracted towards the surface of the earth.
If we increase the velocity to such an extent that the object which is thrown up will never fall back. This velocity is known as escape velocity.


Ball is thrown up but it falls down because of force of gravitation.


The same ball is thrown with a velocity that itescapes the force of gravitation of earth and does not come back. This velocity is known as escape velocity.

Mathematically:
Suppose we throw a ball and the initial velocity of the ball is equal to the escape velocity such that ball never comes back.

Final Position will be infinity.
At Final Position: At Infinity
Total Energy $(\infty)=$ kinetic Energy $(\infty)+$ PotentialEnergy $(\infty)$
KineticEnergy $(\infty)=1 / 2 \mathrm{mv}_{\mathrm{f}}{ }^{2}$ where $\mathrm{v}_{\mathrm{f}}$ =final velocity
Potential Energy $(\infty)=-G M m / r+V_{0}$
where $M=$ mass of the earth, $m=$ mass of the ball,
$\mathrm{V}_{0}=$ potential energy at surface of earth, $r=\infty r=$ distance from the centre of the earth.
Therefore: - Potential Energy $(\infty)=0$
Total Energy $(\infty)=1 / 2 \mathrm{mv}_{\mathrm{f}}{ }^{2}(1)$
At initial position:-
E. $=1 / 2 \mathrm{mv}_{\mathrm{i}}{ }^{2}$
$E=-G M m /\left(R_{e}+h\right)+V_{0}$
Where $\mathrm{h}=$ height of the ball from the surface of the earth.

Total Energy (initial) $=1 / 2 \mathrm{mv}_{\mathrm{i}}{ }^{2}-\mathrm{GMm} /\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)(2)$
According to law of conservation of energy
Total Energy $(\infty)=$ Total Energy (initial)
$1 / 2 m v_{f}^{2}=1 / 2 m v_{i}^{2}-G M m /\left(R_{e}+h\right)$
As L.H.S = positive
$1 / 2 m v_{\mathrm{i}}{ }^{2}-\mathrm{GMm} /\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right) \geq 0$
$1 / 2 m v_{\mathrm{i}}{ }^{2}=G M m /\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)$
By calculating
$v_{\mathrm{i}}^{2}=2 \mathrm{GM} /\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)$
Assume Ball is thrown from earth surface $h \ll R_{e}$
This implies $R_{e}+h$ is same as $R e$ as we can neglect $h$.
Therefore, $\mathrm{v}_{\mathrm{i}}{ }^{2}=2 \mathrm{GM} /\left(\mathrm{R}_{\mathrm{e}}\right)$
Or $\mathrm{v}_{\mathrm{i}}=\sqrt{ }\left(2 G M / \mathrm{R}_{\mathrm{e}}\right)$
This is the initial velocity with which if the ball is thrown it will never fall back on the earth surface.
In terms of ' $g$ '
$\mathrm{g}=\mathrm{GM} / \mathrm{Re}^{2}$
Escape velocity can be written as

$$
V_{e}=\sqrt{ } 2 g R_{e}
$$

## Earth Satellites

Any object revolving around the earth.


## Natural Satellite

Satellite created by nature.
Example: - Moon is the only natural satellite of earth.


## Artificial Satellites:

Human built objects orbiting the earth for practical uses. There are several purposes which these satellites serve.

Example:- Practical Uses of Artificial satellites
Communication
Television broadcasts
Weather observation
Military support
Navigation
Scientific research


## Determining the Time Period of Earth Satellite

Time taken by the satellite to complete one rotation around the earth.
As satellites move in circular orbits there will be centripetal force acting on it.
$F_{c}=\frac{m v^{2}}{R_{e}+h}$ It is towards the centre.
Where,
$\mathrm{H}=$ distance of satellite forms the earth
$\mathrm{F}_{\mathrm{c}}=$ centripetal force
$F_{G}=\frac{G m M_{e}}{\left(R_{e}+h\right)^{2}}$
Where,
$\mathrm{F}_{\mathrm{g}}=$ Gravitation force
$M=$ mass of the satellite
$\mathrm{M}_{\mathrm{e}}=$ mass of the earth
$\mathrm{F}_{\mathrm{c}}=\mathrm{F}_{\mathrm{G}}$
$m v^{2} / R_{e}+h=G m M_{e} /\left(R_{e}+h\right)^{2}$
$v^{2}=G M_{e} / R_{e}+h$
$v=\sqrt{ } G_{e} / R_{e}+h(1)$
This is the velocity with which satellite revolve around the earth.
The satellite covers distance $=2 \pi\left(R_{e}+h\right)$ with velocity v .
$\mathrm{T}=2 \pi\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right) / v$
$2 \pi\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right) / \sqrt{ } \mathrm{GM}_{\mathrm{e}} / \mathrm{R}_{\mathrm{e}}+\mathrm{h}$ From (1)
$\left.\mathrm{T}=2 \boldsymbol{\pi}\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)^{3 / 2} / \sqrt{ } \mathrm{GM}_{\mathrm{e}}\right)$
Special Case:-
$h \ll R_{e}$ (satellite is very near to the surface of the earth)
Then $T=2 \pi \sqrt{ } R_{e}{ }^{3} / G M_{e}$
After calculating

$$
\mathrm{T}=2 \pi \sqrt{ } \mathrm{R}_{\mathrm{e}} / \mathrm{g}
$$

Energy of an orbiting satellite
$m=$ mass of the satellite, $v=$ velocity of the satellite
E. $=\frac{1}{2} m v^{2}$
$=1 / 2 \mathrm{~m}\left(\mathrm{GM}_{\mathrm{e}} / \mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)$ by using (1)
E. $=1 / 2 \mathrm{GM}_{\mathrm{e}} /\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)$
$E .=-G M_{e} m /\left(R_{e}+h\right)$
Total Energy = K.E. + P.E.
$=1 / 2 G M_{e} /\left(R_{e}+h\right)+-G M_{e} m /\left(R_{e}+h\right)$
$E .=G M_{e} m / 2\left(R_{e}+h\right)$
P.E. $=2 \times$ K.E.

Total energy is negative. This means the satellite cannot escape from the earth's gravity.
Geostationary Satellite:


Geo means earth and stationary means at rest. This means something which is stationary. Satellites orbiting around the Earth in equatorial plane with time period equal to 24 hours.
Appear to be stationary with respect to earth. They also rotate around earth with time period of 24 hours.
These satellites can receive telecommunication signals and broadcast them back to a wide area on earth.
Example: INSAT group of satellites.

## Polar Satellites

These are low altitude satellites. This means they orbit around earth at lower heights. They orbit around the earth in North-South direction. Whereas earth is moving from East to West.
A camera is fixed above this type of satellite so they can view small strips of earth.
As earth also moves, so at each instance different types of stripes of earth can be viewed.
Adjacent stripes of earth are viewed in subsequent orbits.
They are useful in remote sensing, meteorology and environmental studies of the earth.

In the above image we can see that the orbit of polar satellites is from north to south direction.

## Weightlessness

Weightlessness is a condition of free fall, in which the effect of gravity is cancelled by the inertial (e.g., centrifugal) force resulting from orbital flight. There is no force of gravity acting on the objects.
It is the condition in which body does not feel its weight at all.
When an apple falls from a tree it won't feel its weight. This condition experienced by anybody while in free-fall is known as weightlessness.


Examples: -When we throw an object from the top of building, the object experiences free fall, that is the object is not under any force. This is weightlessness.

## Weightlessness in the orbital motion of satellites

- In case of a satellite that is rotating around the earth.
- There is an acceleration which is acting towards the centre of the Earth.
- This acceleration is known as centripetal acceleration (ac).
- There is also earth's acceleration which is balancing this centripetal acceleration. $g=a_{c}$ they are equal in magnitude and they are balancing each other.
- Inside the satellites there is no acceleration which means everything is moving with uniform velocity.
- Inside an orbiting satellite weightlessness is experienced.


## Top Formulae

| Newton's law of gravitation | $\begin{aligned} & \mathrm{F}=\frac{\mathrm{Gm}_{1} \mathrm{~m}_{2}}{\mathrm{r}^{2}}, \\ & \mathrm{G}=6.67 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2} \end{aligned}$ |
| :---: | :---: |
| Acceleration due to gravity | $\mathrm{g}=\frac{\mathrm{GM}}{\mathrm{R}^{2}}=\frac{4}{3} \pi \mathrm{GR} \rho$ |
| Variation of $\mathbf{g}$ | (a) Altitude (height) effect $g^{\prime}=g\left(1+\frac{h}{R}\right)^{-2}$ <br> If $h \ll R$, then $g^{\prime}=g^{( }\left(1-\frac{2 h}{R}\right)$ <br> (b) Effect of depth $\mathrm{g}^{\prime \prime}=\left(1-\frac{\mathrm{d}}{\mathrm{R}}\right)$ |
| Intensity of gravitational field | $\stackrel{\rightharpoonup}{E}_{g}=\frac{G M}{r^{2}}(-\stackrel{\rightharpoonup}{r})$ <br> For the Earth, $\mathrm{E}_{\mathrm{g}}=\mathrm{g}=9.86 \mathrm{~m} / \mathrm{s}^{2}$ |
| Gravitational potential | $v_{g}=-\int_{\infty}^{r} \hat{E} \cdot \overrightarrow{d r}$ <br> For points outside ( $r>R$ ), $v_{g}=-\frac{G M}{r}$ <br> For points inside it, $r<R$ $v_{g}=-G M\left\lceil\frac{3 R^{2}-r^{2}}{2 R^{3}}\right\rfloor$ |
| Change in potential energy in going to a height $h$ above the surface | $\begin{aligned} & \Delta \mathrm{U}_{\mathrm{g}}=\mathrm{mgh} \quad \text { if } \mathrm{h} \ll \mathrm{R}_{\mathrm{e}} \\ & \text { In general, } \Delta \mathrm{U}_{\mathrm{g}}=\frac{\mathrm{mgh}}{\left(1+\frac{\mathrm{h}}{\mathrm{R}}\right)} \end{aligned}$ |


| Orbital velocity of a satellite | $\begin{aligned} & \frac{m v_{0}^{2}}{r}=\frac{G M m}{r^{2}} \\ & v_{0}=\sqrt{\frac{G M}{R+h}} \because r=h+R \\ & \text { If } h \ll R v_{0}=\sqrt{\frac{G M}{R}}=\sqrt{g R}=8 \mathrm{~km} / \mathrm{sec} . \end{aligned}$ |
| :---: | :---: |
| Velocity of projection | $\begin{aligned} & \text { Loss of KE = Gain in PE } \\ & \frac{1}{2} m v_{p}^{2}=-\frac{g M m}{(R+h)}-\left(-\frac{G M m}{R}\right) \\ & v_{P}=\left[\frac{2 G M h}{R(R+h)}\right]^{1 / 2}=\left[\frac{2 g h}{1+\frac{h}{R}}\right]^{1 / 2} \quad\left(\because G M=g R^{2}\right) \end{aligned}$ |
| Period of revolution | $\begin{aligned} & T=\frac{2 \pi r}{v_{0}}=\frac{2 \pi(R+h)^{3 / 2}}{R \sqrt{ }} \\ & \text { Or } T^{2}=\frac{4 \pi^{2} r^{3}}{G M} \\ & \text { If } h \ll R \quad T=\frac{2 \pi R^{3 / 2}}{R \sqrt{ }}=1 \frac{1}{2} h r \end{aligned}$ |
| Kinetic energy of a satellite | $\mathrm{KE}=\frac{\mathrm{GMm}}{2 \mathrm{r}}=\frac{1}{2} \mathrm{mv}^{2}$ |
| Potential energy of a satellite | $\mathrm{U}=-\frac{\mathrm{GMm}}{\mathrm{r}}$ |
| Binding energy of a satellite | $=\frac{1}{2} \underline{\mathrm{GMm}}$ |
| Escape velocity | $\begin{aligned} & \mathrm{v}_{\mathrm{e}}=\sqrt{\frac{2 \mathrm{GM}}{}}=\sqrt{2 \mathrm{gR}}=\mathrm{R} \sqrt{\frac{8 \pi \mathrm{Gd}}{3}} \\ & \mathrm{v}_{\mathrm{e}}=\mathrm{v}_{0} \sqrt{2} \end{aligned}$ |
| Effective weight in a satellite | $w=0$ <br> Satellite behaves like a freely falling body. |
| Kepler's laws for planetary motion | (a) Elliptical orbit with the Sun at one focus <br> (b) Areal velocity constant $\mathrm{dA} / \mathrm{dt}=$ constant <br> (c) $\mathrm{T}^{2} \propto \mathrm{r}^{3} ; \mathrm{r}=\left(\mathrm{r}_{1}+\mathrm{r}_{2}\right) / 2$ |

Class: 11th Physics
Chapter- 8: Gravitation


## Important Questions

## Multiple Choice questions-

1. A body is projected vertically from the surface of the earth of radius $R$ with velocity equal to half of the escape velocity. The maximum height reached by the body is
(a) $R$
(b) $R / 2$
(c) $R / 3$
(d) $R / 4$
2. When the planet comes nearer the sun moves
(a) fast
(b) slow
(c) constant at every point
(d) none of the above
3. Keplers second law regarding constancy of arial velocity of a planet is a consequence of the law of conservation of
(a) energy
(b) angular momentum
(c) linear momentúm
(d) none of these
4. The escape velocity for a body projected vertically upwards from the surface of the earth is $11 \mathrm{~km} / \mathrm{s}$. If the body is projected at an angle of $45^{\circ}$ with the vertical, the escape velocity will be
(a) $11 / \mathrm{V} 2 \mathrm{~km} / \mathrm{s}$
(b) $11 \mathrm{~V} 2 \mathrm{~km} / \mathrm{s}$
(c) $2 \mathrm{~km} / \mathrm{s}$
(d) $11 \mathrm{~km} / \mathrm{s}$
5. The radii of the earth and the moon are in the ratio $10: 1$ while acceleration due to gravity on the earths surface and moons surface are in the ratio $6: 1$. The ratio of escape velocities from earths surface to that of moon surface is
(a) $10: 1$
(b) $6: 1$
(c) $1.66: 1$
(d) $7.74: 1$
6. The escape velocity of a body from the surface of the earth is v . It is given a velocity twice this velocity on the surface of the earth. What will be its velocity at infinity?
(a) $v$
(b) $2 v$
(c) V 2 v
(d) V3v
7. The period of geostationary artificial satellite is
(a) 24 hours
(b) 6 hours
(c) 12 hours
(d) 48 hours
8. If the radius of the earth were to shrink by $1 \%$ its mass remaining the same, the acceleration due to gravity on the earths surface would
(a) decrease by $2 \%$
(b) remain unchanged
(c) increase by $2 \%$
(d) will increase by $9.8 \%$
9. The mean radius of the earth is $R$, its angular speed on its own axis is $w$ and the acceleration due to gravity at earth's surface is $g$. The cube of the radius of the orbit of a geo-stationary satellite will be
(a) $r^{2} g / w$
(b) $R^{2} w^{2} / g$
(c) $R G w^{2}$
(d) $R^{2} g / w^{2}$
10. If escape velocity from the earth's surface is $11.2 \mathrm{~km} / \mathrm{sec}$. then escape velocity from a planet of mass same as that of earth but radius one fourth as that of earth is
(a) $11.2 \mathrm{~km} / \mathrm{sec}$
(b) $22.4 \mathrm{~km} / \mathrm{sec}$
(c) $5.65 \mathrm{~km} / \mathrm{sec}$
(d) $44.8 \mathrm{~km} / \mathrm{sec}$

## Very Short:

1. What velocity will you give to a donkey and what velocity to a monkey so that both escape the gravitational field of Earth?
2. How does Earth retain most of the atmosphere?
3. Earth is continuously pulling the moon towards its center. Why does not then, the moon falls on the Earth?
4. Which is greater out of the following:
(a) The attraction of Earth for 5 kg of copper.
(b) The attraction of 5 kg copper for Earth?
5. Where does a body weigh more - at the surface of Earth or in a mine?
6. How is it that we learn more about the shape of Earth by studying the motion of an artificial satellite than by studying the motion of the moon?
7. If the Earth is regarded as a hollow sphere, then what is the weight of an object below the surface of Earth?
8. What is the formula for escape velocity in terms of $g$ and $R$ ?
9. What is the orbital period of revolution of an artificial satellite revolving in a geostationary orbit?
10.Can we determine the mass of a satellite by measuring its time period?

## Short Questions:

1. Explain how the weight of the body varies en route from the Earth to the moon. Would its mass change?
2. Among the known type of forces in nature, the gravitational force is the weakest. Why then does it play a dominant role in the motion of bodies on the terrestrial, astronomical, and cosmological scale?
3. Show that the average life span of humans on a planet in terms of its natural years is 25 planet years if the average span of life on Earth is taken to be 70 years.
4. Hydrogen escapes faster from the Earth than oxygen. Why?
5. In a spaceship moving in a gravity-free region, the astronaut will not be able to distinguish between up and down. Explain why?
6. Why the space rockets are generally launched from west to east?
7. Explain why the weight of a body becomes zero at the centre of Earth.
8. We cannot move even our little fingers without disturbing the whole universe. Explain why.

## Long Questions:

1. (a) Derive the expression for the orbital velocity of an artificial Earth's satellite. Also, derive its value for an orbit near Earth's surface.
(b) Derive the expression for escape velocity of a body from the surface of Earth and show that it $\sqrt{2}$ times the orbital velocity close to the surface of the Earth. Derive its value for Earth.
2. (a) Explain Newton's law of gravitation.
(b) Define gravitational field intensity. Derive its expression at a point at a distance x from the center of Earth. How is it related to acceleration due to gravity?
3. Discuss the variation of acceleration due to gravity with:
(a) Altitude or height
(b) Depth
(c) Latitude i.e. due to rotation of Earth.

## Assertion Reason Questions:

1. Directions: Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

Assertion: Gravitational potential of earth at every place on it is negative.
Reason: Everybody on earth is bound by the attraction of earth.
2. Directions: Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

Assertion: Planets appear to move slower when they are farther from the sun than when they are nearer.
Reason: All planets move in elliptical orbits with sun at one of the foci of the ellipse.

## $\checkmark$ Answer Key:

## Multiple Choice Answers-

1. Answer: (c) $R / 3$
2. Answer: (a) fast
3. Answer: (b) angular momentum
4. Answer: (d) $11 \mathrm{~km} / \mathrm{s}$
5. Answer: (d) $7.74: 1$
6. Answer: (d) V3v
7. Answer: (a) 24 hours
8. Answer: (c) increase by $2 \%$
9. Answer: (d) $R^{2} g / w^{2}$
10.Answer: (b) 22.4 km/sec

## Very Short Answers:

1. Answer: We will give them the same velocity as escape velocity is independent of the mass of the body.
2. Answer: Due to force of gravity.
3. Answer: The gravitational force between the Earth and the moon provides the necessary centripetal force to the moon to move around the Earth. This centripetal force avoids the moon to fall onto the Earth.
4. Answer: Same.
5. Answer: At the surface of Earth, a body weighs more.
6. Answer: This is because an artificial satellite is closer to the Earth than Moon.
7. Answer: Zero.
8. Answer: $\mathrm{Ve}=\sqrt{2 g R}$.
9. Answer: It is 24 hours.
10.Answer: Yes.

## Short Questions Answers:

1. Answer: When a body is taken from Earth to the moon, then its weight slowly decreases to zero and then increases till it becomes $\frac{1}{6}$ th of the weight of the body on the surface of the moon.

We know that $\mathrm{mgh}=\mathrm{mg}\left(1-\frac{2 h}{R}\right)$
As h increases, gh, and hence mgh, decreases. When $\mathrm{R}=\frac{R}{2}$ the force of attraction of Earth is equal to the force of attraction of the moon.
Then $\mathrm{gh}=0$, so mg becomes zero, and the value of g on the moon's surface is $\frac{1}{6}$ th of its value on the surface of Earth. Hence on increasing h beyond $\frac{R}{2}$, mg starts increasing due to the gravity of the moon. $f$ ts mass remains constant.
2. Answer: Electrical forces are stronger than gravitational forces for a given distance, but they can be attractive as well as repulsive, unlike gravitational force which is always attractive. As a consequence, the forces between massive neutral bodies are predominantly gravitational and hence play a dominant role at long distances. The strong nuclear forces dominate only over a range of distances of the order of $10^{-14}$ m to $10^{-15} \mathrm{~m}$.
3. Answer: Take the distance between Earth and Sun twice the distance between Earth and planet. According to Kepler's third law of planetary motion,

$$
\left(\frac{T_{e}}{T_{p}}\right)^{2}=\left(\frac{R_{e}}{R_{p}}\right)^{3}
$$

where $T_{e}, T_{R}$ is the average life span on Earth and planet respectively.
$\mathrm{R}_{\mathrm{g}}=$ distance between Earth and Sun.
$R_{p}=$ distance between Earth and planet.
Here, $\mathrm{R}_{\mathrm{e}}=2 \mathrm{R}_{\mathrm{p}}$
or

$$
\begin{aligned}
\frac{\mathrm{R}_{\mathrm{e}}}{\mathrm{R}_{\mathrm{p}}} & =2 \\
\mathrm{~T}_{\mathrm{e}} & =70 \text { years } \\
\mathrm{T}_{\mathrm{p}} & =?
\end{aligned}
$$

$$
\therefore \quad\left(\frac{\mathrm{T}_{e}}{\mathrm{~T}_{\mathrm{p}}}\right)^{2}=(2)^{3}
$$

or

$$
\begin{aligned}
\frac{T_{c}}{T_{p}} & =(2)^{\frac{1}{2}} \\
T_{p} & =\frac{T_{e}}{(2)^{\frac{2}{2}}}=\frac{70}{\sqrt{2^{3}}}=\frac{70}{\sqrt{8}}=\frac{70}{2 \sqrt{2}} \\
& =35 \times \frac{\sqrt{2}}{2}=17.5 \times 1.414 \\
& =24.75 \approx 25 \text { planet years. }
\end{aligned}
$$

4. Answer: The thermal speed of hydrogen is much larger than oxygen. Therefore a large number of hydrogen molecules are able to acquire escape velocity than that of oxygen molecules. Hence hydrogen escapes faster from the Earth than oxygen.
5. Answer: The upward and downward sense is due to the gravitational force of attraction between the body and the earth. In a spaceship, the gravitational force is counterbalanced by the centripetal force needed by the satellite to move around the Earth in a circular orbit. Hence in the absence of zero force, the astronaut will not be able to distinguish between up and down.
6. Answer: Since the Earth revolves from west to east around the Sun, so when the rocket is launched from west to east, the relative velocity of the rocket = launching velocity of rocket + linear velocity of Earth. Thus the velocity of the rocket increases which helps it to rise without much consumption of the fuel. Also, the linear velocity of Earth is maximum in the equatorial plane.
7. Answer: We know that the weight of a body at a place below Earth's surface is given by
W = mgd .... (i)
Where gd = acceleration due to gravity at a place at a depth 'd' below Earth's surface and is given

$$
\begin{equation*}
\mathrm{g}_{\mathrm{d}}=\mathrm{g}\left(1-\frac{\mathrm{d}}{\mathrm{R}}\right) \tag{ii}
\end{equation*}
$$

At the centre of Earth, $d=R$,

$$
\begin{aligned}
\therefore \quad \mathrm{g}_{\mathrm{d}} & =\mathrm{g}\left(1-\frac{\mathrm{R}}{\mathrm{R}}\right) \\
& =\mathrm{g}(1-1)=\mathrm{g} \times 0=0
\end{aligned}
$$

From Eqn. (i) $\mathrm{W}=0$ at the center of Earth.
i.e., $g$ decreased with depth and hence becomes zero at the center of Earth, so W = 0 at Earth's center.
8. Answer: According to Newton's law of gravitation, every particle of this universe attracts every other particle with a force that is inversely proportional to the square of the distance between them. When we move our fingers, the distance between the particle's changes, and hence the force of attraction changes which in turn disturbs the whole universe.

## Long Questions Answers:

1. Answer:
2. Let $m=$ mass of the satellite.
$\mathrm{M}, \mathrm{R}=$ mass and radius of Earth.
$h=$ height of the satellite above the surface of Earth.
$r=$ radius of the robot of the satellite
$=R+h$.
$\mathrm{v}_{0}=$ orbital velocity of the satellite.
The centripetal force $\frac{m v_{0}^{2}}{r}$ required by the satellite to move in a circular orbit is proved by the gravitational force between satellite and the Earth.


$$
\frac{\mathrm{mv}_{0}^{2}}{\mathrm{r}}=\frac{\mathrm{GMm}}{\mathrm{r}^{2}}
$$

or

$$
\begin{align*}
v_{0} & =\sqrt{\frac{G M}{r}} \\
& =\sqrt{\frac{G M}{R+h}} \tag{i}
\end{align*}
$$

Also we know that

$$
\mathrm{g}=\frac{\mathrm{GM}}{\mathrm{R}^{2}}
$$

or $\quad G M=\mathrm{gR}^{2}$
$\therefore \quad v_{0}=R \sqrt{\frac{g}{R+h}}$

$$
\begin{equation*}
=R \sqrt{\frac{g}{(R+h)}} \tag{ii}
\end{equation*}
$$

Also $\quad g_{h}=\frac{G M}{(R+h)^{2}}$
from (i) and (iii), we get

$$
\mathrm{v}_{0}=\sqrt{\mathrm{g}_{\mathrm{h}}(\mathrm{R}+\mathrm{h})}
$$

If the satellite is close to the earth's surface, then $h \approx 0$

$$
\therefore \text { from }(i i), \quad v_{0}=\sqrt{\mathrm{gR}}
$$

Putting,

$$
\mathrm{g}=9.8 \mathrm{~ms}^{-2}, \mathrm{R}=6.38 \times 10^{6} \mathrm{~m}
$$

$$
\mathrm{v}_{0}=\sqrt{9.8 \times 6.38 \times 10^{6}}=7.9 \mathrm{kms}^{-1}
$$

$$
\therefore \quad \mathrm{v}_{0}=\sqrt{\mathrm{gR}}=\mathrm{R} \sqrt{\frac{\mathrm{~g}}{\mathrm{R}+\mathrm{h}}}
$$

$$
=\sqrt{\frac{\mathrm{GM}}{\mathrm{R}+\mathrm{h}}}=\sqrt{\mathrm{g}_{\mathrm{h}}(\mathrm{R}+\mathrm{h})}
$$

2. Escape velocity is the minimum velocity with which a body is projected from Earth's surface so as to just escape its gravitational pull or of any other planet. It is denoted by $\mathrm{V}_{\mathrm{e}}$.
Expression: Consider the earth to be a homogenous sphere of radius $R$, mass $M$, center $O$, and density $p$.

Let $m=$ mass of the body projected from point A on the surface of Earth with vel. V .
$\therefore$ K.E. of the body at point $A=\frac{1}{2} m v_{e}{ }^{2} \ldots$ (i)
Let it reaches a point $P$ at a distance $x$ from $O$. If $F$ be the gravitational force of attraction on the body at $P$, then


$$
\begin{equation*}
\mathrm{F}=\frac{G M m}{x^{2}} . \tag{ii}
\end{equation*}
$$

Let it further moves to Q by a distance dx .
If dW be the work done in moving from $P$ to $Q$, then

$$
\begin{equation*}
\mathrm{dW}=\mathrm{Fdx}=\frac{\mathrm{GMm}}{\mathrm{x}^{2}} \mathrm{dx} \tag{iii}
\end{equation*}
$$

If $w$ be the total work done in moving the body from $A$ to $\infty$,
Then

$$
\begin{align*}
\mathrm{W} & =\int_{A}^{\infty} \mathrm{dW}=\int_{A}^{\infty} \frac{\mathrm{GMm}}{\mathrm{x}^{2}} \mathrm{dx} \\
& =\mathrm{GMm} \int_{A}^{\infty} \mathrm{x}^{-2} \mathrm{dx}=\mathrm{GMm}\left[-\frac{1}{\mathrm{x}}\right]_{\mathrm{R}}^{\infty} \\
& =-\mathrm{GMm}\left[\frac{1}{\infty}-\frac{1}{\mathrm{R}}\right] \\
& =-\mathrm{GMm}\left(0-\frac{1}{\mathrm{R}}\right) \\
& =\frac{\mathrm{GMm}}{\mathrm{R}} \tag{iv}
\end{align*}
$$

$\therefore$ According to the law of conservation of energy
K.E. $=$ RE
or $\quad \frac{1}{2} \mathrm{mv}_{\mathrm{e}}^{2}=\frac{\mathrm{GMm}}{\mathrm{R}}$
or

$$
v_{e}=\sqrt{\frac{2 G M}{R}}
$$

Also
$\mathrm{g}=\frac{\mathrm{GM}}{\mathrm{R}^{2}}$
$\therefore \quad$
or

$$
v_{e}=\sqrt{\frac{2 g R^{2}}{R}}
$$

$$
\begin{equation*}
v_{e}=\sqrt{2 g R} \tag{vi}
\end{equation*}
$$

Also

$$
\mathrm{M}=\frac{4}{3} \pi \mathrm{R}^{3} \rho
$$

$$
\therefore \quad v_{e}=\sqrt{\frac{2 \mathrm{G}}{\mathrm{R}} \cdot \frac{4}{3} \pi \mathrm{R}^{3} \rho}=\sqrt{\frac{8}{3} \pi \mathrm{G} \rho}
$$

for earth

$$
\begin{aligned}
\mathrm{R} & =6.38 \times 10^{6} \mathrm{~m} \\
\mathrm{~g} & =9.8 \mathrm{~ms}^{-2}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \text { from }(v i), \quad \mathrm{v}_{\mathrm{e}} & =\sqrt{2 \times 9.8 \times 6.38 \times 10^{6}} \\
& =11.2 \mathrm{kms}^{-1}
\end{aligned}
$$

Relation between $\mathrm{v}_{\mathrm{e}}$ and $\mathrm{v}_{\mathrm{o}}$ : Also we know that the orbital velocity around Earth close to its surface is given
by $\mathrm{v}_{0}=\sqrt{g R}$
and ve $=\sqrt{2 g R}=\sqrt{2} \sqrt{g R}$
$=\sqrt{2} \mathrm{v}_{0}$
Hence proved.
2. Answer:
(a) We know that Newton's law of gravitation is expressed mathematically as:

$$
\mathrm{F}=\frac{\mathrm{Gm}_{1} \mathrm{~m}_{2}}{\mathrm{r}^{2}}
$$

or in vector form $\mathbf{F}=\frac{G m_{1} m_{2}}{\mathrm{r}^{3}} \hat{r}$,
where $\hat{r}=$ unit vector along $F$
It was found that law is equally applicable anywhere in the universe between small and big objects like stars and galaxies. The value of G remains the same everywhere. (Some scientists have claimed that as the size of the object under consideration becomes big like a galaxy, the value of G also changes). Hence this law of Newton is also called Newton's universal law of gravitation.

The force of attraction is called the force of gravitation or gravitational force. This force is only attractive and is never repulsive. The force is both ways i.e., particle 1 attracts particle 2 and so does particle 2 attracts particle 1.

Hence $F_{12}=-F_{21}$.
The law is a direct outcome of the study of acceleration of bodies. Newton wondered how Moon revolves around the Earth or other planets revolve around the Sun. His calculations showed that the Moon is accelerated by the same amount as does any other object towards the Earth.

His famous narration of the apple falling from the tree and noticing every other object fall towards Earth led to the announcement of his famous law of gravitation about 50 years later in his book 'Principia'.

Out of the known forces in nature, the Gravitational force is the weakest, yet it is the most apparent one as it acts for long distances and between objects which are visible to us. The law of gravitation has been used to determine the mass of heavenly bodies. It has been used to study the atmosphere of planets. Manmade satellites remain in the orbits due to gravitation.
(b) The gravitational field intensity at a point is defined as the force acting on a unit mass placed at that point in the field.

Thus, the gravitational field intensity is given by:
$\mathrm{E}=\frac{F}{m}$
Now at distance x from the centre of Earth, the gravitational force is

$$
\begin{aligned}
\mathbf{F} & =\frac{\mathrm{GMm}}{\mathrm{x}^{2}} \hat{\mathrm{x}} \\
\mathbf{E} & =\frac{\mathbf{F}}{\mathrm{m}}=\frac{\mathrm{GMm}}{\mathrm{x}^{2}} \hat{\mathrm{x}}=\frac{\mathrm{GM}}{\mathrm{x}^{2}} \hat{\mathrm{x}} \\
|\mathbf{E}| & =\frac{\mathrm{GM}}{\mathrm{x}^{2}} \\
\text { or } \quad & \mathbf{E}
\end{aligned}=\frac{\mathrm{GM}}{\mathrm{x}^{2}},
$$

On Earth

$$
\begin{aligned}
& \mathbf{E}=\frac{\mathbf{F}}{\mathrm{m}}=\frac{\text { Force }}{\text { mass }}=\text { acceleration } \\
& \mathbf{E}=\frac{\mathbf{F}}{\mathrm{m}}=\mathbf{g}
\end{aligned}
$$

So, the intensity of the gravitational field at the surface of Earth is equal to the acceleration due to gravity.
3. Answer:

Let $M, R$ be the mass and radius of the earth with centre $O$.
$\mathrm{g}=$ acceleration due to gravity at a point
An on Earth's surface.

(a) Variation of $g$ with height: Let gOh be the acceleration due to gravity at a point B at a height $h$ above the earth's surface
$\frac{\text { (2) }}{(1)}$ gives, $\quad \frac{g_{h}}{g}=\frac{R^{2}}{(R+h)^{2}}=\frac{1}{\left(1+\frac{h}{R}\right)^{2}}$
or

$$
g_{h}=g\left(1+\frac{h}{R}\right)^{-2}
$$

If $h \ll R$, then using Binomial Expansion, we get

$$
\begin{align*}
\mathrm{g}_{\mathrm{h}} & =\mathrm{g}\left(1-\frac{2 \mathrm{~h}}{\mathrm{R}}\right) \\
& =\mathrm{g}-\frac{2 \mathrm{gh}}{\mathrm{R}} \tag{3}
\end{align*}
$$

Thus, from Eqn. (3), we conclude that acceleration due to gravity decreases with height.
(b) With depth: Let the Earth be a uniform sphere.


Let $\mathrm{gd}=$ acceleration due to gravity at a depth d below earth's surface i.e., at point B.

Let $\rho=$ density of Earth of mass $M$.

$$
\begin{equation*}
\therefore \quad \mathrm{g}=\frac{\mathrm{GM}}{\mathrm{R}^{2}} \tag{i}
\end{equation*}
$$

where

$$
M=\frac{4}{3} \pi R^{3} \rho
$$

Also, let $\mathrm{M}^{\prime}=$ mass of Earth at a depth d, then

$$
\begin{align*}
& M^{\prime}=\frac{4}{3} \pi(R-d)^{3} \rho \\
& \therefore \quad \mathrm{~g}=\frac{\mathrm{G}}{\mathrm{R}^{2}} \frac{4}{3} \pi \mathrm{R}^{3} \rho \\
& =\frac{4}{3} \pi \mathrm{G} \rho \mathrm{R}  \tag{ii}\\
& \text { Similarly } \quad \mathrm{g}_{\mathrm{d}}=\frac{\mathrm{GM}^{\prime}}{(\mathrm{R}-\mathrm{d})^{2}} \\
& =\frac{4}{3} \pi \mathrm{G} \rho(\mathrm{R}-\mathrm{d})  \tag{iiii}\\
& \therefore \quad \frac{\mathrm{g}_{\mathrm{d}}}{\mathrm{~g}}=\frac{\mathrm{R}-\mathrm{d}}{\mathrm{R}}=1-\frac{\mathrm{d}}{\mathrm{R}} \\
& \text { or } \\
& g_{d}=g\left(1-\frac{d}{R}\right) \tag{iv}
\end{align*}
$$

From equation (iv), we see that acceleration due to gravity decreases with depth.
Special case: At the centre of Earth, $d=R$
$\therefore \mathrm{g}_{\mathrm{d}}=0$
Hence an object at the centre of Earth is in a state of weightlessness.
(c) Variation of g with latitude:

Let $m=$ mass of a particle at a place $P$ of latitude $X$.
$\omega=$ angular speed of Earth about axis NS.


As the earth rotates about the NS axis, the particle at P also rotates and describes a horizontal circle of radius $r$,
where $r=P C=O P \cos \lambda,=R \cos \lambda$
Let $\mathrm{g}^{\prime}$ be the acceleration due to gravity at P when the rotation of Earth is taken into account. Now due to the rotation of the earth, two forces that act on the particle at P are:
Its weight mg , acting along with PO.
Centrifugal force mroo2 along PO'.
$\therefore$ The angle between them $=180-\lambda$
$\therefore$ According to the parallelogram law of vector addition

$$
\begin{aligned}
\mathrm{mg}^{\prime} & =\sqrt{(\mathrm{mg})^{2}+\left(\mathrm{mr} \omega^{2}\right)^{2}+2(\mathrm{mg})\left(\mathrm{mr} \omega^{2}\right) \cos \left(180^{\circ}-\lambda\right)} \\
& =\sqrt{\mathrm{m}^{2} \mathrm{~g}^{2}+\mathrm{m}^{2} \mathrm{r}^{2} \omega^{4}-2 \mathrm{~m}^{2} \mathrm{gr} \omega^{2} \cos \lambda} \\
\text { or } \quad \mathrm{g}^{\prime} & =\mathrm{g} \sqrt{1+\frac{\mathrm{r}^{2} \omega^{4}}{\mathrm{~g}^{2}}-\frac{2 \mathrm{r} \omega^{2}}{\mathrm{~g}} \cos \lambda} \\
& =\mathrm{g} \sqrt{1+\frac{\omega^{4} \mathrm{R}^{2} \cos ^{2} \lambda}{\mathrm{~g}^{2}}-\frac{2 \mathrm{R} \omega^{2}}{\mathrm{~g}} \cos ^{2} \lambda} \\
& =\mathrm{g} \sqrt{1-\frac{2 R \omega^{2}}{\mathrm{~g}} \cos \lambda}
\end{aligned}
$$

[As $\frac{\mathrm{R} \omega^{2}}{\mathrm{~g}}$ is very small $\left(=\frac{1}{289}\right)$ so its square and higher powers are neglected.]

$$
\therefore g^{\prime}=\left(1-2 \frac{R \omega^{2}}{g} \cos ^{2} \lambda\right)^{\frac{1}{2}}
$$

Using binomial expansion, we get

$$
\begin{aligned}
& g^{\prime}=\left(g-\frac{1}{2} \times \frac{2 R \omega^{2}}{g} \times g \cos ^{2} \lambda\right) \\
& g^{\prime}=g-R \omega^{2} \cos ^{2} \lambda
\end{aligned}
$$

$\Rightarrow \mathrm{g}$ decreases with the rotation of the earth.
At poles, $\lambda=90^{\circ}, \therefore \mathrm{g}^{\prime}=\mathrm{g}_{\mathrm{p}}=\mathrm{g}$
At equator, $\lambda=0, g^{\prime}=g_{e}=g-R \omega^{2}$.

Clearly $\mathrm{g}_{\mathrm{p}}>\mathrm{g}_{\mathrm{e}}$.

## Assertion Reason Answer:

1. (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

## Explanation:

Because gravitational force is always attractive in nature, and everybody is bound by this gravitational force of attraction of earth.
2. (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion.

## Explanation:

Both assertion and reason are true, but reason is not correct explanation of the assertion.

## Case Study Questions-

1. If a stone is thrown by hand, we see it falls back to the earth. Of course using machines we can shoot an object with much greater speeds and with greater and greater initial speed, the object scales higher and higher heights. A natural query that arises in our mind is the following: can we throw an object with such high initial speeds that it does not fall back to the earth ? Thus minimum speed required to throw object to infinity away from earth's gravitational field is called escape velocity.
$\mathrm{v}_{\mathrm{e}}=\sqrt{2 g r}$
Where g is acceleration due to gravity and r is radius of earth and after solving $\mathrm{v}_{\mathrm{e}} 11.2$ $\mathrm{km} / \mathrm{s}$. This is called the escape speed, sometimes loosely called the escape velocity. This applies equally well to an object thrown from the surface of the moon with g replaced by the acceleration due to Moon's gravity on its surface and $r$ replaced by the radius of the moon. Both are smaller than their values on earth and the escape speed for the moon turns out to be $2.3 \mathrm{~km} / \mathrm{s}$, about five times smaller. This is the reason that moon has no atmosphere. Gas molecules if formed on the surface of the moon having velocities larger than this will escape the gravitational pull of the moon. Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them. In particular, their orbits around the earth are circular or elliptic. Moon is the only natural satellite of the earth with a near circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis.

## i. Time period of moon is

a. 27.3 days
b. 20 days
c. 85 days
d. None of these
ii. Escape velocity from earth is given by
a. $20 \mathrm{~km} / \mathrm{s}$
b. $11.2 \mathrm{~km} / \mathrm{s}$
c. $2 \mathrm{~km} / \mathrm{s}$
d. None of these
iii. Define escape velocity. Give its formula
iv. Why moon don't Have any atmosphere?

## v. What is satellite? Which law governs them?

2. Satellites in a circular orbits around the earth in the equatorial plane with $T=24$ hours are called Geostationary Satellites. Clearly, since the earth rotates with the same period, the satellite would appear fixed from any point on earth. It takes very powerful rockets to throw up a satellite to such large heights above the earth but this has been done in view of the several benefits of many practical applications. Thus radio waves broadcast from an antenna can be received at points far away where the direct wave fails to reach on account of the curvature of the earth. Waves used in television broadcast or other forms of communication have much higher frequencies and thus cannot be received beyond the line of sight. A Geostationery satellite, appearing fixed above the broadcasting station can however receive these signals and broadcast them back to a wide area on earth. The INSAT group of satellites sent up by India is one such group of geostationary satellites widely used for telecommunications in India. Another class of satellites is called the Polar satellites. These are low altitude ( 500 to 800 km ) satellites, but they go around the poles of the earth in a north-south direction whereas the earth rotates around its axis in an east-west direction. Since its time period is around 100 minutes it crosses any altitude many times a day. However, since its height $h$ above the earth is about 500-800 km, a camera fixed on it can view only small strips of the earth in one orbit. Adjacent strips are viewed in the next orbit, so that in effect the whole earth can be viewed strip by strip during the entire day. These satellites can view polar and equatorial regions. at close distances with good resolution. Information gathered from such satellites is extremely useful for remote sensing, meterology as well as for environmental studies of the earth.

## i. Time period of geospatial satellite is

a. 24 hours
b. 48 hours
c. 72 hours
d. None of these
ii. Polar satellites are approximately revolving at height of
a. 500 to 800 km
b. 1500 to 2000 km
c. 3000 to 4000 km
d. None of these
iii. Which satellite used to view polar and equatorial regions?
iv. Write note on polar satellites
v. Write a note on geostationary satellite. Give its applications.

## Case Study Answer-

## 1. Answer

i. (a) 27.3 days
ii. (b) 500 to 800 km
iii. Polar satellites are used to view polar and equatorial regions as they rotate on poles of earth.
iv. The escape speed for the moon turns out to be $2.3 \mathrm{~km} / \mathrm{s}$, about five times smaller than that of earth. Therefore all atmospheric gas can go easily out of atmosphere of moon. This is the reason that moon has no atmosphere.
v. Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them.

## 2. Answer

i. (a) 24 hours
ii. (a) Pascal's law
iii. Polar satellites are used to view polar and equatorial regions as they rotate on poles of earth.
iv. Polar satellites are low altitude ( 500 to 800 km ) satellites, but they go around the poles of the earth in a north-south direction. Since its time period is around 100 minutes it crosses any altitude many times a day. Information gathered from such satellites is extremely useful for remote sensing, meterology as well as for environmental studies of the earth.
v. Satellites in circular orbits around the earth in the equatorial plane with time period same as earth are called Geostationary Satellites.
Applications:- Radio waves broadcast. Satellites widely used for telecommunications in India. GPS system, navigation system, defence etc.

